# Analysis of Quasi-ML Multiuser Detection of DS/CDMA Systems in Asynchronous Channels

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***Abstract*— In this paper, we consider the quasi-maximum-likelihood (quasi-ML) detectors of the reverse link system that uses antenna arrays in asynchronous channels when the channel vector is time-invariant and time-varying. A channel vector estimation method based on eigendecomposition for time-invariant channels and its adaptive version suitable for the time-varying channels are also proposed. It is shown that the proposed quasi-ML detector can be regarded as a beamformer followed by a decorrelating filter and that the proposed system performs better than the conventional decorrelator scheme. It is also observed that the performance gain of the proposed scheme over the conventional decorrelator system increases as the numbers of active users and antenna arrays increase.**

***Index Terms*—Antenna array, CDMA system, multiuser detection.**

## I. Introduction

**R**ECENTLY, the direct-sequence code-division multiple-access (DS/CDMA) technique became a much highlighted research area because of its unique features and large capacity for accommodation of users [1], [2]. Although the CDMA technology offers higher spectral efficiency than others [2]–[5] and has many other desirable features, DS/CDMA systems have major drawbacks of interuser interference and the near–far problem. It is also anticipated that even their large capacity will not be sufficient enough to satisfy the explosive demands for mobile communication in the near future. Thus, some additional method to increase the spectral efficiency should be considered. For example, antenna array diversity techniques have been investigated in [6]–[9] to increase the capacity of DS/CDMA systems. Diversity reception is an effective way to reduce the fading effects of wireless systems; however, it is insufficient to eliminate the interuser interference and fully restore the CDMA system capacity [10].

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In order to eliminate interuser interference and resist the near–far problem, multiuser detection has been considered in [11]–[16]. While the performance of an optimum multiuser detector gives the lower bound of the error probability, the complexity of the decision algorithm increases exponentially as the number of users. Because of this complexity of the optimum multiuser detector, some suboptimum detectors are considered [17]–[20], and the decorrelating approach is considered as a simple suboptimum multiuser detector in [15]. In addition, multiuser receivers with antenna arrays are considered to enhance the performance of the system in [21]–[24]. The performance of these systems is better than that of conventional systems because the multiuser interference can be eliminated at the expense of noise correlation and enhancement, when the cross-correlation matrix of signature waveforms is known perfectly. These systems, however, have a drawback that the decorrelator enhances and correlates noise, and the enhancement and correlation of noise gets larger as the cross correlation gets higher and the number of users increases. In [23], the optimum and suboptimum multiuser detectors employing antenna arrays are studied in synchronous and finite packet-size asynchronous channel cases. It has been shown that the system proposed in [23] has a higher asymptotic efficiency than others. This system, however, is not useful when the packet size is large because of intensive computations in the matrix inversion and the large-size memory required. In this case, a decorrelating filter can be used [15], [16].

In this paper, we propose a quasi-maximum-likelihood (quasi-ML) detector in the reverse link from a mobile to the base station, which uses antenna arrays in asynchronous channels employing a decorrelating filter. In Section II, the system model will be described. We will consider the quasi-ML multiuser detector in a time-invariant channel in Section III, and extend it to the case where the channel is time-varying in Section IV. In Section V, we will investigate the performance of the proposed systems. Some numerical examples and simulation results will be shown in Section VI, and concluding remarks will be given in Section VII.

## II. System Model

Consider the reverse link from a mobile station to the base station with an antenna array in asynchronous channels. The channel is assumed to be time-nonselective fading described by a wide-sense stationary uncorrelated scattering model. We also assume the characteristic of the channel is slowly varying

Rayleigh; in other words, the coherence time of the channel is much longer than the data symbol duration.

In mobile stations, information bits are multiplied by a spreading sequence. After being multiplied by a carrier, the signal is transmitted. We assume that we use binary phase-shift keying. Let the $k$th user's baseband information signal waveform be

$$x_k(t) = \sum_{p=-\infty}^{\infty} x_k(p) P_{T_s}(t - pT_s) \tag{1}$$

where $T_s$ is the symbol period, $P_{T_s}(t)$ is a rectangular pulse with duration $T_s$, and $x_k(p)$ is the baseband symbol of the $k$th user during the $p$th symbol period. Similarly, the signature waveform is defined as

$$c_k(t) = \sum_{n=-\infty}^{\infty} c_k(n) P_{T_c}(t - nT_c) \tag{2}$$

where $c_k(n)$ is the $n$th chip of the $k$th user, and $P_{T_c}(t)$ is a time-limited chip waveform with duration $T_c$ ($\ll T_s$). Then, the transmitter signal modulated by a carrier frequency $f_c$ ($\gg (1/T_c)$) is

$$u_k(t) = \sqrt{2P_k}\,\mathrm{Re}\{x_k(t)c_k(t)\exp[j(\omega_c t + \psi_k)]\} \tag{3}$$

where $\omega_c = 2\pi f_c$, $\psi_k$ is the random phase of the $k$th carrier, and $P_k$ is the $k$th user's transmitted power.

Let us assume that an $M$-element antenna array is employed at the base station. Then, the equivalent complex baseband received signal vector of size $M \times 1$ at the receiver of the base station can be written as

$$\mathbf{r}(t) = \sum_{k=1}^{K}\sum_{l=1}^{L} s_{k,l}(t - \tau_{k,l})c_k(t - \tau_{k,l})\mathbf{a}_{k,l} + \mathbf{n}(t) \tag{4}$$

where $K$ is the number of users in a cell, $s_{k,l}(t) = \sqrt{P_k}\alpha_{k,l}x_k(t)e^{j\phi_{k,l}}$, $\alpha_{k,l}$ is the attenuation factor of the $l$th path of the $k$th user, $L$ is the number of resolvable multipath components, $\phi_{k,l} = \psi_k - \omega_c\tau_{k,l}$ (in coherent reception, we can assume that $\psi_k = 0$), $\mathbf{a}_{k,l}$ is the $M \times 1$ channel vector of the $l$th path of the $k$th user, and $\mathbf{n}(t)$ is the $M \times 1$ additive temporally and spatially white complex Gaussian noise vector with covariance matrix $\sigma_n^2\mathbf{I}$.

The receiver schemes considered in this paper for the time-invariant channel and time-varying channel are shown in Figs. 1 and 2, respectively. The $M \times KL$ output matrix $\mathbf{Y}(n)$ from the rake correlator can be written as

$$\mathbf{Y}(n) = [\mathbf{y}_{1,1}(n) \;\cdots\; \mathbf{y}_{1,L}(n) \;\cdots\; \mathbf{y}_{K,1}(n) \;\cdots\; \mathbf{y}_{K,L}(n)] \tag{5}$$

where

$$\mathbf{y}_{p,q}(n) = \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=-1}^{1} s_{k,l}(n+m)\gamma_{p,q,k,l}^{(m)}\mathbf{a}_{k,l} + \mathbf{n}_{p,q}(n) \tag{6}$$

is the $M \times 1$ column vector representing the output of the rake correlator of the $q$th path of the $p$th user

$$\gamma_{p,q,k,l}^{(-1)} = \begin{cases} \dfrac{1}{T_s}\displaystyle\int_{\tau_{p,q}}^{\tau_{k,l}} c_k(t-\tau_{k,l})c_p(t-\tau_{p,q})\,dt, & \tau_{k,l} > \tau_{p,q} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

$$\gamma_{p,q,k,l}^{(0)} = \frac{1}{T_s}\int_{\max(\tau_{p,q},\tau_{k,l})}^{\min(\tau_{p,q},\tau_{k,l})+T_s} c_k(t-\tau_{k,l})c_p(t-\tau_{p,q})\,dt \tag{8}$$

