Design and Analysis of Concatenated Coded DS/CDMA Systems in Asynchronous Channels

Kwang Soon Kim, *Member, IEEE,* Sun Yong Kim, *Member, IEEE,*
Suk Chan Kim, and Iickho Song, *Senior Member, IEEE*

***Abstract*—In this paper, we propose and analyze concatenated coding schemes for direct-sequence code-division multiple-access (DS/CDMA) systems in asynchronous channels. In the concatenated coding, bandwidth-efficient 2^{2L-2}-state $L/(L+1)$-rate 2-MTCM with biorthogonal signal constellation is used for the inner code, and $(2^L-1, \lceil(2^L-1)/L/2\rceil)$ RS code is used for the outer code.**

It is shown that we can get considerable performance gain over the uncoded system without sacrificing the data transmission rate. The proposed system can be used as a coding scheme for reliable and high-speed integrated information services of mobile communication systems.

***Index Terms*— Concatenated code, direct-sequence code-division multiple access, trellis-coded modulation.**

I. Introduction

RECENTLY, the design and analysis of direct-sequence code-division multiple-access (DS/CDMA) systems has become one of the most highlighted research areas because of its highly useful applications to mobile communication. Although the capacities of CDMA systems are higher than those of others [1], [2], some additional techniques are required to meet the explosively increasing demand for mobile communication. In addition, transmitted data in the near future will be composed not only of voice data, but also of images and integrated information. Thus, faster and more reliable communication techniques are desired.

In order to get more reliable communication systems, various coding schemes have been widely considered. Among these coding schemes, convolutional codes have been frequently used for DS/CDMA systems. For voice communications, the required bit error rate (BER) is about 10^{-3}, which can be achieved with convolutional codes with moderate values of constraint length. In mobile data communication systems, however, the required BER is in the range of 10^{-6}–10^{-9}. Thus, convolutional codes with a very long constraint length are necessary to meet the required BER, and the complexity of the decoder accordingly grows so high that the implementation of the decoder becomes impractical. In addition, the bandwidth efficiency of the coded DS/CDMA systems becomes very poor because of the low code rate.

In order to achieve large coding gain and good bandwidth efficiency with practically acceptable complexity, concatenated coding schemes have been studied in [3]–[7]. It has been shown that concatenated coding can not only achieve considerable coding gain over uncoded systems without bandwidth expansion but also resist both random and burst errors. It can be implemented with bandwidth-efficient trellis-coded modulation (TCM) schemes as the inner code and burst error resistant Reed–Solomon (RS) codes as the outer code.

In wireless DS/CDMA systems, the amplitude of a received signal is severely distorted because of fading. Thus, such signal constellations based on the amplitude of a signal as the quadrature amplitude modulation are not suitable. Among the constant amplitude signal constellations, phase shift keying (PSK) signal constellations are widely used. In DS/CDMA systems, however, the existence of interuser interference severely degrades the performance of the inner TCM using PSK signal constellations even when the signal-to-noise ratio (SNR) is very high, which becomes more severe as the code rate of the inner TCM increases: this is due to the reduced Euclidean distance between the signal points in the PSK signal constellation. In [8]–[11], TCM in DS/CDMA systems using biorthogonal signal constellations was investigated. In this system, the Euclidean distance is not reduced when the code rate increases if we allow as many signature sequences as the number of the signal points of the signal constellation of the trellis code, since the signal points are biorthogonal. The cost for the increased code rate is the increased number of signature sequences used for each user.

In this paper, we propose concatenated coding schemes in DS/CDMA systems using bandwidth-efficient two-multidimensional TCM (MTCM) with biorthogonal signal constellations as the inner code and the burst error resistant RS code as the outer code. The performance of the system is investigated and some properties of the proposed system are described.

II. System Model

To use concatenated coding in a DS/CDMA system, we first select its inner and outer codes. For the inner code, TCM is widely used because of its bandwidth efficiency. For the outer code, RS code can be used to combat the burst errors which may have occurred in the inner code decoding.

Manuscript received March 15, 1998; revised November 20, 1998. This work was supported by the Hallym Academy of Science, Hallym University, Korea, under a 1998 grant.

K. S. Kim, S. C. Kim, and I. Song are with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, Daejeon, Korea.

S. Y. Kim is with the Department of Electronics Engineering, Hallym University, Chuncheon, Kangwon Do, Korea.

Publisher Item Identifier S 0018-9545(99)05735-7.

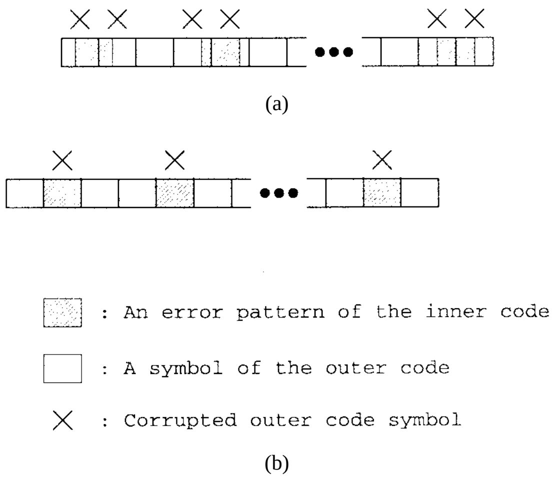

Fig. 1. Relation between the symbols of the outer code and the error patterns of the inner code: (a) the symbols of the outer code are not matched to the error patterns of the inner code and (b) the symbols of the outer code are matched to the error patterns of the inner code.

