Delay and Energy Constrained Random Access Transport Capacity

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***Abstract*—In this paper, we consider a delay and energy constrained wireless *ad hoc* network with node density of λ_n, where a packet should be delivered to the destination within $D(\lambda_n)$ seconds using at most $E(\lambda_n)$ energy in joules while satisfying the target outage probability. The performance metric for analyzing the network is the delay and energy constrained random access transport capacity (DE-RATC), i.e., $C_\epsilon(D(\lambda_n), E(\lambda_n))$, which quantifies the maximum end-to-end distance weighted rate per unit area of a delay and energy constrained network using a random access protocol. It is shown that a slotted ALOHA protocol is order-optimal under any delay and energy constraints if equipped with additional features such as power control, multihop control, interference control, and rate control, and the delay and energy constraints can be divided into three regions according to the relation between the physical quantities due to the constraints and those due to the node density and network size. The three regions are the non-constrained (NC) region, where the DE-RATC is given by $\Theta(\sqrt{\lambda_n/\log \lambda_n})$; the delay-constrained (DC) region, where the DE-RATC depends only on the delay constraint as $\Theta(D(\lambda_n))$; and the non-achievable (NA) region where a packet delivery under the given constraints is impossible. Also, it is shown that an arbitrary tradeoff between the rate of each source node and the number of source nodes can be achieved while keeping the optimal capacity scaling as long as $\lambda_s = \Omega(\min(\sqrt{\lambda_n/\log \lambda_n}, D(\lambda_n)))$.**

***Index Terms*—*Ad hoc* networks, random access transport capacity, capacity scaling, delay constraint, energy constraint.**

I. Introduction

A wireless *ad hoc* network is an uncoordinated network in which nodes communicate with each other without a centralized control or wired infrastructure. Those wireless *ad hoc* networks have a wide range of applications in real time services such as battlefield in military communication, vehicular *ad hoc* networks (VANET), disaster relief, etc. [1], [2] and consist of nodes powered by batteries. Thus, in wireless *ad hoc* networks, it is an important issue to deliver data reliably to the destinations on time using a limited energy.

Ad hoc network performance has been typically analyzed in terms of the capacity scaling [3]–[14] which denotes the order of the growth of a scalar performance metric as the number of source nodes in the network increases. In [3], the optimal capacity scaling of an arbitrary static network was shown to be $\Theta(\sqrt{n})$, where n is the number of source nodes in a network and this can be obtained by using the nearest-neighbor routing. Also, the optimal capacity scaling of a random static network was shown to be $\Theta(\sqrt{n/\log n})$ which can be obtained by turning nodes on and off in a time division multiple access (TDMA) manner to reduce interference from other transmitters. After that seminal work, the capacity scaling of various network models and assumptions have been proposed and the media access control (MAC) protocols for achieving the optimal capacity scaling have been developed [4]–[11]. Specifically, it has been shown in literature that the capacity scaling of a random static network is affected by a delay constraint or an energy constraint, which are typically important for practical applications [12]–[18].

In [12], [13], the capacity scaling with a delay constraint was shown to be $\Theta(\bar{D})$ if the average packet transmission time, $\bar{D}$, is allowed to be $O(\sqrt{n/\log n})$ and the average energy used for packet transmission, $\bar{E}$, is allowed to be $\Theta(\bar{D})$. In [12], the capacity scaling was obtained in the fluid model, where the packet size can be arbitrarily scaled down so that the queueing delay at each node and the media access fairness issue among nodes within the maximum delay can be ignored. In [13], the capacity scaling was obtained in the constant packet size model, where the queueing delay and the fairness issue should be considered, and it was shown that the optimal capacity scaling in the fluid model can be achieved when the packet arrival rate of each node is $\Theta(\sqrt{1/(n \log n)})$. In [14], it was shown that, for a given constant delay constraint, the optimal capacity scaling in [12] can be obtained by using a *slotted ALOHA (SA) protocol* with interference control and rate control. In [15], the power efficiency (bit per second per Watt) of an *ad hoc* network using the protocol in [3] was shown to be $\Theta(n^{\alpha/2-1}/(\log n)^{\alpha/2})$, where $\alpha > 2$ denotes the path-loss exponent, which implies that $\bar{E} = \Theta((n/\log n)^{-(\alpha-1)/2})$ when the packet size is constant and the packet arrival rate at the source nodes is $\Theta(\sqrt{1/(n \log n)})$.

The delay constraint and the energy constraint can be given in various ways for different applications of *ad hoc* networks. However, the results in [12], [13] can be used only for the case when the energy constraint is $\Theta(\bar{D})$, the results in [14]

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is obtained only for the case when the delay constraint is $\Theta(1)$, and the results in [15] can be used only for the case when the average delay constraint is $\Theta(\sqrt{n/\log n})$. Thus, those previous work do not provide a full understanding of delay-energy constrained *ad hoc* networks. In addition to that, the media access control (MAC) protocol considered in [12], [13], [15] is a Genie-aided time division multiple access (TDMA) protocol, which is unrealistic for *ad hoc* networks. From the above observations, the following questions arise:

1) How does the capacity scaling change under various delay and energy constraints?
2) Is it possible to achieve the optimal scaling under various delay and energy constraints by using a MAC scheme based on a simple random access? If so, what will be the required additional features?

In this paper, we consider a random access *ad hoc* network with various delay and energy constraints in the constant packet size model. The MAC protocol considered in this paper is a slotted ALOHA protocol with additional features, such as power control (P), multi-hop control (M), rate control (R), and interference control (I) and those control schemes can be used individually or together as in [14]. For example, the SA with multi-hop, rate, and interference control (SA-MRI) protocol controls the number of hops, the transmission rate of each end-to-end (e2e) link, and the media access probability of the whole network. The SA with power, multi-hop, rate, and interference control (SA-PMRI) protocol controls the transmit power at each node, the number of hops, the transmission rate of each e2e link, and the media access probability of the whole network. The performance metric used in this paper is the delay-energy-constrained random access transport capacity (DE-RATC), $C_\epsilon(D(\lambda_s), E(\lambda_s))$, which quantifies the maximum e2e rate per unit area of a delay-energy constrained network where packets generated at each source should be delivered within $D(\lambda_s)$ seconds using at most $E(\lambda_s)$ joules satisfying the target outage probability ϵ, where λ_s denotes the average number of source nodes per unit area. The main contributions of this paper can be summarized as follows.