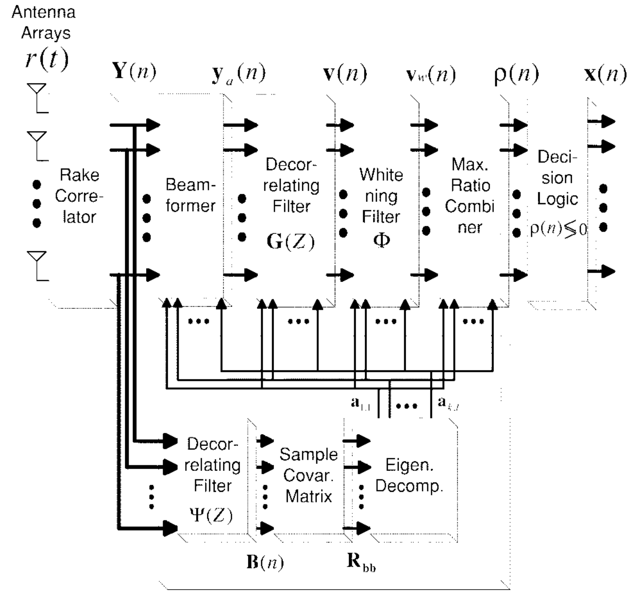


Fig. 1. The receiver system architecture: the time-invariant channel case.

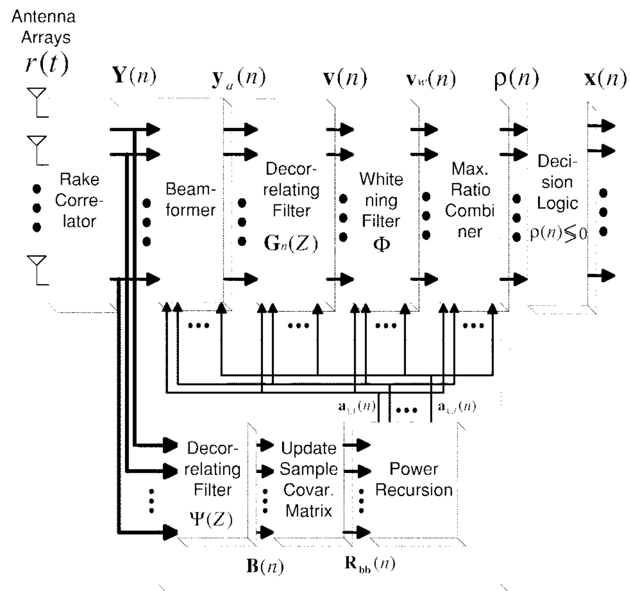


Fig. 2. The receiver system architecture: the time-varying channel case.

and

$$\gamma_{p,q,k,l}^{(1)} = \begin{cases} \dfrac{1}{T_s}\displaystyle\int_{\tau_{k,l}}^{\tau_{p,q}} c_k(t-\tau_{k,l})c_p(t-\tau_{p,q})\,dt, & \tau_{k,l} < \tau_{p,q} \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

are the cross correlations between the $p$th user signature waveform through the $q$th path during the $n$th symbol of the $p$th user signal through the $q$th path and the $k$th user signature waveform through the $l$th path during the $(n-1)$th, $n$th, and $(n+1)$th symbol of the $k$th user through the $l$th path, respectively, and

$$\mathbf{n}_{p,q}(n) = \frac{1}{T_s}\int_{\tau_{p,q}+(n-1)T_s}^{\tau_{p,q}+nT_s} \mathbf{n}(t)c_p(t-\tau_{p,q})\,dt \tag{10}$$

is an $M \times 1$ column noise vector.

We can see that $\gamma^{(-1)}_{p,q,k,l}$, $\gamma^{(0)}_{p,q,k,l}$, and $\gamma^{(1)}_{p,q,k,l}$ can be computed from the prior knowledge of user signature waveforms and time delays. The noise cross covariance matrix can be obtained as

$$\begin{aligned} &E\{\mathbf{n}_{p,q}(n)\mathbf{n}^H_{k,l}(n+m)\} \\ &= \frac{1}{T_s^2}\int_{\tau_{p,q}+(n-1)T_s}^{\tau_{p,q}+nT_s}\int_{\tau_{k,l}+(n+m-1)T_s}^{\tau_{k,l}+(n+m)T_s} E\{\mathbf{n}(t)\mathbf{n}^H(\sigma)\} \\ &\quad\cdot c_p(t-\tau_{p,q})c_k(\sigma-\tau_{k,l})\,d\sigma\,dt \\ &= \frac{\sigma_n^2\mathbf{I}}{T_s^2}\int_{\tau_{p,q}}^{\tau_{p,q}+T_s}\int_{\tau_{k,l}}^{\tau_{k,l}+T_s}\delta(t-\sigma-mT_s) \\ &\quad\cdot c_p(t-\tau_{p,q})c_k(\sigma-\tau_{k,l})\,d\sigma\,dt \\ &= \begin{cases}\gamma^{(m)}_{p,q,k,l}\sigma'^2_n\mathbf{I}, & m=-1,0,1\\ 0 & \text{otherwise}\end{cases} \end{aligned} \tag{11}$$

where $\sigma'^2_n = (\sigma_n^2/T_s)$.

## III. Quasi-ML Detection

In this section, we will investigate the quasi-ML detection of user signals and compare it to the conventional decorrelating approach.

Let $\hat{\mathbf{s}}(n)$ denote all $\hat{s}_{k,l}(n+m)$, for $k=1,\cdots,K$, $l=1,\cdots,L$, and $m=-1,0,1$. Then, the likelihood function $L_{p,q}(\hat{\mathbf{s}}(n))$ of $\mathbf{y}_{p,q}(n)$ can be written as (12), shown at the bottom of the page, where $\mathbf{R}_{nn,p,q} = E\{\mathbf{n}_{p,q}(n)\mathbf{n}^H_{p,q}(n)\} = \sigma'^2_n\mathbf{I}$, $p=1,\cdots,K$, and $q=1,\cdots,L$. Taking the derivative of the logarithm of (12) and setting the result to zero yields

$$\begin{aligned} 0 &= \frac{\partial \log L_{p,q}(\hat{\mathbf{s}}(n))}{\partial \hat{s}_{p,q}(n)} \\ &= \gamma_{p,q,p,q}\Bigg[\mathbf{a}^H_{p,q}\mathbf{R}^{-1}_{nn,p,q}\mathbf{y}_{p,q}(n) - \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=-1}^{1}\hat{s}_{k,l}(n+m) \\ &\qquad\cdot\gamma^{(m)}_{p,q,k,l}\mathbf{a}^H_{p,q}\mathbf{R}^{-1}_{nn,p,q}\mathbf{a}_{k,l}\Bigg]. \end{aligned} \tag{13}$$

Since $\mathbf{n}_{p,q}(n)$ is temporally and spatially white, we can simplify (13) as

$$\begin{aligned} \hat{s}_{p,q}(n) &= \left(\gamma^{(0)}_{p,q,p,q}\|\mathbf{a}_{p,q}\|\right)^{-1}\Bigg[\mathbf{a}^H_{p,q}\mathbf{y}_{p,q}(n)/\|\mathbf{a}_{p,q}\| \\ &\quad - \sum_{\substack{k=1\\(k,l)\neq(p,q)}}^{K}\sum_{l=1}^{L}\sum_{m=-1}^{1}\hat{s}_{k,l}(n+m)\gamma^{(m)}_{p,q,k,l}\mathbf{a}^H_{p,q}\mathbf{a}_{k,l}/\|\mathbf{a}_{p,q}\|\Bigg]. \end{aligned} \tag{14}$$