To maximize the performance of the concatenated coding system, the symbol size of the outer code should be matched with the error pattern of the inner code. Otherwise, the same errors occurred in the inner code can cause more outer code errors (see Fig. 1). The byte-oriented nature of the MTCM can be used to match the outer code symbol size with the error pattern of the inner code [3], [12].

To implement the multidimensionality of MTCM, time division is frequently used. In DS/CDMA systems, nearly orthogonal signature sequences can instead be used. A $2^{r(q+1)}$-ary biorthogonal constellation can be constructed by assigning r time slots and 2^q signature sequences to each user. If r is too small, the number of signature sequences required for a user increases, which results in capacity restriction of the system. On the contrary, if r is too large, the bandwidth required for the system increases, which can also restrict the performance of the system. In this paper, we consider a $2^{2(q+1)}$-ary biorthogonal modulation scheme with two time slots and 2^q signature sequences for each time slot.

Let 2^m-state $L/(L+1)$-rate 2-MTCM be used for the inner code. Then, the 2^{L+1}-ary signal constellation is required. Since the number of time slots is two, we can implement this scheme with a ($2^{(L+1)/2}$-ary) $\times$ ($2^{(L+1)/2}$-ary) signal constellation if L is an odd number. If L is an even number, we can select 2^{L+1} signal points from a ($2^{(L/2+1}$-ary) $\times$ ($2^{L/2+1}$-ary) signal constellation as follows: let the signature sequence set for a user be divided into two groups. When we transmit a signal, we can choose a signature sequence set only from the first (second) group in the second time slot if we have chosen a signature sequence set in the first (second) group in the first time slot.

Let $q = \lfloor L/2 \rfloor$, where $\lfloor a \rfloor$ is the largest integer not exceeding a. Then, the 2^{q+1}-ary biorthogonal signal constellation can be constructed by assigning sets of signature sequences to each user which satisfy:

- signature sequences in a set satisfy the distance properties of a biorthogonal signal constellation;
- signature sequences in different sets are orthogonal.

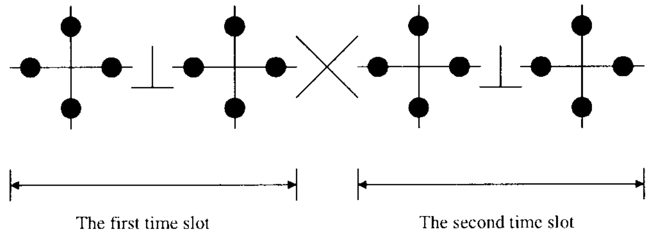

Fig. 2. The 64-ary biorthogonal constellation.

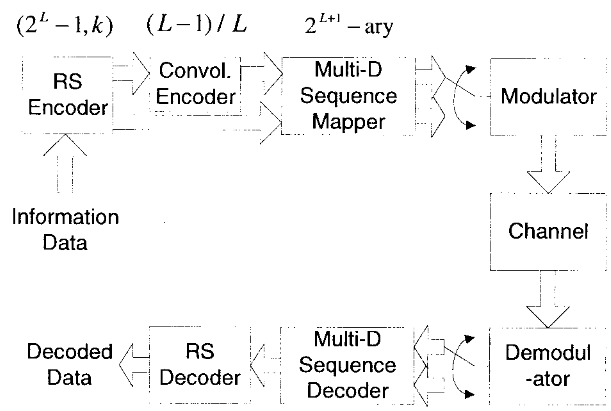

Fig. 3. A block diagram of the proposed system.

A construction method of such signature sequences are discussed in [8]: this method, however, is not useful in asynchronous system. In this case, we can implement the biorthogonal constellation by assigning 2^q nearly orthogonal pseudonoise (PN) sequences to each user. An example of the 64-ary biorthogonal constellation is shown in Fig. 2.

Let the (n, k) RS code be used for the outer code. To maximize the performance of the concatenated coding system, the matching between the outer code symbols with the error patterns of the inner code mentioned earlier is required. In this case

$$n = 2^L - 1 \tag{1}$$

and since we use two time slots for a symbol, the effective code rate for the overall concatenated coding system is

$$R_{\text{eff}} = \frac{kL/2}{2^L - 1}. \tag{2}$$

The concatenated coding scheme with the $(2^L - 1, k)$ outer RS code and 2^m-state $L/(L+1)$-rate 2-MTCM inner code in DS/CDMA systems considered in this paper is shown in Fig. 3.

III. Concatenated Coded DS/CDMA System

To design a concatenated coded DS/CDMA system, we should determine some parameters (n and k for the outer RS code and L and m for the inner TCM). In some cases, we may want to transmit the information at least as fast as the uncoded system. In other words, the effective code rate should be equal to or greater than one. If we select the effective code rate R_{eff}, the parameter k can be obtained as

$$k = \left\lceil \frac{(2^L - 1)R_{\mathrm{eff}}}{L/2} \right\rceil \tag{3}$$

where $\lceil a \rceil$ is the smallest integer not less than a. If the exact value of the effective code rate is not of our interest but is only required that $R_{\mathrm{eff}} \geq 1$, k can be chosen to minimize the RS decoding error probability as

$$k = \left\lceil \frac{2^L - 1}{L/2} \right\rceil. \tag{4}$$

From (1) and (3) [or (4)], we can determine n and k of the outer RS code from the value of L. Then, we are to choose only two parameters, m and L of the inner TCM.