1) For the SA-PMRI protocol, the delay-energy constraint space is divided into three regions: the non-constrained (NC) region, the delay-constrained (DC) region, and the non-achievable (NA) region as will be shown in Fig. 2. In the NC region, the DE-RATC scaling is $\Theta(\sqrt{\lambda_n/\log\lambda_n})$ while the DE-RATC scaling depends only on the delay constraint and is given as $\Theta(D(\lambda_n))$ in the DC region. In the NA region, packets cannot be delivered under the delay and energy constraints.
2) The SA protocol is shown to be order-optimal under all delay and energy constraints with additional features such as power control, multi-hop control, interference control, and rate control.
3) Even when only a part of node act as source nodes and the other nodes just help them, the DE-RATC of the SA-PMRI protocol does not change as long as $\lambda_s = \Omega(\min(\sqrt{\lambda_n/\log\lambda_n}, D(\lambda_n)))$, where λ_s denotes the average number of source nodes per unit area.

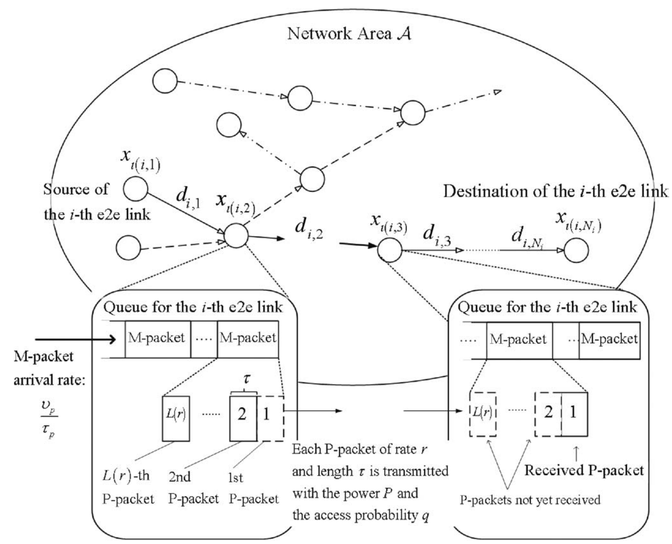

Fig. 1. Network model.

The remainder of this paper is organized as follows. In Section II, the network model and MAC protocols are introduced. In Section III, the DE-RATC and an upper bound of the DE-RATC are derived. In Section IV, the DE-RATC scaling of the SA-PMRI protocol is described. Finally, concluding remarks are given in Section V.

Notations:

1) $f(n) = O(g(n))$ if there exists a constant c and integer N such that $f(n) \le cg(n)$ for $n > N$.
2) $f(n) = \Omega(g(n))$ if there exists a constant c and integer N such that $f(n) \ge cg(n)$ for $n > N$.
3) $f(n) = \Theta(g(n))$ means that $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.
4) $f(n) = o(g(n))$ means that $\lim_{n\to\infty} f(n)/g(n) = 0$.
5) $f(n) = \omega(g(n))$ means that $\lim_{n\to\infty} g(n)/f(n) = 0$.

II. System Model

We consider a large wireless *ad hoc* network of area $\mathcal{A}$ with node density of λ_n. The source node location is assumed to follow a homogeneous Poisson point process (PPP) Φ_s with density of $\lambda_s = \rho_s\lambda_n$, where ρ_s is the ratio of the source node density to the node density. For each source node, there is a corresponding destination node randomly located in the area $\mathcal{A}$, i.e., the destination node location follows a binomial point process over the network area. Each source or destination node can also serve as a relay of other source-destination pair but there are additional pure relay nodes whose location follows a homogeneous PPP Φ_r with density of $\lambda_n(1-2\rho_s)$. In this paper, a source-destination pair with its relays is called an end-to-end (e2e) link and each hop in the e2e link is called a constituent point-to-point (p2p) link as in [14]. Let $\mathcal{N}$ be the index set of participating nodes (source, destination, and relay) in a network realization as shown in Fig. 1 similarly as in [14], $\mathcal{E}(\lambda_s)$ be the index set of the e2e links in the network, $\mathcal{P}_i = \{1, 2, \ldots, N_i\}$ be the index set of the p2p links in the ith e2e link where N_i is the number of hops in the ith e2e link. Also, let $x_{\iota(i,j)} \in \Phi_n$ and $x_{\iota(i,j+1)} \in \Phi_n$, where $\iota(i,j) \in \mathcal{N}$, $i \in \mathcal{E}(\lambda_s)$ and $j \in \mathcal{P}_i$, respectively be the locations of the transmitter and the receiver of the jth p2p link in the ith e2e link. Then, the ith e2e link can also be represented by the locations of its constituent nodes as $\eta_i = (x_{\iota(i,1)}, x_{\iota(i,2)}, \ldots, x_{\iota(i,N_i+1)})$ and the jth p2p link in the ith e2e link can be similarly represented as $\varsigma_{i,j} = (x_{\iota(i,j)}, x_{\iota(i,j+1)})$. Also, the hop distance vector of the ith e2e link is defined as $\mathbf{d}_i = (d_{i,1}, d_{i,2}, \ldots, d_{i,N_i})$ where $d_{i,j} = |x_{\iota(i,j)} - x_{\iota(i,j+1)}|$ and $d_i = |x_{\iota(i,1)} - x_{\iota(i,j+N_i)}|$ and the set of link vector (i,j) connected with transmitter x_k is defined as $\mathcal{H}_k \in \{(i,j)|\iota(i,j) = k, \text{ where } i \in \mathcal{E}(\lambda_s) \text{and } j \in \mathcal{P}_i\}$.

The source of each e2e link generates a media access control layer packet (M-packet) of b bits in each *M-packet slot* of length τ_p seconds according to a Bernoulli process with success probability ν_p. Each node is assumed to have multiple unlimited-length queues and assign one to each participating e2e link. The ratio of the time that the queue of the jth p2p link of the ith e2e link is occupied to the available time for occupancy, $\rho_{i,j}$, is called the traffic intensity and then the average traffic intensity, ρ_p, is defined as the average of $\rho_{i,j}$ over all queues in all possible network realizations. The average number of queues per node, κ, can also be obtained by averaging $|\mathcal{H}_k|$ over all possible network realizations. Then, the average number of queues per unit area, λ_q, is given by $\kappa\lambda_n$.