If we consider all the likelihood functions for each path and user, we can obtain

$$\mathbf{y}_a(n) = \mathbf{QW}\hat{\mathbf{x}}_a(n) \tag{15}$$

where

$$\mathbf{Q} = \mathbf{\Lambda}^{(-1)}\cdot D + \mathbf{\Lambda}^{(0)} + \mathbf{\Lambda}^{(1)}\cdot D^{-1} \tag{16}$$

$$\begin{aligned}\lambda^{(m)}_{p,q,k,l} &= \left[\mathbf{\Lambda}^{(m)}\right]_{p\circ q,k\circ l} \\ &= \gamma^{(m)}_{p,q,k,l}d_{p,q,k,l}\end{aligned} \tag{17}$$

$\mathbf{W} = \operatorname{diag}([w_{1,1}w_{1,L}\cdots w_{2,1}\cdots w_{2,L}\cdots w_{K,1}\cdots w_{K,L}])$, $w_{k,l} = \sqrt{P_k}\alpha_{k,l}e^{j\phi_{k,l}}$, $\mathbf{x}_a(n) = [x_{a,1,1}(n)\cdots x_{a,1,L}(n)\cdots x_{a,K,1}(n)\cdots x_{a,K,L}(n)]^T$, $x_{a,k,l}(n) = \|\mathbf{a}_{k,l}\|x_k(n)$, $\hat{\mathbf{x}}_a(n)$ is the estimate of $\mathbf{x}_a(n)$, $d_{p,q,k,l} = \mathbf{a}^H_{p,q}\mathbf{a}_{k,l}/\|\mathbf{a}_{p,q}\|\|\mathbf{a}_{k,l}\|$, $\mathbf{y}_a(n) = [\mathbf{a}^H_{1,1}\mathbf{y}_{1,1}(n)/\|\mathbf{a}_{1,1}\| \ \cdots\ \mathbf{a}^H_{1,L}\mathbf{y}_{1,L}(n)/\|\mathbf{a}_{1,L}\| \ \cdots\ \mathbf{a}^H_{K,1}\mathbf{y}_{K,1}(n)/\|\mathbf{a}_{K,1}\| \ \cdots\ \mathbf{a}^H_{K,L}\,\mathbf{y}_{K,L}(n)/\|\mathbf{a}_{K,L}\|]^T$ for $m=-1,0,1$, $[\mathbf{R}]_{i,j}$ is the $ij$th element of $\mathbf{R}$, and $p\circ q = p(L-1)+q$. Note that $\mathbf{y}_a(n)$, the decorrelator input, is the output of the beamformer whose weight is the channel vectors, and $\mathbf{\Lambda}^{(m)}$, $m=-1,0,1$, are the cross-correlation matrices in which both the cross correlations of signature waveforms and channel vectors are taken into account.

Since the elements of $\mathbf{Q}$ are polynomials of $D$, we can see that the numerator and denominator of an element of $\mathbf{Q}^{-1}$ are also polynomials of $D$ in general. Thus, we can design an inverse filter whose transfer function, input, and output are $\mathbf{Q}^{-1}|_{D=Z}$, $\mathbf{y}_a(n)$, and $\mathbf{W}\hat{\mathbf{x}}_a(n)$, respectively. The coefficients of the filter can be computed once the signature waveforms and the channel vectors are given. In summary, the rake correlator output $\mathbf{Y}(n)$ enters the beamformer whose weights are the channel vectors. Then, the beamformer output $\mathbf{y}_a(n)$ is the input of the inverse filter.

In the proposed scheme, the channel vectors should be estimated since they are required for calculating not only the beamformer weights but also the filter coefficients. Let us consider the $KL$-input $KL$-output decorrelating filter whose transfer function is $\mathbf{\Upsilon}(Z) = (\mathbf{\Gamma}^{(-1)}Z^{-1} + \mathbf{\Gamma}^{(0)} + \mathbf{\Gamma}^{(1)}Z)^{-1}$, where the $(p\circ q, k\circ l)$th element of $\mathbf{\Gamma}^{(m)}$ is $\gamma^{(m)}_{p,q,k,l}$, for each antenna output as studied in [15], [24]. Then, the filter output $\mathbf{B}(n)$ is

$$\mathbf{B}(n) = [\mathbf{b}_{1,1}(n)\ \cdots\ \mathbf{b}_{1,L}(n)\ \cdots\ \mathbf{b}_{K,1}(n)\ \cdots\ \mathbf{b}_{K,L}(n)] \tag{18}$$

where

$$\mathbf{b}_{p,q}(n) = s_{p,q}(n)\mathbf{a}_{p,q} + \mathbf{n}'_{p,q}(n) \tag{19}$$

$$\begin{aligned} L_{p,q}(\hat{\mathbf{s}}(n)) &= f_{\mathbf{n}_{p,q}}(\mathbf{y}_{p,q}(n)\mid\hat{\mathbf{s}}(n)) \\ &= \frac{1}{(2\pi)^{(M/2)}(\det[\mathbf{R}_{nn,p,q}])^{(1/2)}}\exp\Bigg[-\frac{1}{2}\left(\mathbf{y}_{p,q}(n) - \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=-1}^{1}\hat{s}_{k,l}(n+m)\gamma^{(m)}_{p,q,k,l}\mathbf{a}_{k,l}\right)^H\mathbf{R}^{-1}_{nn,p,q} \\ &\qquad\cdot\left(\mathbf{y}_{p,q}(n) - \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=-1}^{1}\hat{s}_{k,l}(n+m)\gamma^{(m)}_{p,q,k,l}\mathbf{a}_{k,l}\right)\Bigg] \end{aligned} \tag{12}$$

$$E\{\mathbf{n}'_{p,q}(n)\mathbf{n}'^H_{p,q}(n)\} = [\mathbf{F}(0)]_{p \circ q, p \circ q}\sigma'^2_n \mathbf{I} \tag{20}$$

$$\sum_{k=-\infty}^{\infty} \mathbf{F}(k)Z^{-k} = \mathbf{\Upsilon}(Z)\mathbf{R}_n(Z)\mathbf{\Upsilon}^H(1/Z^*) \tag{21}$$

and $\mathbf{R}_n(Z) = \mathbf{\Gamma}^{(-1)}Z^{-1} + \mathbf{\Gamma}^{(0)} + \mathbf{\Gamma}^{(1)}Z$ is the $Z$-transform of the covariance matrix of the noise in the rake correlator output. Note that $\mathbf{F}(k)$ is the covariance matrix of the noise in the output of the decorrelating filter whose transfer function is $\mathbf{\Upsilon}(Z)$. Then, the covariance matrix of $\mathbf{b}_{p,q}(n)$ is

$$\begin{aligned}\mathbf{R}_{\mathbf{bb},p,q} &= E\{\mathbf{b}_{p,q}(n)\mathbf{b}^H_{p,q}(n)\} \\ &= P_p\alpha^2_{p,q}\mathbf{a}_{p,q}\mathbf{a}^H_{p,q} + [\mathbf{F}(0)]_{p \circ q, p \circ q}\sigma'^2_n\mathbf{I}.\end{aligned} \tag{22}$$

Thus, we can obtain the sample covariance matrix $\mathbf{R}_{\mathbf{bb},p,q}$ from the decorrelating filter outputs and estimate the channel vector of the $q$th multipath component of the $p$th user from the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_{\mathbf{bb},p,q}$. If we assume that the signature waveforms are known *a priori*, we can compute $d_{p,q,k,l}$ and the filter coefficients. Then, it is easily seen that the inverse filter has the following form:

$$\mathbf{G}(Z) = \left[\mathbf{\Lambda}^{(-1)}Z^{-1} + \mathbf{\Lambda}^{(0)} + \mathbf{\Lambda}^{(1)}Z\right]^{-1}. \tag{23}$$