Now, let us consider the 2^{L+1}-ary biorthogonal signal constellation. We assume that the orthogonality among the signature sequences is perfect for simplicity. Then, the number of different chips is N for antipodal signals and $N/2$ for orthogonal signals, where N is the length of a signature sequence. Let p be the desired signal point, $\tilde{p}$ be an arbitrary signal point, and

$$d_{p,\tilde{p}}^2 = \frac{2}{N} N_{\tilde{p}} \tag{5}$$

be the normalized Euclidean distance $d_{p,\tilde{p}}^2$ between p and $\tilde{p}$, where $N_{\tilde{p}}$ is the number of different chips between p and $\tilde{p}$. (We will use d^2 instead of $d_{p,\tilde{p}}^2$ for simplicity whenever it does not cause any confusion.) Then, the following properties hold.

Property 1: The normalized Euclidean distance d^2 between signal points can have a value in $\{0, 1, 2, 3, 4\}$.

Property 2: The number of signal points whose normalized Euclidean distance is d^2 is

$$\begin{cases} 1, & \text{when } d^2 = 0 \\ 2^{\lceil L/2 \rceil + 1} - 4, & \text{when } d^2 = 1 \\ 2^{L+1} - 2^{\lceil L/2 \rceil + 2} + 6, & \text{when } d^2 = 2 \\ 2^{\lceil L/2 \rceil + 1} - 4, & \text{when } d^2 = 3 \\ 1, & \text{when } d^2 = 4. \end{cases} \tag{6}$$

From Properties 1 and 2, we can see that only one signal point has the maximum distance from the desired signal. That gives us a motivation to use a TCM scheme with one uncoded bit. In this paper, we will consider the 2^m-state $L/(L+1)$-rate 2-MTCM with one uncoded bit. The $L-1$ information bits are convolutionally encoded to select a coset and the one uncoded bit selects a signal point in the coset. In [8], it has been shown that there is an analogous trellis code based on a Q-ary biorthogonal signal constellation for every trellis code based on Q-ary PSK signal constellation such that the free distance of the trellis code using the biorthogonal signal constellation is at least as large as that of the trellis code using the PSK signal constellation. Thus, we can construct a trellis code using biorthogonal signal constellation with well-known good trellis codes using PSK signal constellation. As an example, the eight-state $\frac{3}{4}$-rate two-dimensional (2-D) trellis code is shown in Fig. 4.

Let d_f^2 be the free distance, C_f be the set of error events whose distance is the free distance, N_f be the number of paths

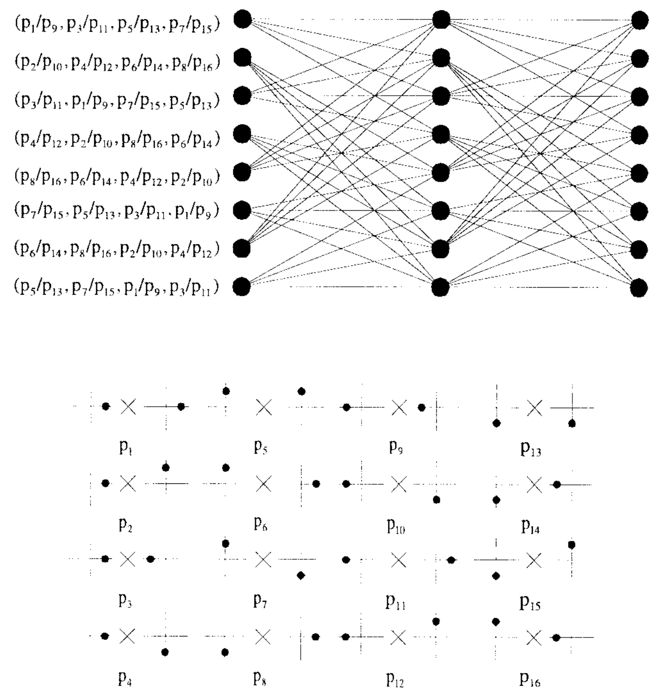

Fig. 4. The eight-state 3/4-rate trellis code over 16-ary biorthogonal signal constellation.

in C_f, and $W_{p,\tilde{p}}$ be the number of different branches between p and $\tilde{p}$. Then, the following properties hold.

Property 3: Let $L \leq m \leq 2L-2$, then $d_f^2 = 4$, $N_f = 2^{2L-m} - 3$, and $\sum_{\tilde{p} \in C_f} W_{p,\tilde{p}} = 2^{2L-m+1} - 7$.

Property 4: Let $m \geq 2L-2$, then $d_f^2 = 4$, $N_f = 1$, and $\sum_{\tilde{p} \in C_f} W_{p,\tilde{p}} = 1$.

Properties 3 and 4 imply that we can improve the performance of the TCM by increasing the number of states until $m = 2L-2$, and there will be no asymptotic advantage when $m > 2L-2$.

Now, consider the complexity of the decoder of the proposed concatenated system. The complexity of the decoder of the 2^m-state $L/(L+1)$-rate TCM is proportional to 2^m. Thus, the complexity of the decoder of the concatenated system increases as m increases. The complexity of the decoder of the (n, k) RS code depends mostly on two factors: one is the value of $n-k$ and the other is the value of n. In the proposed $(2^L-1, \lceil (2^L-1)/(L/2) \rceil)$ RS code, the complexity of the decoder depends only on L. Thus, the complexity of the decoder of the concatenated system increases as L increases. Let us assume that the value of m is fixed. Then, our next goal is to find the value of L which provides the best performance of the proposed concatenated code. In other words, for a given number of states of the inner TCM, we should determine the most efficient value of L in the bit error probability sense. We have shown in Properties 3 and 4 that it is wasteful to choose $m > 2L-2$. In addition, in most cases of trellis codes practically used, m is larger than L. Thus, it is reasonable to restrict the value of L to the range of $(m+2)/2 \leq L \leq m$ in our consideration. It will be shown in Section IV that the smallest value of L among the possible values is the most efficient one in the bit error probability sense in most practical