M-packets generated at each source should be delivered to its destination within $D(\lambda_n)$ seconds at most using $E(\lambda_n)$ joules with success probability of at least $1-\epsilon$. Each M-packet is transmitted in a *transmission slot* of length τ seconds with transmission rate of r (bps) and transmission power of P (J/s) as shown in Fig. 1, i.e., an M-packet is divided into $L(r) = \lceil b/r\tau \rceil$ separatively-encoded physical layer packets (P-packets) which can be transmitted at most $\hat{E}(\lambda_n) = \lfloor E(\lambda_n)/P\tau \rfloor$ times within $\hat{D}(\lambda_n) = \lfloor D(\lambda_n)/\tau \rfloor$ transmission slots. On each hop, each P-packet is transmitted to the receiver with automatic repeat request (ARQ) scheme until all P-packets of an M-packet are delivered[1] and an M-packet is dropped if it is not delivered to the destination within $\hat{E}(\lambda_n)$ transmissions during $\hat{D}(\lambda_n)$ transmission slots. The slotted ALOHA protocol considered in this paper sets the media access probability of each p2p link to q so that the set of the transmitters' locations at transmission slot t, $\Phi_t \subset \Phi_n$, can be considered to follow a homogeneous PPP of density $q\rho_p\lambda_q$ by assuming that typically $\nu_p \ll 1$ for $\lambda_n \gg 1$ and each queue can be served independently with probability ρ_p. Let $d_{xy}^{-\alpha}$, $\alpha > 2$, and $h_{xy}[t] \sim \mathrm{Exp}(1)$, respectively be the large-scale and the small-scale fadings between nodes x and y. Then, the received SINR at the receiver $x_{\iota(i,j+1)}$ of the jth p2p link of the ith e2e link in slot t, $\gamma_{i,j}[t]$, is given by

$$\gamma_{i,j}[t] = \frac{P d_{i,j}^{-\alpha} h_{x_{\iota(i,j)}x_{\iota(i,j+1)}}[t]}{P\sum_{z\in\Phi_t\setminus\{x_{\iota(i,j)}\}} d_{zx_{\iota(i,j+1)}}^{-\alpha} h_{zx_{\iota(i,j+1)}}[t] + N_0}, \quad (1)$$

where $N_0/2$ is the two-sided power spectral density of the additive white Gaussian noise (AWGN). Strictly speaking, $\gamma_{i,j}[t]$ is spatially and temporally correlated in a network using ALOHA protocol [19]. However, as shown in [19], the spatial and temporal correlations are proportional to the media access probability of a node, which is $q\rho_p$ for each queue in this paper. Also, it was shown that such a media access probability should be controlled (roughly speaking, inverse proportionally to the number of nodes) to achieve the optimal capacity scaling [14]. Thus, the spatial and temporal correlations can be ignored as long as the capacity scaling is of interest. Then, as given in [20], the complementary cumulative distribution function of $\gamma_{i,j}[t]$ is obtained as

$$\begin{aligned} &p(P, d_{i,j}, r, q) \\ &\quad\triangleq \Pr\{\gamma_{i,j}[t] > 2^r - 1\} \\ &\quad= \exp\left(-\frac{2^r-1}{d_{i,j}^{-\alpha}P/N_0} - q\rho_p\lambda_q(2^r-1)^{\frac{2}{\alpha}}K_\alpha d_{i,j}^2\right), \quad (2) \end{aligned}$$

where $K_\alpha = 2\pi^2/(\alpha\sin(2\pi/\alpha))$. Note that, (2) is not valid if $\alpha = 2$ because the interferers' signals do not decay fast enough to keep the cumulated interference power finite [21].

Let $Q_{i,j}$ be the number of P-packets in the queue of the jth p2p link of the ith e2e link when an M-packet arrives, $S_{i,j,k}(P, d_{i,j}, r, q)$, $k = 1, \ldots, L(r) + Q_{i,j}$, be the number of transmissions required for a successful transmission of the kth P-packet at the queue in the jth p2p link of the ith e2e link, which is a geometric random variable with success probability $p(P, d_{i,j}, r, q)$, $W(m)$ be the number of waiting transmission slots while transmitting P-packets m times, which is a negative binomial distribution with m and $1-q$, and $\bar{W}(m) = W(m) + m$. Then, the number of P-packet transmissions and the number of transmission slots required for an M-packet delivery at the jth p2p link of the ith e2e link, $S_{i,j}(P, d_{i,j}, r, q)$ and $T_{i,j}(P, d_{i,j}, r, q)$, are, respectively given by

$$S_{i,j}(P, d_{i,j}, r, q) = \sum_{k=1}^{L(r)} S_{i,j,k}(P, d_{i,j}, r, q), \quad (3)$$

$$\begin{aligned} &T_{i,j}(P, d_{i,j}, r, q) \\ &\quad= \bar{W}\left(S_{i,j}(P, d_{i,j}, r, q) + \sum_{k=L(r)+1}^{L(r)+Q_{i,j}} S_{i,j,k}(P, d_{i,j}, r, q)\right). \quad (4) \end{aligned}$$

Also, the number of P-packet transmissions and the number of transmission slots required for an M-packet delivery to its destination of the ith e2e link, $T_i(\mathbf{d}_i, r, q)$ and $S_i(\mathbf{d}_i, r, q)$, are, respectively given by

$$S_i(P, \mathbf{d}_i, r, q) = \sum_{j=1}^{N_i} S_{i,j}(P, d_{i,j}, r, q), \quad (5)$$

$$T_i(P, \mathbf{d}_i, r, q) = \sum_{j=1}^{N_i} T_{i,j}(P, d_{i,j}, r, q). \quad (6)$$

However, the number of transmission slots actually used for an M-packet delivery to its destination of the ith e2e link is given by $\min(T_i(P, \mathbf{d}_i, r, q), \hat{D}(\lambda_n))$ because of the delay constraint and the number of P-packet transmissions actually used for an M-packet delivery is given by $\min(S_i(P, \mathbf{d}_i, r, q), \hat{E}(\lambda_n))$ because of the energy constraint. From (6), the e2e outage

[1]We assume appropriate ACK/NACK signalings for P-packet transmissions among the transmitters and receivers and the overhead for the ACK/NACK signaling is ignored because it does not affect the capacity scaling [12], [14].

probability and the throughput of the ith e2e link can be, respectively given by

$$P_{\text{out}}(D, E|P, \mathbf{d}_i, r, q) = 1 - \Pr\left\{T_i(P, \mathbf{d}_i, r, q) \le \hat{D}, S_i(P, \mathbf{d}_i, r, q) \le \hat{E}\right\}, \quad (7)$$

$$R(D, E|\nu_p, P, \mathbf{d}_i, r, q) = b\frac{\nu_p}{\tau_p}\left(1 - P_{\text{out}}(D, E|P, \mathbf{d}_i, r, q)\right). \quad (8)$$

Remark: In the sequel, we abbreviate $D(\lambda_n)$ as D, $E(\lambda_n)$ as E, $\hat{D}(\lambda_n)$ as $\hat{D}$, and $\hat{E}(\lambda_n)$ as $\hat{E}$ if it does not cause any confusion.