Now, let us consider the realization of the inverse filter $\mathbf{G}(Z)$. A stable realization of the filter exists if and only if

$$\det\left[\mathbf{\Lambda}^{(-1)}e^{-j\omega} + \mathbf{\Lambda}^{(0)} + \mathbf{\Lambda}^{(1)}e^{j\omega}\right] \neq 0 \qquad \forall\omega \in [0, 2\pi]. \tag{24}$$

The above condition is equivalent to the linear independence of the infinite sequence of multiuser signals [13], i.e., for real numbers $\epsilon_{i,j}$

$$\sum_{i=1}^{K}\sum_{j=1}^{L}\epsilon_{i,j}c_i(t_0-\tau_{i,j})\mathbf{a}_{i,j} = 0, \qquad \text{for some } t_0 \in [-\infty, \infty] \tag{25}$$

should imply $\forall\epsilon_{i,j} = 0$. If we assume that at least one channel vector is nonzero, it can be easily seen that a stable realization of the filter exists when $c_i(t)$ and $\tau_{i,j}$, $i = 1, 2, \cdots, K$, $j = 1, 2, \cdots, L$, are such that

$$\sum_{i=1}^{K}\sum_{j=1}^{L}\epsilon_{i,j}c_i(t_0-\tau_{i,j}) = 0, \qquad \text{for some } t_0 \in [-\infty, \infty] \tag{26}$$

implies $\forall\epsilon_{i,j} = 0$. Thus, a stable realization of the filter exists if the delays are uniformly distributed, which is the case in the asynchronous channel used by noncooperating users [13]. (Note that even if (26) does not hold, (25) may hold in some cases.) Hence, we can implement a stable causal inverse filter if we use an appropriately long delay and truncate the noncausal part of the filter [16].

Note that in the systems proposed in [15] and [24], the filter whose transfer function is $\Upsilon(Z)$ is used for asynchronous multiuser detection. In the system proposed in this paper, on the other hand, we use a filter with the same structure, but the coefficients of the filter are dependent on the channel vectors. Therefore, we can implement the proposed filter by adding some more memories and multipliers to the filters used in [15] and [24]. Although the complexity of the filter may increase slightly, as it will be observed later in this paper, we get some performance gain with the proposed system.

In the conventional decorrelating approach with a base station antenna array (for example, [24]), decorrelation precedes beamforming. In the proposed system, on the other hand, the received signal is beamformed first and decorrelated later. With the proposed approach, we can get not only diversity combining but also the advantage of reduced noise correlation and enhancement. The decorrelating filter proposed in this paper can also be considered as an extension of the conventional one; we can see that the former can be regarded as the later with the cross correlation of signature waveforms lowered due to beamforming. We would also like to note that the space diversity is used not only to get diversity combining gain but also to reduce the effective cross correlation of user signature waveforms (i.e., to reduce the noise enhancement and correlation which are unavoidable in the conventional approach).

## IV. Quasi-ML Detection in Time-Varying Channels

In practical situations, mobile stations move in the propagation environment. The effective scatterer contributing to the received signal at the base station antenna array can thus vary. Thus, the channel vector of the user will change with time. In such a case, a (recursive) method for updating the estimates of the channel vector is required.

To begin with, we assume that the channel vector of a user can change only at the beginning of each symbol of the user. This means the channel vector remains constant during the symbol period [25]. In the IS-95 standard, the symbol rate is 9600 b/s. If a mobile moves at 72 km/h, the mobile moves only 2 cm during a symbol period. Thus, the assumption makes sense in most practical situations.

The sampled rake correlator output vector of the $q$th path of the $p$th user can be written as

$$\begin{aligned}\mathbf{y}_{p,q}(n) = \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=-1}^{1} s_{k,l}(n+m)\gamma^{(m)}_{p,q,k,l} \\ \cdot \mathbf{a}_{k,l}(n+m) + \mathbf{n}_{p,q}(n).\end{aligned} \tag{27}$$

Then, the ML estimate $\hat{s}_{p,q}(n)$ can be obtained as

$$\begin{aligned}\hat{s}_{p,q}(n) = \left(\gamma^{(0)}_{p,q,p,q}\|\mathbf{a}_{p,q}(n)\|\right)^{-1}\Bigg[\mathbf{a}^H_{p,q}(n)\mathbf{y}_{p,q}(n)/\|\mathbf{a}_{p,q}(n)\| \\ - \sum_{\substack{k=1 \\ (k,l)\neq(p,q)}}^{K}\sum_{l=1}^{L}\sum_{m=-1}^{1}\hat{s}_{k,l}(n+m)\gamma^{(m)}_{p,q,k,l}\mathbf{a}^H_{p,q}(n) \\ \cdot \mathbf{a}_{k,l}(n+m)/\|\mathbf{a}_{p,q}(n)\|\Bigg].\end{aligned} \tag{28}$$

If we consider all the likelihood functions for each path and user, we can obtain

$$\mathbf{y}_a(n) = \mathbf{Q}(n)\mathbf{W}\hat{\mathbf{x}}_a(n) \tag{29}$$

where

$$\mathbf{Q}(n) = \mathbf{\Lambda}_n^{(-1)} \cdot D + \mathbf{\Lambda}_n^{(0)} + \mathbf{\Lambda}_n^{(1)} \cdot D^{-1} \tag{30}$$

$$\lambda_{p,q,k,l}^{(m)}(n) = \left[\mathbf{\Lambda}_n^{(m)}\right]_{p\circ q, k\circ l} = \gamma_{p,q,k,l}^{(m)} d_{p,q,k,l}^{(m)}(n) \tag{31}$$

$\mathbf{x}_a(n) = [x_{a,1,1}(n) \cdots x_{a,1,L}(n) \cdots x_{a,K,1}(n) \cdots x_{a,K,L}(n)]^T$, $x_{a,k,l}(n) = \|\mathbf{a}_{k,l}(n)\|\ x_k(n)$, $\hat{\mathbf{x}}_a(n)$ is the estimate of $\mathbf{x}_a(n)$, and $d_{p,q,k,l}^{(m)}(n) = \mathbf{a}_{p,q}^H(n)\mathbf{a}_{k,l}(n+m)/\|\mathbf{a}_{p,q}(n)\|\ \|\mathbf{a}_{k,l}(n+m)\|$. Since the elements of $\mathbf{Q}(n)$ are polynomials of $D$, we can see that the numerator and denominator of an element of $\mathbf{Q}^{-1}(n)$ are also polynomials of $D$ in general. Thus, we can design a time-varying inverse filter whose transfer function, input, and output are $\mathbf{Q}^{-1}(n)|_{D=Z}$, $\mathbf{y}_a(n)$, and $\mathbf{W}\hat{\mathbf{x}}_a(n)$, respectively. The coefficients of the filter are time-varying and can be computed once the signature waveforms and the sequence $d_{p,q,k,l}^{(m)}(n)$, $m = -1, 0, 1$ are given. If we can estimate the channel vector, we can compute $d_{p,q,k,l}^{(m)}(n)$, $m = -1, 0, 1$ and the filter coefficients. The inverse filter derived is a time-varying filter and we can treat it as a filter whose coefficients are varying at each time epoch. Let the impulse response of the filter at time epoch $n$ be $\mathbf{H}_n(k)$, and define the transfer function of the filter at time epoch $n$ as $\mathbf{G}_n(Z) = \sum_{k=-\infty}^{\infty} \mathbf{H}_n(k) Z^{-k}$. Then, it is easy to see that the inverse filter has the following form:

$$\mathbf{G}_n(Z) = \left[\mathbf{\Lambda}_n'^{(-1)} Z^{-1} + \mathbf{\Lambda}_n'^{(0)} + \mathbf{\Lambda}_n'^{(1)} Z\right]^{-1} \tag{32}$$

where $\mathbf{\Lambda}_n'^{(m)}$ is the $KL \times KL$ matrix whose elements consist of $\gamma_{p,q,k,l}^{(m)}$ and $d_{p,q,k,l}^{(m)}(n)$, $m = -1, 0, 1$, $k = 1, 2, \cdots K$, and $p = 1, 2, \cdots, K$, in the above-mentioned manner.