TABLE I
THE SYSTEM PARAMETERS OF THE PROPOSED SYSTEM

L	$m = 2L - 2$	$n = 2^L - 1$	$k = \lceil \frac{2^L - 1}{L/2} \rceil$
3	4	7	5
4	6	15	8
5	8	31	13
6	10	63	21

cases. Furthermore, it is also the best choice in the complexity sense since the complexity of the decoder of the RS code decreases as L decreases. Therefore, the most efficient value of L is $\lceil (m+2)/2 \rceil$ in both the bit error probability and complexity senses, if the value of m is given. Conversely, the value of m can be given as $m = 2L - 2$, when the value of L is given. Since one is determined by the other, from now on, only L is taken into account. If L increases, the corresponding value of m also increases. Thus, it is easily seen that the complexity of the proposed concatenated system increases as L increases. The system parameters of the proposed system are summarized in Table I for easy reference.

IV. PERFORMANCE ANALYSIS

The RS symbol error probability P_s can be written as

$$P_s \approx \sum_{\tilde{p} \in C_f} W_{p,\tilde{p}} P_{p \to \tilde{p}} \tag{7}$$

where $P_{p \to \tilde{p}}$ is the probability of selecting $\tilde{p}$ when p is the transmitted path. Then, the decoded RS symbol error probability P_e can be approximated as [13]

$$P_e \approx \sum_{t=\lfloor (n-k)/2 \rfloor + 1}^{n} \frac{t}{n} \binom{n}{k} P_s^t (1 - P_s)^{n-t} \tag{8}$$

and the upper bound of the bit error probability P_b is

$$P_b \leq k P_e. \tag{9}$$

We assume that the channel is slow Rayleigh fading and the fading process is constant over the duration of an error event and independent of the fading processes of other users. In the following analysis, we use the notations described in [8]. Without loss of generality, we can assume that the first user is the desired user. Let $z_{p,\tilde{p}}(j)$ be the decision statistic of the jth time slot of the first user, $j = 1, 2$. Then, the decision statistic to be used in deciding between the paths p and $\tilde{p}$ can be written as

$$z_{p,\tilde{p}} = \sum_{j=1}^{2} z_{p,\tilde{p}}(j). \tag{10}$$

According to the results in [8], [14], we have

$$E\{z_{p,\tilde{p}}(j) | \rho_1\} = \sqrt{\frac{P}{2}}\, \rho_1 N_{\tilde{p}}(j) T_c \tag{11}$$

and

$$\mathrm{Var}\{z_{p,\tilde{p}}(j) | \rho_1\} = \sum_{k=2}^{K} \frac{P T_c^2 E\{\rho_k^2\} N_{\tilde{p}}(j)}{6} + \frac{N_0 T_c N_{\tilde{p}}(j)}{4} \tag{12}$$

where P is the transmitted power, T_c is the chip duration of the signature sequence, $N_{\tilde{p}}(j)$ is the number of different chips between paths p and $\tilde{p}$ in the jth time slot, the Rayleigh random variable ρ_k is the fading process of the kth user, K is the number of users, and $N_0/2$ is the two-sided power spectral density of the background noise. Since we use a biorthogonal constellation, $N_{\tilde{p}} = N$ for antipodal signals and $N_{\tilde{p}} \approx N/2$ for orthogonal signals. If we assume an appropriate power control, we can set $E\{\rho_k^2\} = 1$, for $k = 1, 2, \cdots, K$. Then, the conditional expectation and variance of $z_{p,\tilde{p}}$ are

$$E\{z_{p,\tilde{p}} | \rho_1\} = \sqrt{\frac{P}{2}}\, \rho_1 N_{\tilde{p}} T_c \tag{13}$$

and

$$\mathrm{Var}\{z_{p,\tilde{p}} | \rho_1\} = \frac{(K-1) P T_c^2 N_{\tilde{p}}}{6} + \frac{N_0 T_c N_{\tilde{p}}}{4} \tag{14}$$

where $N_{\tilde{p}} = \sum_{j=1}^{2} N_{\tilde{p}}(j)$ and the instantaneous SNR $\nu(\rho_1)$ is

$$\nu(\rho_1) = \frac{\dfrac{N_{\tilde{p}} \rho_1^2}{N}}{\dfrac{N_0}{E_s} + \dfrac{2(K-1)}{3N}} \tag{15}$$

where $E_s = PNT_c$. Note that the first term in the right-hand side of (14) is due to the multiple access and the second term is due to the background Gaussian noise. Since the interuser interference can be approximated as a Gaussian variable by the central limit theorem, the conditional error probability $P_{p \to \tilde{p}}(\rho_1)$ can be obtained as

$$\begin{aligned} P_{p \to \tilde{p}}(\rho_1) &= \Pr(z_{p,\tilde{p}} < 0 | \rho_1) \\ &= \tfrac{1}{2}\, \mathrm{erfc}\left(\sqrt{\nu(\rho_1)}\right) \end{aligned} \tag{16}$$

and the error probability can be obtained by averaging (16)

$$\begin{aligned} P_{p \to \tilde{p}} &= E\{P_{p \to \tilde{p}}(\rho_1)\} \\ &= \int_0^{\infty} P_{p \to \tilde{p}}(x) f_{\rho_1}(x)\, dx \\ &= \frac{1}{2}\left(1 - \sqrt{\frac{\kappa}{1+\kappa}}\right) \end{aligned} \tag{17}$$

where $\kappa = \{(N_{\tilde{p}}/N)/(N_0/E_s) + [2(K-1)/3N]\}$.