The outage probability (7) or the throughput (8) in a wireless *ad hoc* network is highly dependent on its communication protocol and most typical control mechanisms used for wireless *ad hoc* networks in literature, including [22]–[25], can be classified into a transmission rate (rate control) and a transmit power (power control), the route from a source to its destination (multi-hop control), and a scheme to schedule the transmission of each node (interference control). Similarly as in [14], it is assumed in this paper that power control enables controlling P, multi-hop control enables an e2e link to control the vector $\mathbf{d}_i$, rate control enables controlling r, and interference control enables controlling q. Also, it is assumed that each source-destination pair selects its relays using the routing protocol in [12] by dividing the network into square cells having area of $B(\lambda_s)$ and selecting one node each in $N_i = \Theta(d_i/\sqrt{B(\lambda_s)})$ connecting cells including the source and the destination nodes. Note that, $B(\lambda_s)$ should be greater than or equal to $\log\lambda_n/\lambda_n$ to avoid an empty cell [12] so that $N_i = O(d_i\sqrt{\lambda_n/\log\lambda_n})$.

Let network ψ denote a random network using the SA-ψ protocol under the above assumptions, where $\psi \in \{\text{MRI}, \text{PMRI}\}$. Then, for protocol ψ, the set of all controllable parameter vectors of the ith e2e link satisfying the constraints, $\mathbf{U}_\epsilon^\psi(D, E|i_i)$, is given by

$$\mathbf{U}_\epsilon^\psi(D, E|\mathbf{d}_i) = \big\{(P, \mathbf{d}_i, r, q) \in \mathcal{M}^\psi(d_i) \times \mathcal{R}^\psi \times \mathcal{Q}^\psi | \; P_{\text{out}}(D, E|P, \mathbf{d}_i, r, q) \le \epsilon\big\}, \quad (9)$$

where

$$\mathcal{M}^\psi(d_i) = \begin{cases} \left\{(P, \mathbf{d}_i) \,\middle|\, P \in \left(0, \frac{E}{\tau}\right], \mathbf{d}_i \in \bigcup_{N_i=1}^{N_{\max}(P)} \mathcal{D}(N_i, d_i)\right\}, & \text{if } \Psi = \text{PMRI}, \\ \left\{(P, \mathbf{d}_i) \,\middle|\, \text{a given } P, \mathbf{d}_i \in \bigcup_{N_i=1}^{N_{\max}(P)} \mathcal{D}(N_i, d_i)\right\}, & \text{if } \Psi = \text{MRI}, \end{cases} \quad (10)$$

$\mathcal{R}^\psi = (0, \infty)$, and $\mathcal{Q}^\psi = (0, 1]$. Here, $N_{\max}(P) = \lfloor\min(D/(\tau L(r)), E/(P\tau L(r)), c_N d_i\sqrt{\lambda_n/\log\lambda_n})\rfloor$, for some constant $c_N > 1$ and

$$\mathcal{D}(N_i, d_i) = \left\{(d_{i,1}, d_{i,2}, \ldots, d_{i,N_i}) \,\middle|\, d_{i,j} = c_{i,j}\frac{d}{N} \text{ for some } 1 \le c_{i,j} \le \sqrt{2}\right\}. \quad (11)$$

Notations used in this paper are summarized in Table I.

TABLE I
NOTATIONS USED IN THIS PAPER

	Description
$\mathcal{A}$	network area
λ_n	node density
λ_s	source node density
ρ_s	ratio of the source node density to the node density(λ_s/λ_n)
a_s	scaling exponent of λ_s
D	maximum delay allowed for an M-packet delivery
a_D	scaling exponent of D
E	maximum energy used for an M-packet delivery
a_E	scaling exponent of E
τ_p	M-packet slot length (seconds)
ν_p	M-packet generation probability in each M-packet slot
P	transmit power
$\mathbf{d} = (d_1, \cdots, d_N)$	multi-hop vector of an N-hop e2e link
r	transmission rate of a P-packet
τ	P-packet transmission slot length (seconds)
$L(r)$	number of P-packets for an M-packet when the transmission rate is r
q	media access probability for P-packet transmission
ρ_p	average traffic intensity
κ	average number of queues per node
λ_q	average number of queues per unit area
$S(P, \mathbf{d}, r, q)$	number of P-packet transmissions required for an M-packet delivery
$T(P, \mathbf{d}, r, q)$	number of transmission slots required for an M-packet delivery
$R(D, E\|\nu_p, P, \mathbf{d}, r, q)$	e2e throughput
$P_{\text{out}}(D, E\|P, \mathbf{d}, r, q)$	e2e outage probability
ϵ	target outage probability
$\mathbf{U}_\epsilon^\psi(D, E\|d)$	set of the controllable parameter vectors satisfying the delay-energy constraints
$C_\epsilon^\psi(D, E\|\rho_s)$	DE-RATR of protocol ψ for a given ρ_s
$C_\epsilon^\psi(D, E)$	DE-RATC of protocol ψ
$a_C^\psi(k)$	capacity scaling exponent of protocol ψ
$a_C(k)$	capacity scaling exponent of the optimal protocol

III. Performance Metrics

In this section, two performance metrics, which are *delay and energy-constrained random access transport rate* (DE-RATR) and *delay and energy-constrained random access transport capacity* (DE-RATC), are considered.

Definition 1: (DE-RATR & DE-RATC) The DE-RATR is the distance-weighted maximum e2e rate per unit area of a random access network where the ratio of λ_s to λ_n, ρ_s, is given and packets generated at each source should be delivered to its destination within D seconds using at most E joules satisfying the target outage probability ϵ, which is given by

$$C_\epsilon^\psi(D, E|\rho_s) \triangleq \frac{1}{\mathcal{A}} \max_{\nu_p \in (0,1]} E_{\mathcal{E}(\lambda_s)}\left[\sum_{i \in \mathcal{E}(\lambda_s)} d_i \cdot \max_{(P, \mathbf{d}_i, r, q) \in \mathbf{U}_\epsilon^\psi(D, E|\mathbf{d}_i)} R^\psi(D, E|\nu_p, P, \mathbf{d}_i, r, q)\right], \quad (12)$$

where $\mathcal{A}$ denotes the network area. Also, the DE-RATC, which is the maximum DE-RATR, is given by

$$C_\epsilon^\psi(D, E) \triangleq \max_{\rho_s \in (0,1]} C_\epsilon^\psi(D, E|\rho_s). \quad (13)$$

For better mathematical tractability, a typical e2e link with distance of d rather than the average over the whole network is considered, similarly as in [14], [26], [27]. Then, the typical DE-RATR and DE-RATC are given as follows.

Definition 2: (typical DE-RATR & DE-RATC) The typical DE-RATR and DE-RATC are, respectively defined as the DE-RATR and DE-RATC under the assumption that each source has its destination at a fixed distance d, given by

$$C_\epsilon^\psi(D,E|d,\rho_s) = d\lambda_n\rho_s \max_{\nu_p\in(0,1]} \max_{(P,\mathbf{d},r,q)\in\mathbf{U}_\epsilon^\psi(D,E|\mathbf{d})} R^\psi(D,E|\nu_p,P,\mathbf{d},r,q), \quad (14)$$

$$C_\epsilon^\psi(D,E|\mathbf{d}) \triangleq \max_{\rho_s\in(0,1]} C_\epsilon^\psi(D,E|\mathbf{d},\rho_s), \quad (15)$$

where $\mathbf{d}=(d_1,\dots,d_N)$ and r, respectively denote the hop distance vector and the transmission rate of the typical e2e link.