*Example:* Consider a two-user system with $L = 1$. Here, the subscripts representing paths are omitted for notational convenience. The correlation matrix of signature waveforms are computed after estimating the delay of each user. Then $\mathbf{Q}(n)$ can be written as

$$\mathbf{Q}(n) = \begin{bmatrix} \lambda_{1,1}^{(0)}(n) & \lambda_{1,2}^{(0)}(n) + \lambda_{1,2}^{(-1)}(n) D \\ \lambda_{2,1}^{(0)}(n) + \lambda_{2,1}^{(1)}(n) D^{-1} & \lambda_{2,2}^{(0)}(n) \end{bmatrix}. \tag{33}$$

Next we compute the inverse matrix of $\mathbf{Q}(n)$ as

$$\mathbf{Q}^{-1}(n) = \frac{\mathrm{adj}[\mathbf{Q}(n)]}{\det[\mathbf{Q}(n)]} \tag{34}$$

where

$$\begin{aligned}\det[\mathbf{Q}(n)] = \lambda_{1,1}^{(0)}(n)\lambda_{2,2}^{(0)}(n) - \lambda_{1,2}^{(0)}(n)\lambda_{2,1}^{(0)}(n) \\ - \lambda_{1,2}^{(-1)}(n)\lambda_{2,1}^{(1)}(n-1) - \lambda_{1,2}^{(-1)}(n) \\ \cdot \lambda_{2,1}^{(0)}(n-1) D - \lambda_{1,2}^{(0)}(n)\lambda_{2,1}^{(1)}(n) D^{-1}\end{aligned} \tag{35}$$

and

$$\mathrm{adj}[\mathbf{Q}(n)] = \begin{bmatrix} \lambda_{2,2}^{(0)}(n) & -\lambda_{1,2}^{(0)}(n) - \lambda_{1,2}^{(-1)}(n) D \\ -\lambda_{2,1}^{(0)}(n) - \lambda_{2,1}^{(1)}(n) D^{-1} & \lambda_{1,1}^{(0)}(n) \end{bmatrix}. \tag{36}$$

Then, the inverse filter is

$$\mathbf{W}\hat{\mathbf{x}}(Z) = \frac{1}{D(Z)} \mathbf{A}(Z) \mathbf{y}_a(Z) \tag{37}$$

where

$$\mathbf{A}(Z) = \begin{bmatrix} \lambda_{2,2}^{(0)}(n) & -\lambda_{1,2}^{(0)}(n) - \lambda_{1,2}^{(-1)}(n) Z^{-1} \\ -\lambda_{2,1}^{(0)}(n) - \lambda_{2,1}^{(1)}(n) Z & \lambda_{1,1}^{(0)}(n) \end{bmatrix} \tag{38}$$

and

$$\begin{aligned}D(Z) = \lambda_{1,1}^{(0)}(n)\lambda_{2,2}^{(0)}(n) - \lambda_{1,2}^{(0)}(n)\lambda_{2,1}^{(0)}(n) \\ - \lambda_{1,2}^{(-1)}(n)\lambda_{2,1}^{(1)}(n-1) - \lambda_{1,2}^{(-1)}(n) \\ \cdot \lambda_{2,1}^{(0)}(n-1) Z^{-1} - \lambda_{1,2}^{(0)}(n)\lambda_{2,1}^{(1)}(n) Z.\end{aligned} \tag{39}$$

Since we can compute $d_{p,k}^{(m)}$ for $k = 1, 2$, $p = 1, 2$, and $m = -1, 0, 1$, from the estimated channel vector $\hat{\mathbf{a}}_k(n+m)$, $k = 1, 2$, and $m = -1, 0, 1$, we can obtain the first user signal from the output of the filter by updating the filter coefficients, if we use an appropriately long delay and truncate the noncausal part of the filter [16].

Now, consider the estimation of the channel vector. Let us consider the $KL$-input $KL$-output decorrelating filter whose transfer function is $\mathbf{\Upsilon}(Z)$ for each antenna output again. Then, the $M \times 1$ filter output vector of the $p$th user in the $q$th multipath signal is

$$\mathbf{b}_{p,q}(n) = s_{p,q}(n)\mathbf{a}_{p,q}(n) + \mathbf{n}'_{p,q}(n) \tag{40}$$

and the covariance matrix is

$$\begin{aligned}\mathbf{R}_{\mathbf{bb},p,q}(n) &= E\{\mathbf{b}_{p,q}(n)\mathbf{b}_{p,q}^H(n)\} \\ &= P_p \alpha_{p,q}^2 \mathbf{a}_{p,q}(n)\mathbf{a}_{p,q}^H(n) + [\mathbf{F}(0)]_{p\circ q, p\circ q}\sigma_n'^2 \mathbf{I}.\end{aligned} \tag{41}$$

Note that the covariance matrix is time-varying. Thus, we should first update the sample covariance matrix with the decorrelator outputs, and then estimate the channel vector. Since the channel is slowly time-varying with respect to the symbol rate, a fading memory can be used [26]. Then, the estimate of the covariance matrix $\mathbf{R}_{\mathbf{bb},p,q}(n)$ can be written as

$$\hat{\mathbf{R}}_{\mathbf{bb},p,q}(n) = \beta \hat{\mathbf{R}}_{\mathbf{bb},p,q}(n-1) + \mathbf{b}_{p,q}(n)\mathbf{b}_{p,q}^H(n) \tag{42}$$

where $\beta$ is the forgetting factor, a positive constant between 0 and 1.

Now, we are ready to estimate the channel vector $\mathbf{a}_{p,q}(n)$ from $\hat{\mathbf{R}}_{\mathbf{bb},p,q}(n)$. Since the eigendecomposition is a highly time-consuming task, we will use the power method [27]. Let

$$\boldsymbol{\eta}_{r+1} = \frac{\hat{\mathbf{R}}_{\mathbf{bb},p,q}(n)\boldsymbol{\eta}_r}{\|\hat{\mathbf{R}}_{\mathbf{bb},p,q}(n)\boldsymbol{\eta}_r\|}. \tag{43}$$

Then, we can get the estimate of the channel vector and user received power from the largest eigenvalue and the corresponding eigenvector as

$$P_p E\{\alpha_{p,q}^2\} = \lim_{r\to\infty} \|\hat{\mathbf{R}}_{\mathbf{bb},p,q}(n)\boldsymbol{\eta}_r\| \tag{44}$$

and

$$\hat{\mathbf{a}}_{p,q}(n) = \lim_{r\to\infty} \boldsymbol{\eta}_r. \tag{45}$$

The convergence rate of the iteration depends upon the ratio of the largest to the second largest eigenvalues, which is the same as the *signal-to-noise ratio (SNR)* of the filtered output. Thus, we can get fast enough convergence rate if the *SNR* is high. Since the time-correlation of the channel vector is close to 1, we can use $\hat{\mathbf{a}}_{p,q}(n-1)$ as the initial vector of the recursion to estimate $\hat{\mathbf{a}}_{p,q}(n)$, which will result in fast convergence.