In Figs. 5–7, the bit error probability of the proposed system for various values of L is plotted in the asymptotic case ($N_0 \to 0$) when $R_{\mathrm{eff}} = 1$, the length N of the signature sequence is 1024, and the number K of users is 5, 10, and 20, respectively. We can clearly see that the smallest value of L becomes the most efficient value as the channel becomes worse (i.e., K increases) for a fixed value of m. This can be explained as follows. For a given number of states, the performance of the inner TCM degrades while that of the outer RS code improves as L increases and the performance improvement of the outer RS code can overcome the degradation in the inner TCM only when the number of users is small and the SNR is high. Thus, the smallest value of L is a good choice in the bit error probability sense for most practical situations. In addition, the smallest value of L is also the best choice in the complexity sense for fixed m, as we explained

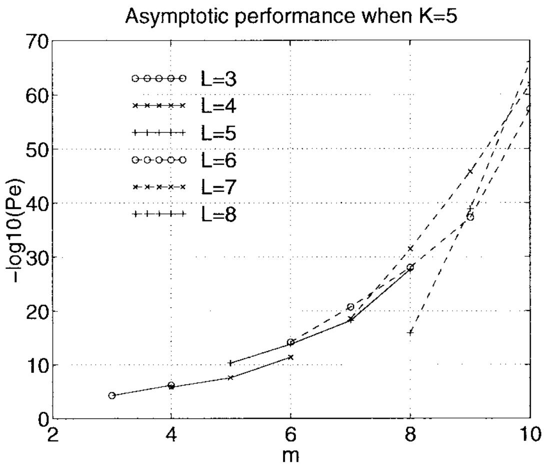

Fig. 5. Asymptotic performance of the proposed system when the number K of users is five.

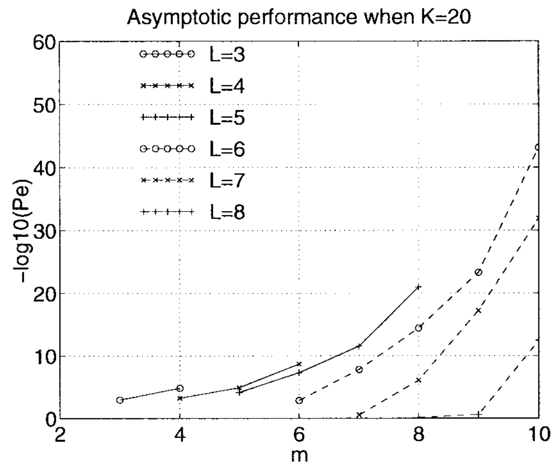

Fig. 7. Asymptotic performance of the proposed system when the number K of users is 20.

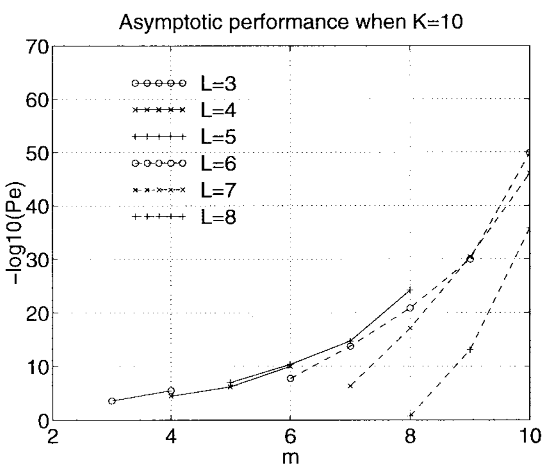

Fig. 6. Asymptotic performance of the proposed system when the number K of users is ten.

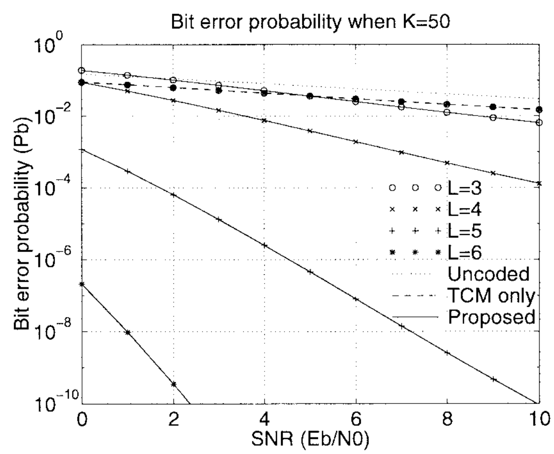

Fig. 8. The bit error probability of the proposed system for various values of L when $K = 50$.

in Section III also. Therefore, 2^{2L-2}-state $L/(L+1)$-rate 2-MTCM is selected for the inner code.

In Fig. 8, the upper bounds of the bit error probabilities of the $(2^L - 1, \lceil (2^L - 1)/(L/2) \rceil)$ RS code and 2^{2L-2}-state $L/(L+1)$-rate 2-MTCM with one uncoded bit are plotted for various values of L when $R_{\text{eff}} = 1$, $N = 1024$, and $K = 50$. We can see that considerable coding gain can be achieved without sacrificing the data rate and that the coding gains over the uncoded system and over the system using only the inner TCM get larger as L increases. The cost for the performance gain are the increased complexity and increased number of signature sequences for each user.