The proposed DE-RATR can be considered as the sum of the distance-weighted throughput capacity with a random access protocol and an extension of the transmission capacity [27] to a multi-hop retransmission scenario in which a routing adaptation is allowed. If there is no constraint on D or E, $C_\epsilon^{\mathrm{M}}(D,E|\rho_s)$ is equal to the random access transport capacity for simple MAC schemes introduced in [26]. Also, the delay constraint considered in this paper is stricter than the constraint on the average packet transmission time used in [12], [13] and the energy constraint per packet, E, is actually the inverse of the energy efficiency used in [16] if divided by the constant packet size b. Although the DE-RATR is a performance metric for a network with a given ρ_s, the DE-RATC is a performance metric for a network where ρ_s is also controllable ($\lambda_s=O(\lambda_n)$). For example, a sensor network may control the number of active nodes in order to maximize the information flow of the network.

Definition 3: (*scaling exponent of the DE-RATC*) The scaling exponent of the DE-RATC, $a_C^\psi(k)$, is defined as

$$a_C^\psi(k)=\max_a\left\{a+\delta\,\middle|\,C_\epsilon^\psi(D,E|\mathbf{d})=\Omega\left(\lambda_n^a(\log\lambda_n)^k\right)\right\}-\delta \quad (16)$$

for a small $\delta>0$ that makes the maximum exist. Also, the scaling exponents of λ_s, D, and E are, respectively defined as

$$a_s=\lim_{\lambda_n\to\infty}\frac{\log\lambda_s}{\log\lambda_n}, \quad (17)$$

$$a_D=\lim_{\lambda_n\to\infty}\frac{\log D(\lambda_n)}{\log\lambda_n}, \quad (18)$$

$$a_E=\lim_{\lambda_n\to\infty}\frac{\log E(\lambda_n)}{\log\lambda_n}. \quad (19)$$

Note that each of the above scaling exponents represents the asymptotic growth rate as a power law of λ_n, i.e., $\lambda_s\sim\lambda_n^{a_s}$, $D(\lambda_n)\sim\lambda_n^{a_D}$, and $E(\lambda_n)\sim\lambda_n^{a_E}$. Also, $a_C^\psi(k)=a$ implies that $C_\epsilon^\psi(D,E|i)=\Omega(\lambda_n^a(\log\lambda_n)^k)$ and $a_C^\psi(l)$ for $l>k$ becomes infinitely close to but less than a since $C_\epsilon^\psi(D,E|i)=o(\lambda_n^a(\log\lambda_n)^l)$.

As described in Section II, the delay and/or energy constraints limit some physical quantities in a random *ad hoc* network such as the maximum hops, the maximum p2p distance, and the maximum e2e distance.

- The *maximum hops* denotes the maximum number of hops that an e2e link could have and is given as $\Theta(D)$ as long as the transmit power is set to $P=O(E/D)$. If the number of hops is set to be $o(D)$, the residual delay may be used to increase the packet size by increasing the transmission time at each hop. However, the e2e throughput cannot be improved because the packet transmission time also increases in directly proportional to the packet size.
- The *maximum p2p distance* denotes the maximum one-hop transmission range of a finite size packet with E joules and is given as $\Theta(E^{1/\alpha})$. If the typical p2p distance, denoted as d_T, is smaller than the maximum p2p distance, the residual energy may be used to increase the packet size by increasing the transmission rate.
- The *maximum e2e distance* denotes the maximum multi-hop transmission range of a finite size packet with E joules and is given as $\Theta(D^{1-1/\alpha}E^{1/\alpha})$ when the number of hops is set to $\Theta(D)$ and the transmit power is set to $\Theta(E/D)$. It determines the possibility of an M-packet delivery under the delay and energy constraints: if the maximum e2e distance vanishes, the DE-RATC of a finite-length typical e2e link vanishes.

IV. DE-RATC Scaling of Random *ad hoc* Networks

In this section, the DE-RATR and DE-RATC of wireless *ad hoc* networks are derived and it will be shown that the DE-RATC is order optimal, i.e., the order of the growth of the DE-RATC according to λ_n is the same to that of the maximum achievable transport capacity at given delay and energy constraints. The following theorem gives the DE-RATR and DE-RATC of the SA-PMRI protocol.

Theorem 1: For $0<\rho_s=\lambda_s/\lambda_n\le 1$, the DE-RATR of the SA-PMRI protocol is given as

$$C_\epsilon^{\mathrm{PMRI}}(D,E|\rho_s) = \begin{cases} \Theta\left(\min\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}},\rho_s\lambda_n\right)\right), \\ \quad \text{if } D=\Omega\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E=\Omega\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right), \\ \Theta\left(\min(D,\rho_s\lambda_n)\right), \\ \quad \text{if } D=\Omega(1),\ D=o\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E=\Omega(D^{-\alpha+1}), \\ 0, \text{if } D=o(1) \text{ or } E=o\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right) \\ \quad \text{or } E=o(D^{-\alpha+1}). \end{cases} \quad (20)$$

Proof: See Appendix. ■

1) NC Region: The non-constrained region (NC region) denotes the region satisfying $D = \Omega(\sqrt{\lambda_n/\log\lambda_n})$ and $E = \Omega((\lambda_n/\log\lambda_n)^{-(\alpha-1)/2})$. Theorem 1 says that, in the NC region, the DE-RATR scaling is not changed by the delay and energy constraints and the optimal capacity scaling in [12] can be achieved by the SA-PMRI protocol. Note that, $B(\lambda_s) = \Omega(\log\lambda_n/\lambda_n)$ for avoiding isolated nodes [3], [12] so that the typical p2p distance is given as $d_T = \Theta(\sqrt{\log\lambda_n/\lambda_n})$, i.e., the typical hops is $N_T = \Theta(\sqrt{\lambda_n/\log\lambda_n})$. Also, under the given energy constraint per packet for an e2e link, the e2e throughput monotonically increases as the number of hops increases up to N_T. Thus, in the NC region, the additional delay does not improve the DE-RATR while it may be used to increase the packet size b. In the region of $E = \Omega((\lambda_n/\log\lambda_n)^{-(\alpha-1)/2})$, the energy per packet allowed for a p2p link is E/N_T so that the maximum p2p distance is greater than or equal to d_T, i.e., $(E/N_T)^{1/\alpha} = \Omega(d_T)$. However, the residual energy cannot be used to improve the DE-RATR because the interference power from other nodes also increases as the transmit power increases. Thus, the DE-RATR is obtained by setting the number of hops as $\Theta(\sqrt{\lambda_n/\log\lambda_n})$ and the transmit power between $\Omega((\lambda_n/\log\lambda_n)^{-(\alpha/2)})$ and $O(E/\sqrt{\lambda_n\log\lambda_n})$. Note that, although interference and rate controls themselves are indispensable MAC functions to satisfy the constraints for any given λ_n and ν_p, the DE-RATR is achieved when $\nu_p = \Theta(\min(1/(\rho_s\sqrt{\lambda_n\log\lambda_n}), 1))$ and q and r can be set to $\Theta(1)$ in this case.