Now, the remaining problem is how to choose the optimal forgetting factor. If the forgetting factor is too large, it is difficult to update the covariance matrix as fast as the change of the channel vector. If it is too small, the effective number of data samples for the computation of the estimate of the covariance matrix becomes small and the estimate may be erroneous. In general cases, it is not feasible to find an optimal value of the forgetting factor. In addition, the aspect of the change in the channel can be varying with time. In this case, the time-correlation of the channel vector can be used as the forgetting factor since the change in the covariance matrix is due to the change in the channel vector [9].

## V. Performance Analysis

In this section, we analyze the performance of the proposed system in the sense of bit-error probability. Without loss of generality, we assume that the first user is the desired user.

The filter output of the first user is

$$\mathbf{v}_1(n) = \mathbf{w}_1 x_1(n) + \mathbf{n}_{g,1}(n) \tag{46}$$

where $\mathbf{w}_1 = [w_{1,1}\|\mathbf{a}_{1,1}\| \; w_{1,2}\|\mathbf{a}_{1,2}\| \; \cdots \; w_{1,L}\|\mathbf{a}_{1,L}\|]^T$, and $\mathbf{n}_{g,1}(n) = [n_{g,1,1}(n) \; n_{g,1,2}(n) \; \cdots \; n_{g,1,L}(n)]^T$ is the first $L$ elements of $\mathbf{n}_g(n) = [\mathbf{n}_{g,1}^T \; \mathbf{n}_{g,2}^T \; \cdots \; \mathbf{n}_{g,K}^T]^T$. The $Z$-transform of the correlation matrix of the noise vector is $\mathbf{R}_n(Z) = \mathbf{\Lambda}^{(-1)}Z^{-1} + \mathbf{\Lambda}^{(0)} + \mathbf{\Lambda}^{(1)}Z$. Let $\mathbf{G}(Z)\mathbf{R}_n(Z)\mathbf{G}^H(1/Z^*) = \sum_{k=-\infty}^{\infty}\mathbf{T}(k)Z^{-k}$, then the noise covariance matrix can be written as

$$E\{\mathbf{n}_g(n)\mathbf{n}_g^H(n)\} = \sigma_n'^2\mathbf{T}(0). \tag{47}$$

Since the noise vector $\mathbf{n}_{g,1}(n)$ is correlated, a whitening technique can be applied. The correlation matrix of $\mathbf{n}_{g,1}(n)$ is $\sigma_n'^2[\mathbf{T}(0)]_L$, where $[\mathbf{T}(0)]_L$ is the $L\times L$ matrix whose $ij$th element is the $ij$th element of the $KL\times KL$ matrix $\mathbf{T}(0)$. Thus, we can get an $L\times L$ whitening matrix $\Phi_1$, such that $(\mathbf{\Phi}_1^{-1})(\mathbf{\Phi}_1^{-1})^H = [\mathbf{T}(0)]_L$ by Cholesky decomposition [28]. Then, we get the output of the whitening filter as

$$\mathbf{v}_{1,w}(n) = \mathbf{\Phi}_1\mathbf{w}_1 x_1(n) + \mathbf{n}_{1,w}(n) \tag{48}$$

where $E\{\mathbf{n}_{1,w}(n)\mathbf{n}_{1,w}^H(n)\} = \sigma_n'^2\mathbf{I}$. If we assume coherent reception with maximum-ratio combining, we get the decision variable $\rho_1(n)$ as

$$\begin{aligned}\rho_1(n) &= \mathrm{Re}\{\mathbf{w}_1^H\mathbf{\Phi}_1^H\mathbf{v}_{1,w}(n)\}\\ &= \mathbf{w}_1^H\mathbf{\Phi}^H\mathbf{\Phi}\mathbf{w}_1x_1(n) + \mathrm{Re}\{\mathbf{w}_1^H\mathbf{\Phi}_1^H\mathbf{n}_{1,w}(n)\}\end{aligned} \tag{49}$$

and the instantaneous *SNR* $\nu_1$ of $\rho_1(n)$ is

$$\begin{aligned}\nu_1 &= \frac{\left(\mathbf{w}_1^H\left(\mathbf{\Phi}_1^H\mathbf{\Phi}_1\right)\mathbf{w}_1\right)^2}{\sigma_n'^2\mathbf{w}_1^H\left(\mathbf{\Phi}_1^H\mathbf{\Phi}_1\right)\mathbf{w}_1}\\ &= \frac{\mathbf{w}_1^H\left(\mathbf{\Phi}_1^H\mathbf{\Phi}_1\right)\mathbf{w}_1}{\sigma_n'^2}\\ &= \frac{E_1\mathbf{w}_1'^H([\mathbf{T}(0)]_L)^{-1}\mathbf{w}_1'}{\sigma_n^2}\end{aligned} \tag{50}$$

where $\mathbf{w}_1 = \mathbf{w}_1'\sqrt{P_1}$. Here $\mathbf{w}_1'$ is a complex Gaussian vector with mean zero and covariance matrix

$$\mathbf{R}_{\mathbf{w}_1'\mathbf{w}_1'} = \mathrm{diag}(\mu_1, \mu_2, \cdots, \mu_L) \tag{51}$$

where $\mu_i = E\{\alpha_{1,i}^2\}\mathbf{a}_{1,i}^H\mathbf{a}_{1,i}$. Since $\mathbf{R}_{\mathbf{w}_1'\mathbf{w}_1'}([\mathbf{T}(0)]_L)^{-1}$ is symmetric, by using the result in [29], the characteristic function of $\nu_1(n)$ can be obtained as

$$\Phi_{\nu_1}(\omega) = \frac{1}{\prod_{j=1}^{L}(1+2j\omega\xi_{1,j})} \tag{52}$$

where $\xi_{1,j}$ are the eigenvalues of $\mathbf{R}_{\mathbf{w}_1'\mathbf{w}_1'}([\mathbf{T}(0)]_L)^{-1}$. Then, by using the result in [30], the bit-error probability can be obtained as

$$P_{b,1} = \int_0^\infty \frac{1}{2}\,\mathrm{erfc}(\sqrt{x})f_{\nu_1}(x)\,dx = \sum_{l=1}^{L}\frac{\pi_{1,l}}{2}\left[1-\sqrt{\frac{\kappa_{1,l}}{1+\kappa_{1,l}}}\right] \tag{53}$$

where $\pi_{1,l} = \prod_{j=1,j\neq l}^{L}(\xi_{1,l}/(\xi_{1,l}-\xi_{1,j}))$ and $\kappa_{1,l} = ((E_1\xi_{1,l})/\sigma_n^2)$. In the time-varying case, we can get the bit-error probability by replacing $\mathbf{T}_n(0)$ for $\mathbf{T}(0)$, where $\mathbf{G}_n(Z)\mathbf{R}_n(Z)\;\mathbf{G}_n^H(1/Z^*) = \sum_{k=-\infty}^{\infty}\mathbf{T}_n(k)Z^{-k}$, provided that the estimation is perfect.