V. Concluding Remark

In this paper, we proposed a concatenated coding scheme for DS/CDMA systems using 2-MTCM with biorthogonal signal constellation as the inner code and RS code as the outer code. In the proposed system, we used the bandwidth-efficient 2^{2L-2}-state $L/(L+1)$-rate 2-MTCM with one uncoded bit for the inner code and $(2^L - 1, \lceil (2^L - 1)/(L/2) \rceil)$ RS code for the outer code. We investigated the properties and performance of the proposed system. The performance of the proposed system increases without loss of data transmission rate at the cost of increased complexity, which is expected to be easily implemented with the progress in microelectronic technology in the near future. Performance analysis results indicate that the proposed system is much more reliable than the uncoded system without sacrificing data rate and that we can achieve bit error probability in the range of 10^{-6}–10^{-9} at SNR 5–9 dB when $L = 5$. Thus, the proposed system is a good method of data transmission with high reliability and high speed in future DS/CDMA systems.

Appendix

Proof of Property 1: Let $p = (p_1 \times p_2)$ be the desired signal point and $\tilde{p} = (\tilde{p}_1 \times \tilde{p}_2)$ be a signal point. Then $\tilde{p}$ should be in one of the following nine sets with respect to p:

$$A_0 = \{\tilde{p} | p_1 = \tilde{p}_1,\, p_2 = \tilde{p}_2\} \tag{18}$$

$$A_1 = \{\tilde{p} | p_1 \perp \tilde{p}_1,\, p_2 = \tilde{p}_2\} \tag{19}$$

$$A_2 = \{\tilde{p} | p_1 = \tilde{p}_1,\, p_2 \perp \tilde{p}_2\} \tag{20}$$

$$A_3 = \{\tilde{p} | p_1 \perp \tilde{p}_1,\, p_2 \perp \tilde{p}_2\} \tag{21}$$

$$A_4 = \{\tilde{p} | p_1 = \tilde{p}_1,\, p_2 = -\tilde{p}_2\} \tag{22}$$
$$A_5 = \{\tilde{p} | p_1 = -\tilde{p}_1,\, p_2 = \tilde{p}_2\} \tag{23}$$
$$A_6 = \{\tilde{p} | p_1 \perp \tilde{p}_1,\, p_2 = -\tilde{p}_2\} \tag{24}$$
$$A_7 = \{\tilde{p} | p_1 = -\tilde{p}_1,\, p_2 \perp \tilde{p}_2\} \tag{25}$$

and

$$A_8 = \{\tilde{p} | p_1 = -\tilde{p}_1,\, p_2 = -\tilde{p}_2\} \tag{26}$$

where $a \perp b$ represents that a and b are orthogonal. Since the normalized Euclidean distance between orthogonal signals is one and that between antipodal signals is two, it is easily seen that the normalized Euclidean distance between p and $\tilde{p}$ is in $\{0, 1, 2, 3, 4\}$ ■.

Proof of Property 2: It is clear that there exists only one desired signal point and one antipodal signal point. First, we assume that L is an odd number. Since there are four signal points in one dimension, the number of dimensions in one time slot is $2^{(L+1)/2} \times \frac{1}{4} = 2^{(L-3)/2}$. Thus, the number of the signal points which is orthogonal to p_1 in the first time slot can be obtained as

$$\begin{aligned}
&(\#\text{ of orthogonal points})\\
&= (\#\text{ of points in one dim.})\\
&\quad \times (\#\text{ of dim.} - 1)\\
&\quad + (\#\text{ of orthogonal points in the same dim.})\\
&= 4 \times (2^{(L-3)/2} - 1) + 2\\
&= 2^{(L+1)/2} - 2.
\end{aligned} \tag{27}$$

Thus, it can be easily seen that

$$\begin{aligned}
\mathrm{Card}(\{\tilde{p} | d^2_{p,\tilde{p}} = 0\}) &= \mathrm{Card}(A_0)\\
&= 1
\end{aligned} \tag{28}$$
$$\begin{aligned}
\mathrm{Card}(\{\tilde{p}|^2_{p,\tilde{p}} = 1\}) &= \mathrm{Card}(A_1 \cup A_2)\\
&= 2^{(L+3)/2} - 4\\
&= 2^{\lceil L/2 \rceil + 1} - 4
\end{aligned} \tag{29}$$
$$\begin{aligned}
\mathrm{Card}(\{\tilde{p} | d^2_{p,\tilde{p}} = 2\}) &= \mathrm{Card}(A_3 \cup A_4 \cup A_5)\\
&= 2^{L+1} - 2^{(L+5)/2} + 6\\
&= 2^{L+1} - 2^{\lceil L/2 \rceil + 2} + 6
\end{aligned} \tag{30}$$
$$\begin{aligned}
\mathrm{Card}(\{\tilde{p} | d^2_{p,\tilde{p}} = 3\}) &= \mathrm{Card}(A_6 \cup A_7)\\
&= 2^{(L+3)/2} - 4\\
&= 2^{\lceil L/2 \rceil + 1} - 4
\end{aligned} \tag{31}$$

and

$$\begin{aligned}
\mathrm{Card}(\{\tilde{p} | d^2_{p,\tilde{p}} = 4\}) &= \mathrm{Card}(A_8)\\
&= 1
\end{aligned} \tag{32}$$

where $\mathrm{Card}(A)$ is the cardinality of a set A. If we assume that L is an even number, it is again clear that the numbers of points whose Euclidean distances are zero and four are both one. The number of points which is orthogonal to p_1 in the first time slot is

$$\begin{aligned}
&(\#\text{ of orthogonal points})\\
&= (\#\text{ of points in one dim.})\\
&\quad \times (\#\text{ of dim. in the first group} - 1)\\
&\quad + (\#\text{ of points in one dim.})\\
&\quad \times (\#\text{ of dim. in the second group})\\
&\quad + (\#\text{ of orthogonal points in the same dim.})\\
&= 4 \times (2^{(L/2)-2} - 1) + 4 \times 2^{(L/2)-2} + 2\\
&= 2^{(L/2)+1} - 2.
\end{aligned} \tag{33}$$