2) DC Region: In Theorem 1, the delay-constrained region (DC region) denotes the region satisfying $D = o(\min(\lambda_s, \sqrt{\lambda_n/\log\lambda_n}))$ and $E = \Omega(D^{-\alpha+1})$. In the DC region, the DE-RATR scaling is affected by D but not by E and the optimal delay-constrained capacity scaling shown in [12], [14] can be achieved by the SA-PMRI protocol. As discussed earlier, the e2e throughput and the energy constraint per packet for an e2e link increases as the number of hops increases up to N_T. However, in the DC region, the maximum hops is smaller than N_T due to the delay constraint so that the optimal multi-hop control policy is to set the number of hops as large as possible while satisfying the delay constraint. Also, the maximum e2e distance should be $\Omega(1)$ to make an M-packet delivery possible for a typical finite-length e2e link, i.e., $E^{1/\alpha} = \Omega(D^{-1+1/\alpha})$. Thus, the DE-RATR can be obtained by setting the number of hops as $\Theta(D)$ and the transmit power between $\Omega(D^{-\alpha})$ and $O(E/D)$. The interference and rate controls are same as in the NC region.

3) NA region: The non-achievable region (NA region) denotes the region of $D = o(1)$ or $E = o((\lambda_n/\log\lambda_n)^{-(\alpha-1)/2})$ or $D^{\alpha-1}E = o(1)$. In the region of $D = o(1)$, M-packets cannot be transmitted due to the vanishing slot length. In the region of $E = o((\lambda_n/\log\lambda_n)^{-(\alpha-1)/2})$, the maximum p2p distance becomes smaller than the typical distance d_T. In the region of $E = o(D^{-1+\alpha})$, the maximum e2e distance vanishes as λ_s increases so that the DE-RATR of a finite-length typical e2e link vanishes.

Remark: Note that, among the three boundaries of the NA region, $D = \Theta(1)$ is straightforward and $E = \Theta((\lambda_n/\log\lambda_n)^{-(\alpha-1)/2})$ can be easily expected from [15]. However, $D^{\alpha-1}E = \Theta(1)$ has not been reported yet and describes the interaction between the delay and energy constraints.

Theorem 2: The optimal ρ_s that maximizes the DE-RATR, $\rho_s^* = \arg\max_{\rho_s\in(0,1]} C_\epsilon^\psi(D, E|i, \rho_s)$, is given as

$$\rho_s^* = \begin{cases} \Omega\left(\frac{1}{\sqrt{\lambda_n\log\lambda_n}}\right) \text{ and } O(1), \\ \quad \text{if } D = \Omega\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right), \\ \Omega\left(\frac{D}{\lambda_n}\right) \text{ and } O(1), \\ \quad \text{if } D = \Omega(1),\ D = o\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega(D^{-\alpha+1}), \end{cases} \tag{21}$$

which is straightforward from (20) and then the DE-RATC, $C_\epsilon^{\mathrm{PMRI}}(D, E) = C_\epsilon^{\mathrm{PMRI}}(D, E|\rho_s^*)$, is given as

$$C_\epsilon^{\mathrm{PMRI}}(D, E) = \begin{cases} \Theta\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right), \\ \quad \text{if } D = \Omega\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right), \\ \Theta(D), \\ \quad \text{if } D = \Omega(1),\ D = o\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega(D^{-\alpha+1}), \\ 0,\ \text{if } D = o(1) \text{ or } E = o\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right) \\ \quad \text{or } E = o(D^{-\alpha+1}), \end{cases} \tag{22}$$

which implies that the proposed SA-PMRI protocol is the order-optimal for any delay and energy constraints.

Proof: It is straightforward to obtain (22) by substituting (21) into (20) and the optimality of the DE-RATC can be proved as follows. For a given delay constraint, a lower bound on the energy for delivering an M-packet to its destination through the ith e2e link using N hops, $E_i(N)$, can be given by

$$E_i(N) \geq N d_{i,j}^\alpha \beta\tau \tag{23}$$
$$\geq N^{-\alpha+1} d_i^\alpha \beta\tau \tag{24}$$
$$\geq \min\left(\frac{D}{\tau}, c_N d_i \sqrt{\frac{\lambda_n}{\log\lambda_n}}\right)^{-\alpha+1} d_i^\alpha \beta\tau, \tag{25}$$

where the first inequality comes from the fact that the received signal power at each hop should exceed a finite threshold β, $d_{i,j}^{-\alpha}P \geq \beta$, where $d_{i,j}$ is the jth hop distance in the ith e2e link, the second inequality comes from $d_{i,j} \geq d_i/N$, and the third inequality comes from $N \leq \min(D/\tau, c_N d_i\sqrt{\lambda_n/\log\lambda_n})$. Thus, an M-packet cannot be delivered to its destination if $E = o(\min(D, \sqrt{\lambda_n/\log\lambda_n})^{-\alpha+1})$, which implies that the DE-RATC is the optimal capacity scaling because the DE-RATC achieves the optimal capacity scaling in [12] at any delay constraint as long as $E = \Omega(\min(D, \sqrt{\lambda_n/\log\lambda_n})^{-\alpha+1})$. ■

Theorem 2 shows that we can use some part of nodes as relays as long as $\lambda_s = \Omega(\min(\sqrt{\lambda_n/\log\lambda_n}, D))$ for increasing the packet arrival rate at each source node to $\nu_p = \Theta(\min(D/\lambda_s, \sqrt{\lambda_n/\log\lambda_n}/\lambda_s, 1))$ while keep achieving the optimal capacity scaling. Note that the packet arrival rate can be increased up to $\nu_p = \Theta(1)$ because the interference level at a receiver is maintained if λ_s is inversely proportional to ν_p

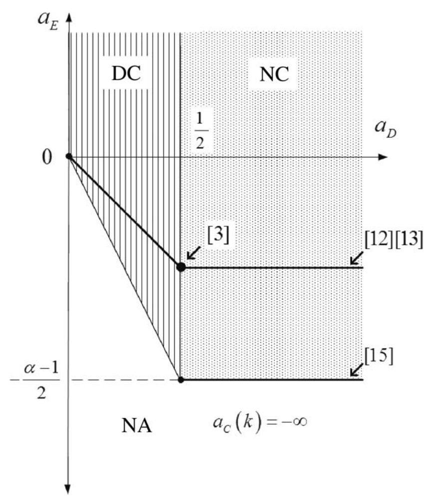

Fig. 2. The NC, DC, and NA regions in terms of (a_D, a_E).