Now, consider the asymptotic efficiency of the proposed system. Since the beamforming does not affect the asymptotic efficiency in the conventional systems, the near–far resistance and asymptotic efficiency of the first user in a conventional system is [20]

$$\bar{\eta}_1^c = (\det([\mathbf{\Gamma}]_L[\mathbf{F}(0)]_L))^{-(1/L)}. \tag{54}$$

Similarly, the near–far resistance and asymptotic efficiency of the $k$th user of the proposed system can be written as

$$\bar{\eta}_1^p = (\det([\mathbf{\Gamma}]_L[\mathbf{T}(0)]_L))^{-(1/L)}. \tag{55}$$

It is easily seen that $\det([\mathbf{T}(0)]_L) < \det([\mathbf{F}(0)]_L)$ from the fact that $\mathbf{\Lambda}^{(1)}$, $\mathbf{\Lambda}^{(0)}$, and $\mathbf{\Lambda}^{(-1)}$ have smaller offdiagonal elements than $\mathbf{\Gamma}^{(-1)}$, $\mathbf{\Gamma}^{(0)}$, and $\mathbf{\Gamma}^{(1)}$, respectively. Thus, the near–far resistance and asymptotic efficiency of the proposed system are clearly higher than those of the conventional system.

## VI. Numerical Examples and Simulation Results

In this section, we consider some examples using randomly generated channel vectors using the channel vector model investigated in [31]. For each resolvable path, we set the channel vector as

$$\mathbf{a} = \left[1 \; e^{-(\eta^2/2)-j\Delta\sin\theta_c} \cdots e^{-(((M-1)^2\eta^2)/2)-j(M-1)\Delta\sin\theta_c}\right]^T \tag{56}$$

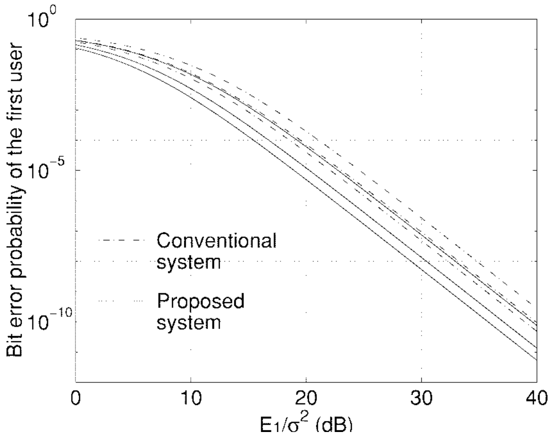


Fig. 3. The bit-error probabilities of the two systems when the number $K$ of active users is 20, the number $L$ of resolvable path is 3, the number $M$ of antenna arrays is 2, 3, and 4, the period of Gold sequence is 63, and the Rayleigh distribution parameter is 0.1.

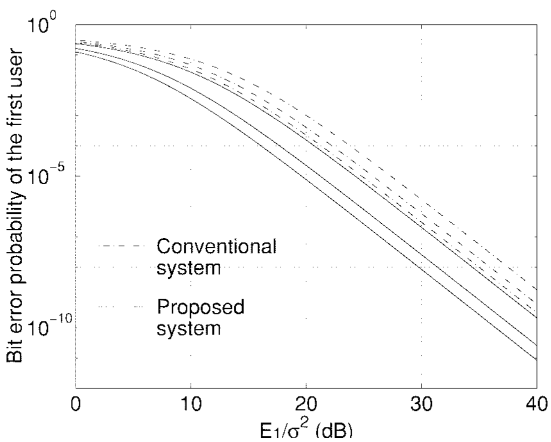


Fig. 4. The bit-error probabilities of the two systems when the number $K$ of active users is 40, the number $L$ of resolvable path is 3, the number $M$ of antenna arrays is 2, 3, and 4, the period of Gold sequence is 63, and the Rayleigh distribution parameter is 0.1.

in which the dispersion parameter $\eta$ (representing the degree of dispersion of angle of arrival) is 0.2, $\Delta = \pi$ (the antenna spacing is half of the carrier wavelength), and the center angle $\theta_c$ is uniformly distributed over $[0, 2\pi]$. The time-delay $\tau_{k,l}$ of each resolvable path is uniformly distributed over $[0, T_s]$. We use a Gold code of length 63 and set the Rayleigh distribution parameter to 0.1 (i.e., $E\{\alpha^2\} = 0.1$) for all paths.

In Figs. 3–5, the bit-error probabilities of the two systems are plotted when the number of resolvable paths is 3, and the number of active users is 20, 40, and 60, respectively. In each of the figures, we have three solid lines and three dotted lines. The solid lines are the bit-error-probability curves when the number $M$ of antennas is 2, 3, and 4 for the proposed system, and the dotted lines are those for the conventional system. We can see that we get more gain over the conventional system as the number of antennas increases and as the number of active users increases. It is clearly seen that higher spectral efficiency can be achieved by using the proposed system.

To show the performance of the estimation of the time-varying channel vector, some simulations are performed with the following scenario. The mobile station moves a distance

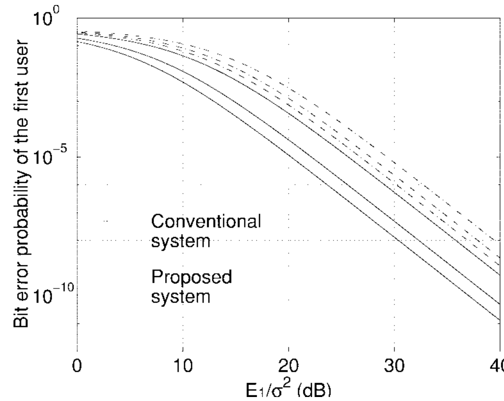


Fig. 5. The bit-error probabilities of the two systems when the number $K$ of active users is 60, the number $L$ of resolvable path is 3, the number $M$ of antenna arrays is 2, 3, and 4, the period of Gold sequence is 63, and the Rayleigh distribution parameter is 0.1.

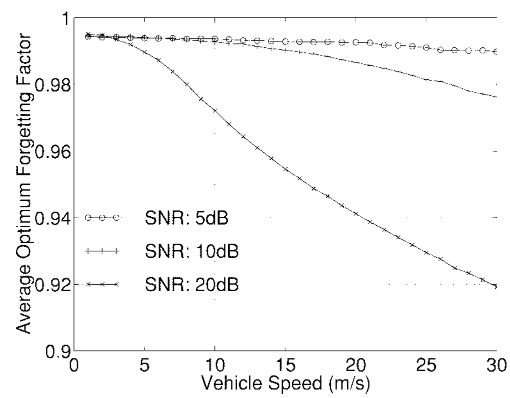


Fig. 6. Optimum values of the forgetting factors when the *SNR* is 5, 10, and 20 dB.

$d$ at speed $v$ from a starting point to an end point. We set the channel vector of the starting point to $\mathbf{a}_s = [1\ \ -0.02 + 0.089j\ \ 0.475 - 0.338j\ \ -0.087 + 0.076j]^T$ and the channel vector of the ending point to $\mathbf{a}_f = [1\ \ 0.5 - 0.527j\ \ 0.266 - 0.285j\ \ 0.509 - 0.464j]^T$. The channel vectors are assumed to change linearly as the mobile moves, i.e., the channel vector when the mobile is $r$ meters from the starting point is $(1 - (r/d))\mathbf{a}_s + (r/d)\mathbf{a}_f$. We set the symbol rate to 9600 b/s and get the results by averaging the mismatch loss during 100 symbols.