Then, it is also easily seen that

$$\begin{aligned}
\mathrm{Card}(\{\tilde{p} | d^2_{p,\tilde{p}} = 0\}) &= \mathrm{Card}(A_0)\\
&= 1
\end{aligned} \tag{34}$$
$$\begin{aligned}
\mathrm{Card}\big(\{\tilde{p} | d^2_{p,\tilde{p}} = 1\}\big) &= \mathrm{Card}(A_1 \cup A_2)\\
&= 2^{(L+2)/2} - 4\\
&= 2^{\lceil L/2 \rceil + 1} - 4
\end{aligned} \tag{35}$$
$$\begin{aligned}
\mathrm{Card}(\{\tilde{p} | d^2_{p,\tilde{p}} = 2\}) &= \mathrm{Card}(A_3 \cup A_4 \cup A_5)\\
&= 2^{L+1} - 2^{(L+4)/2} + 6\\
&= 2^{L+1} - 2^{\lceil L/2 \rceil + 2} + 6
\end{aligned} \tag{36}$$
$$\begin{aligned}
\mathrm{Card}(\{\tilde{p} | d^2_{p,\tilde{p}} = 3\}) &= \mathrm{Card}(A_6 \cup A_7)\\
&= 2^{(L+2)/2} - 4\\
&= 2^{\lceil L/2 \rceil + 1} - 4
\end{aligned} \tag{37}$$

and

$$\begin{aligned}
\mathrm{Card}(\{\tilde{p} | d^2_{p,\tilde{p}} = 4\}) &= \mathrm{Card}(A_8)\\
&= 1.
\end{aligned} \tag{38}$$

Thus, Property 2 holds for both odd and even values of L. ■

Proof of Property 3: The number of states is 2^m and the number of transitions leaving a state is 2^L. Since there is one parallel transition, one branch consists of two transitions. Thus, the number of branches leaving a state is 2^{L-1}. Let us assume that the states are indexed from zero to $2^m - 1$ and the starting state is the zeroth state. Since the normalized Euclidean distance between the parallel transitions in each branch is four, from Property 2, it is easily seen that the number of branches, in which the normalized Euclidean distances of the two transitions are both two, is $2^L - 2^{\lceil L/2 \rceil + 1} + 3$. Thus, the number of this kind of branches is greater than the number of branches leaving a state, and we can assign each transition from the starting state to another state with the transition whose normalized Euclidean distance is two. Similarly, we can assign each transition from one state to the starting state with the transition whose normalized Euclidean distance is two. Then, it is clear that an error event consisting of more than three branches is not a free distance error event. Therefore, the free distance is $d_f^2 = 4$ and such error events are: 1) one parallel transition and 2) $2^{2L-m-2} - 1$ error events which consist of two branches. Since every branch consists of two transitions, $N_f = 1 + 4 \times (2^{2L-m-2} - 1) = 2^{2L-m} - 3$ and $\sum_{\tilde{p} \in C_f} W_{p,\tilde{p}} = 2 \times 4 \times (2^{2L-m-2} - 1) + 1 = 2^{2L-m+1} - 7$. ■

Proof of Property 4: From Property 3, we can see that $N_{free} = 1$ and $\sum_{\tilde{p} \in C_{free}} W_{p,\tilde{p}} = 1$ when $m = 2L - 2$. The only closest error event is the parallel path, and then, we cannot increase d_f^2 more by increasing the number of states. Therefore, $d_f^2 = 4$ and $N_f = \sum_{\tilde{p} \in C_f} W_{p,\tilde{p}} = 1$. ■

ACKNOWLEDGMENT

The authors also wish to thank the anonymous reviewers for their constructive and helpful comments and suggestions.

REFERENCES

[1] K. S. Gilhousen, I. M. Jacobs, R. Padovani, A. J. Viterbi, L. A. Weaver, and C. Wheatly, "On the capacity of a cellular CDMA system," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 303–312, May 1991.

[2] R. L. Pickholtz, L. B. Milstein, and D. L. Schilling, "Spread spectrum for mobile communications," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 313–322, May 1991.

[3] L.-N. Lee, "Concatenated coding systems employing a unit-memory convolutional code and a byte-oriented decoding algorithm," *IEEE Trans. Commun.*, vol. COM-25, pp. 1064–1074, Oct. 1977.

[4] J. Justesen, C. Thommesen, and V. V. Zyablov, "Concatenated codes with convolutional inner codes," *IEEE Trans. Inform. Theory*, vol. 34, pp. 1217–1225, Sept. 1988.

[5] T. Kasami, T. Takata, T. Fujiwara, and S. Lin, "A concatenated coded modulation scheme for error control," *IEEE Trans. Commun.*, vol. 38, pp. 752–763, June 1990.

[6] B. Vucetic, "Bandwidth efficient concatenated coding scheme for fading channels," *IEEE Trans. Commun.*, vol. 41, pp. 50–61, Jan. 1993.

[7] S. Rajpal, D. J. Rhee, and S. Lin, "Low complexity, high performance and bandwidth efficient concatenated coded 8-PSK schemes for reliable data communications," *IEEE Trans. Commun.*, vol. 43, pp. 785–794, Feb./Mar./Apr. 1995.

[8] B. D. Woerner, "Coded modulation for direct sequence spread spectrum communication," Ph.D. dissertation, Univ. Michigan, Ann Arbor, 1991.