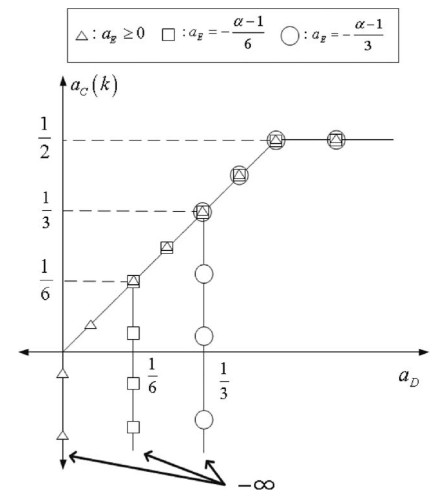

Fig. 3. The capacity scaling $a_C(k)$ according to a_D.

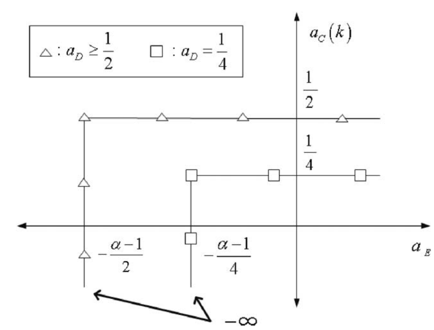

Fig. 4. The capacity scaling $a_C(k)$ according to a_E.

while keeping the typical p2p distance the same when $\lambda_s = \Omega(\min(\sqrt{\lambda_n/\log\lambda_n}, D))$. However, further reduction in λ_s (i.e., $\lambda_s = o(\min(\sqrt{\lambda_n/\log\lambda_n}, D))$) may be utilized to increase the M-packet size but the size increment is log-scale so that the optimal capacity scaling cannot be achieved.

Let $a_C(k) = \max_\psi a_C^\psi(k)$ be the capacity scaling exponent of the optimal protocol at the given delay and energy constraints in an *ad hoc* network with the source node density of λ_s. Then, from Theorems 1 and 2, $a_C(k) = a_C^{\mathrm{PMRI}}(k)$ for any delay and energy constraints, which can be easily represented as a function of a_D, a_E, and a_S from Theorem 1 and Definition 3 as follows.

Corollary 1: The capacity scaling exponent $a_C(k)$ of the optimal protocol in an *ad hoc* network having source node density of $\lambda_s \sim \lambda_n^{a_S}$ at the given delay constraint of $D \sim \lambda_n^{a_D}$ and energy constraint of $E \sim \lambda_n^{a_E}$ is given by

$$a_C(k) = \begin{cases} \min\left(\frac{1}{2} - \delta I\left(k > -\frac{1}{2}\right), a_s\right), \\ \quad \text{if } a_D \geq \frac{1}{2},\ a_E \geq -\frac{\alpha-1}{2}, \\ \min(a_D, a_S), \\ \quad \text{if } 0 \leq a_D < \frac{1}{2},\ (\alpha-1)a_D + a_E \geq 0, \\ -\infty, \\ \quad \text{if } a_D < 0, \text{ or } a_E < -\frac{\alpha-1}{2}, \text{ or } (\alpha-1)a_D + a_E < 0, \end{cases} \tag{26}$$

for an arbitrarily small $\delta > 0$, where $I(k > x)$ denotes the indicator function (1 if $k > x$ and 0 if $k \leq x$).

Proof: For $k \leq -1/2$,

$$a_C(k) = \begin{cases} \min\left(\frac{1}{2}, a_s\right), \text{ if } a_D \geq \frac{1}{2},\ a_E \geq -\frac{\alpha-1}{2}, \\ \min(a_D, a_s), \\ \quad \text{if } 0 \leq a_D < \frac{1}{2},\ (\alpha-1)a_D + a_E \geq 0, \\ -\infty, \text{ Otherwise,} \end{cases} \tag{27}$$

from Definition 3 and Theorem 1. When $k > -(1/2)$ and $a_s \geq (1/2)$, $a_C(k)$ is infinitely close to but strictly less than $(1/2)$ in the first case, which concludes the proof. ■

In Fig. 2, the NC, DC, and NA regions are drawn in terms of (a_D, a_E) in comparison with previously known results in [3], [12], [13], [15] which reveals that previously known results are special-cases (a line or a point) and the proposed DE-RATC provides the full understanding on the capacity scaling of delay and energy constrained *ad hoc* networks. Here, the deciding factors of the boundaries among regions can be redescribed as follows.

- Boundary between the NC and the DC regions: The deciding factor is the maximum hops. If the maximum hops for given delay and energy constraints is greater than or equal to N_T, it belongs to the NC region. If otherwise, it belongs to the DC region.
- Boundary between the NC and the NA regions: The deciding factor is the maximum p2p distance. If the maximum p2p distance for given delay and energy constraints is greater than or equal to d_T, it belongs to the NC region. If otherwise, it belongs to the NA region.
- Boundary between the DC region and the NA region: The deciding factors are the maximum e2e distance and the maximum delay. If the maximum e2e distance for a given delay and energy constraint and the maximum delay are not vanished as λ_s increases, it belongs to the DC region. If otherwise, it belongs to the NA region.

In Figs. 3–5, $a_C(k)$ is respectively shown according to a_D for various values of a_E, according to a_E for various values of a_D, and according to a_D with a constraint on various values of $(\alpha-1)a_D + a_E$. As can be expected from Theorem 1, Fig. 3 shows that $a_C(k) = -\infty$ and $(a_D, a_E) \in$ *NA region*

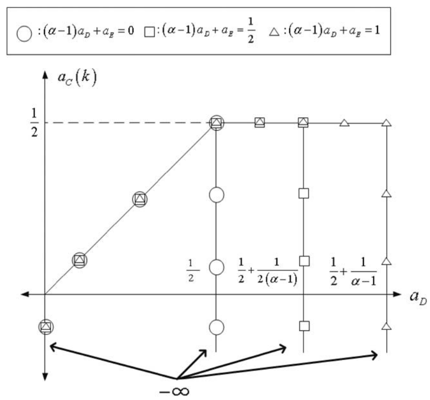

Fig. 5. The capacity scaling $a_C(k)$ according to a_D.