In Fig. 6, the averaged optimum values of the forgetting factors from 100 trials are plotted when the *SNR* is 5, 10, and 20 dB; by optimum value, we mean a value which minimizes the mismatch loss. The mismatch loss, defined as $20\log_{10}(E\{(|\hat{\mathbf{a}}^H(n)\mathbf{a}(n)|)/(\|\hat{\mathbf{a}}(n)\|\|\mathbf{a}(n)\|)\})$, is the loss in the received *SNR* due to the channel vector estimation error. We can see that the optimum value of the forgetting factor decreases as the mobile moves faster. The result can be explained as follows. There are two factors determining the optimum value of the forgetting factor: one is the mobile speed and the other is the effective number of samples when we compute the covariance matrix. If the mobile speed increases, the optimum value of the forgetting factor decreases to follow the change in the channel vector. If the *SNR* decreases, the optimum value of the forgetting factor increases to enlarge the

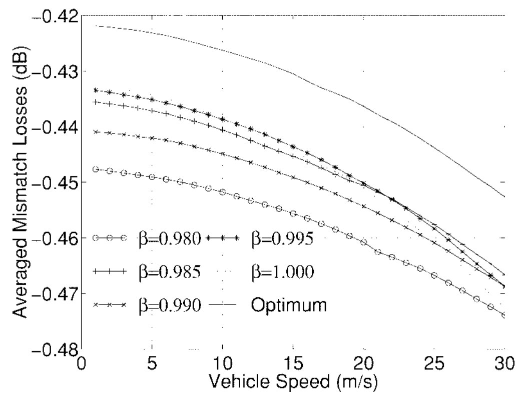


Fig. 7. Averaged mismatch losses for various values of the forgetting factors when the *SNR* is 5 dB.

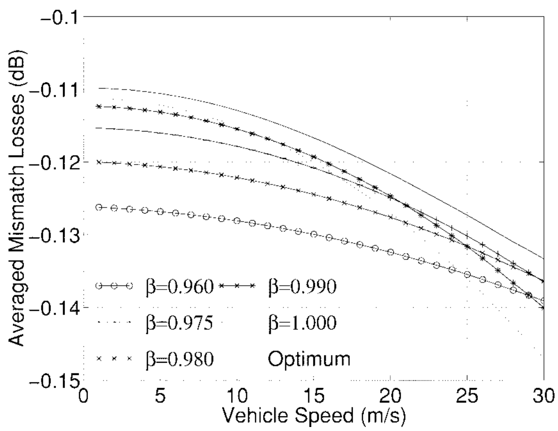


Fig. 8. Averaged mismatch losses for various values of the forgetting factors when the *SNR* is 10 dB.

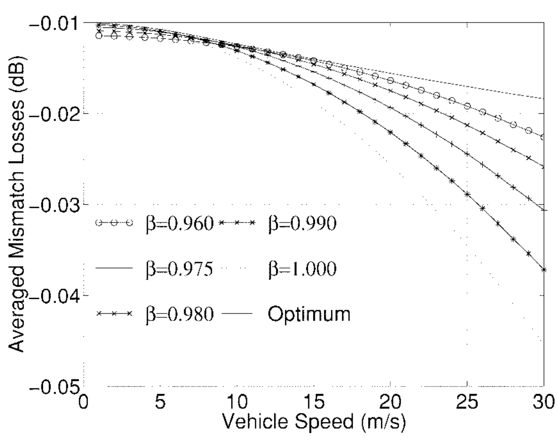


Fig. 9. Averaged mismatch losses for various values of the forgetting factors when the *SNR* is 20 dB.

effective number of samples. Thus, the optimum value of the forgetting factor decreases as the mobile moves faster and the variation decreases as the *SNR* decreases.

In Figs. 7–9, the mismatch losses for various values of the forgetting factor are plotted when the *SNR* is 5, 10, and 20 dB, respectively. We can see that the loss increases as the mobile moves faster and the *SNR* decreases. It is also observed that we can estimate the time-varying channel vectors quite exactly by the proposed algorithm, and that the degradation is not severe even if we do not know the optimum value of the forgetting factor.

## VII. Concluding Remarks

In this paper, we proposed a quasi-ML detector employing antenna arrays in asynchronous channels. We first considered the case where the channel vector was time-invariant and extended it to the case where the channel vector was time-varying. The proposed quasi-ML detector can be considered as a system which performs beamforming first and decorrelating later, while the conventional system performs decorrelating first and beamforming later.

We proposed a method based on the eigendecomposition of the correlation matrix of an inverse-filtered signal to estimate the channel parameters. A fast recursive algorithm was also proposed to estimate the time-varying channel vectors. The simulation results showed that the proposed algorithm estimated the channel vector within a 0.5-dB loss even in severe propagation environments.

We analyzed the performance of the proposed system and observed that the enhancement and correlation of noise, the major drawbacks of the conventional decorrelator system, could be reduced. We obtained some performance gain over the conventional decorrelator system, and the gain got larger as the numbers of active users and antennas increased. Thus, we expect considerably higher capacity with the proposed system than that with the conventional decorrelator system.

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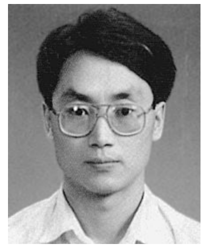

**Iickho Song** (S'80–M'87–SM'96) was born in Seoul, Korea, on February 20, 1960. He received the B.S. (magna cum laude) and M.S.E. degrees in electronics engineering from Seoul National University, Seoul, Korea, in 1982 and 1984, respectively. He also received the M.S.E. and Ph.D. degrees in electrical engineering from the University of Pennsylvania, Philadelphia, PA, in 1985 and 1987, respectively.

He was a member of Technical Staff at Bell Communications Research, Morristown, NJ, from March 1987 to February 1988. In March 1988, he joined the Department of Electrical Engineering, the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, as an Assistant Professor, where he became an Associate Professor and Professor, in 1991 and 1998, respectively. From June through November 1992, he was a Teaching Fellow at the National University of Singapore, Singapore. From December 1996 through December 1997, he was a Visiting Associate Professor at the Communications Research Laboratory, Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON. He served as an Editor for the Journal of the Acoustical Society of Korea (ASK) in 1990, an Associate Editor for the Journal of the Korean Institute of Communication Sciences (KICS) since February 1995, an Associate Editor for the Journal of the ASK (English Edition) since January 1996, and an Editor for the Journal of Communications and Networks since January 1998. He was listed in the 15th and 16th editions of *Marquis Who's Who in the World*, in 1998 and 1999, respectively. His research interests include detection and estimation theory, statistical signal and image processing, and statistical communication theory.

Dr. Song is also a member of the ASK, Institution of Electrical Engineers (IEE), Korean Institute of Electronics Engineers of Korea (IEEK), and KICS. In 1989, Dr. Song served as the Treasurer of the IEEE Korea Section, He was a recipient of the Korean Honor Scholarship in 1985 and 1986, and the Korean American Scholarship in 1986. He received Academic Awards from the KICS in 1991 and 1996, the Best Research Award from the ASK in 1993, and the LG Academic Award from the KICS in 1998.

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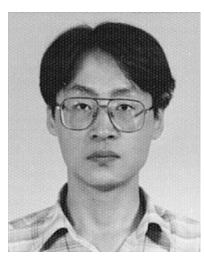
Her current research interests include mobile communications, spread-spectrum systems, and statistical signal processing.

Ms. Kim received the Minister of Science and Technology Award in February 1995.



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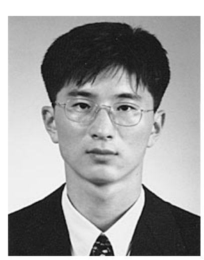
He was a member of Technical Staff at Samsung Electronics Company Ltd., Seoul, Korea, from September 1986 to January 1998. Since March 1998, he has been with the Division of Electronics Engineering, Hallym University, Chunchon, Korea, where he is currently an Assistant Professor. His research interests include array signal processing, adaptive array antenna, and mobile communication theory.



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His research interests include signal estimation and array signal processing.