[9] B. D. Woerner and W. E. Stark, "Trellis-coded direct-sequence spread-spectrum communications," *IEEE Trans. Commun.*, vol. 42, pp. 3161–3170, Dec. 1994.

[10] K. S. Kim, H. G. Kim, I. Song, S. R. Park, S. H. Yoon, and H. M. Kim, "A decorrelating multiuser receiver for DS/CDMA systems using TCM," in *Proc. European Signal Process. Conf.*, Rhodes, Greece, Sept. 1998, pp. 1985–1988.

[11] K. S. Kim, I. Song, H. G. Kim, and Y. H. Kim, "A multiuser receiver for trellis coded DS/CDMA systems in asynchronous channels," *IEEE Trans. Veh. Technol.*, to be published.

[12] S. S. Pietrobon, R. H. Deng, A. Lafanechere, G. Ungerboeck, and D. J. Costello, Jr., "Trellis-coded multidimensional phase modulation," *IEEE Trans. Inform. Theory*, vol. 36, pp. 63–89, Jan. 1990.

[13] S. Lin and D. J. Costello, Jr., *Error Control Coding.* Englewood Cliffs, NJ: Prentice-Hall, 1983.

[14] M. B. Pursley, "Performance evaluation of phase-coded spread spectrum multiple-access communication—Part I: System analysis," *IEEE Trans. Commun.*, vol. COM-25, pp. 795–799, Aug. 1977.

Kwang Soon Kim (S'95–M'99) was born in Seoul, Korea, on September 20, 1972. He received the B.S. (*summa cum laude*), M.S.E., and Ph.D. degrees in electrical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 1994, 1996, and 1999, respectively.

He was a Teaching and Research Assistant at the Department of Electrical Engineering, KAIST, from March 1994 to February 1999. He is currently a Research Fellow at the Department of Electrical Engineering, KAIST. Since March 1999, he has also been with the Department of Electrical and Computer Engineering, University of California at San Diego, La Jolla, as a Post-Doctoral Researcher. His research interests include detection and estimation theory, channel coding, array signal processing, mobile communications, and wireless/optical CDMA systems.

Dr. Kim is a member of the Korean Institute of Communication Sciences (KICS) and Institute of Electronics Engineers of Korea (IEEK). He was the recipient of the Silver Prizes from the Humantech Paper Contest and LG Information and Communications Paper Contest in 1998. He was the recipient of a Postdoctoral Fellowship from the Korea Science and Engineering Foundation (KOSEF) in 1999.

Sun Yong Kim (S'91–M'96) was born in Inchon, Korea, on January 30, 1968. He received the B.S.E. (*summa cum laude*), M.S.E., and Ph.D. degrees in electrical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejon, Korea, in 1990, 1993, and 1995, respectively.

From April 1995 to March 1996, he was a Visiting Researcher at the University of Tokyo, Tokyo, Japan. Since March 1996, he has been with the Department of Electronics Engineering, Hallym University, Chuncheon, Kangwon Do, Korea, where he is currently an Assistant Professor. His research interests include wireless communications and statistical signal processing.

Dr. Kim was a recipient of the IEEE Communication Society Scholarship for 1992/93. He is a member of the Korean Institute of Communication Sciences (KICS).

Suk Chan Kim was born in Namhae, Korea, on February 10, 1971. He received the B.S. (*summa cum laude*) degree in electronics engineering from Pusan National University, Pusan, Korea, in February 1993 and the M.S.E. degree in electrical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in February 1995. He is currently working toward the Ph.D. degree at KAIST.

He has been a Teaching and Research Assistant at the Department of Electrical Engineering, KAIST, since March 1993. His research interests are in mobile communications, signal processing, and networks.

Iickho Song (S'80–M'87–SM'96) was born in Seoul, Korea, on February 20, 1960. He received the B.S. (*magna cum laude*) and M.S.E. degrees in electronics engineering from Seoul National University, Seoul, Korea, in February 1982 and 1984, respectively. He also received the M.S.E. and Ph.D. degrees in electrical engineering from the University of Pennsylvania, Philadelphia, in August 1985 and May 1987, respectively.

He was a Member of Technical Staff at Bell Communications Research, Morristown, NJ, from March 1987 to February 1988. In March 1988, he joined the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, as an Assistant Professor, and became an Associate Professor in September 1991 and a Professor in September 1998. From June through November 1992, he was a Teaching Fellow at the National University of Singapore. From December 1996 through December 1997, he was a Visiting Associate Professor at the Communications Research Laboratory, Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON, Canada. He has served as an Editor of the *Journal of the Acoustical Society of Korea* (ASK) in 1990. He has served as an Associate Editor of the *Journal of the Korean Institute of Communication Sciences* (KICS) since February 1995, as an Associate Editor of the *Journal of the ASK* (English Edition) since January 1996, and as an Editor of the *Journal of Communications and Networks* since January 1998. His research interests include detection and estimation theory, statistical signal and image processing, and statistical communication theory.

Dr. Song served as the Treasurer of the IEEE Korea Section in 1989. He is a Member of the ASK, the Institution of Electrical Engineers (IEE), the Institute of Electronics Engineers of Korea (IEEK), and KICS. He was a recipient of the Korean Honor Scholarship in 1985 and 1986, and of the Korean American Scholarship in 1986. He received Academic Awards from the KICS in 1991 and 1996, the Best Research Award from the ASK in 1993, and the LG Academic Award from the KICS in 1998. He was listed in the 15th and 16th editions of *Marquis Who's Who in the World* in 1998 and 1999, respectively.