when $a_D < \max(-a_E/(\alpha-1), 0)$, $a_C(k)$ increases linearly as a_D increases up to $(1/2)$ and $(a_D, a_E) \in$ *DC region* when $\max(-a_E/(\alpha-1), 0) \leq a_D < 1/2$, and $a_C(k) = 1/2$ and $(a_D, a_E) \in$ *NC region* when $a_D \geq 1/2$. So, it is seen that the capacity scaling depends on the delay constraint as reported in [12] but it is also revealed that the minimum feasible delay constraint changes when the energy constraint becomes tight. Fig. 4 shows that $a_C(k) = -\infty$ and $(a_D, a_E) \in$ *NA region* when $a_E < \max(-a_D(\alpha-1), -(\alpha-1)/2)$, and $a_C(k) = \min(a_D, 1/2)$ and $(a_D, a_E) \in$ *DC region* $\cup$ *NC region* when $a_E \geq \max(-a_D(\alpha-1), -(\alpha-1)/2)$, which implies that the impact of the energy constraint on the capacity scaling is an on-off way and the hard limit on a_E is $-(\alpha-1)/2$ as reported in [15] but more energy is required when the delay constraint becomes tight. Fig. 5 shows $a_C(k)$ according to a_D when the scaling exponent of the maximum e2e distance, $(\alpha-1)a_D + a_E$, is 0, $(1/2)$ and 1. Note that the maximum e2e distance determines the maximum allowable network radius with non-vanishing DE-RATC. When $(\alpha-1)a_D + a_E = 0$ (maximum allowable $\mathcal{A} \sim \lambda_n^0$), allowing more delay improves the DE-RATC up to $1/2$ until the network becomes energy limited. When the values of $(\alpha-1)a_D + a_E$ increases to $(1/2)$ (or 1), the feasible delay and energy constraint region enlarges and although it does not improve the capacity scaling, it helps to increase the maximum allowed network size [maximum allowable $\mathcal{A} \sim \lambda_n^1$ (or λ_n^2)].

V. Conclusion

The capacity scaling of a random *ad hoc* network with various delay and energy constraints is investigated when the MAC protocol is the slotted ALOHA protocol with additional features, such as power control, multi-hop control, rate control, and interference control. It is shown that the DE-RATC scaling is order-optimal for any delay or energy constraints and the proposed DE-RATC provides the full understanding on the capacity scaling of delay and energy constrained *ad hoc* networks with previously known results [3], [12], [13], [15] as its special-cases.

It is shown that the delay-energy constraint space is divided into three regions, the NC region, the DC region, and the NA region, and their boundaries are, respectively determined from the relations between physical quantities due to the delay and energy constraints and those due to network size and node density as follows: the maximum hops and the typical hops, the maximum p2p distance and the typical p2p distance avoiding isolated nodes, and the maximum e2e distance and the network size.

It is also shown that the optimal capacity scaling can be achieved by setting the number of hops as $\Theta(\sqrt{\lambda_n/\log\lambda_n})$ and the transmit power between $\Omega((\lambda_n/\log\lambda_n)^{-(\alpha/2)})$ and $O(E/\sqrt{\lambda_n\log\lambda_n})$ in the NC region, by setting the number of hops as $\Theta(D)$ and the transmit power between $\Omega(D^{-\alpha})$ and $O(E/D)$ in the DC region, in which the packet arrival rate for each source, ν_p, can be increased up to $\Theta(1)$ by reducing ρ_s inverse proportionally as long as $\lambda_s = \Omega(\min(\sqrt{\lambda_n/\log\lambda_n}, D))$.

Appendix

The proof can be sketched as follows. First, upper and lower bounds of the DE-RATR are, respectively shown at Lemmas 1 and 2. Then, the asymptotic growth rate of the DE-RATR is determined by finding that the asymptotic growth rates of the upper and lower bounds are equal. Then, the remaining task is to prove Lemmas 1 and 2. Notations (in addition to those in Table I) are summarized as follows. $\mathbf{U}_\epsilon^{\psi,\mathrm{sup}}(D,E|d) \supset \mathbf{U}_\epsilon^{\psi}(D,E|d)$ and $\mathbf{U}_\epsilon^{\psi,\mathrm{sub}}(D,E|d) \subset \mathbf{U}_\epsilon^{\psi}(D,E|d)$ are, respectively denote a superset and a subset of the controllable parameter vectors, $\hat{E} = \lfloor E/P\tau \rfloor$ and $\hat{D} = \lfloor D/\tau \rfloor$ are, respectively denote the maximum number of transmissions and transmission slots allowed for an M-packet delivery.

The DE-RATR is bounded as in the following Lemmas.

Lemma 1: An upper bound of the DE-RATR, $C_\epsilon^{\psi,\mathrm{ub}}(D, E|\rho_s) \geq C_\epsilon^{\psi}(D,E|\rho_s)$, is given by

$$C_\epsilon^{\psi,\mathrm{ub}}(D,E|\rho_s) = d\rho_s\lambda_n \max_{\nu_p\in(0,1]} \max_{(P,\mathbf{d},r,q)\in\mathbf{U}_\epsilon^{\psi,\mathrm{sup}}(D,E|\mathbf{d})} R^{\mathrm{ub}}(D,E|\nu_p,P,\mathbf{d},r,q), \quad (28)$$

where, for some constant $0 < c_1 < \infty$,

$$R^{\mathrm{ub}}(D,E|\nu_p,P,\mathbf{d},r,q) = \min\left(\frac{rq}{1-\epsilon}\exp\left(-\frac{\nu_p\lambda_s(2^r-1)^{\frac{2}{\alpha}}}{N}c_1K_\alpha d^2\right), b\frac{\nu_p}{\tau_p}\right), \quad (29)$$

$$\mathbf{U}_\epsilon^{\psi,\mathrm{sup}}(D,E|\mathbf{d}) = \left\{(P,\mathbf{d},r,q)\in\mathcal{M}^\psi(d)\times\mathcal{R}^\psi \times\mathcal{Q}^\psi \left| r \leq \log_2\left(1+\beta^{\mathrm{ub}}(D,E|\nu_p,P,N,q)\right)\right.\right\}, \quad (30)$$

$$\beta^{\mathrm{ub}}(D,E|\nu_p,P,N,q) = \min\left(\frac{N^\alpha P}{d^\alpha N_0}\log\left(\frac{\min\left(\left\lfloor\frac{D}{\tau}\right\rfloor,\left\lfloor\frac{E}{P\tau}\right\rfloor\right)e}{N+\log(1-\epsilon)}\right), \left(\frac{N}{\nu_p\lambda_s c_1K_\alpha d^2}\log\left(\frac{\min\left(\left\lfloor\frac{D}{\tau}\right\rfloor,\left\lfloor\frac{E}{P\tau}\right\rfloor\right)e}{N+\log(1-\epsilon)}\right)\right)^{\frac{\alpha}{2}}\right). \quad (31)$$

□

Lemma 2: A lower bound of the DE-RATR, $C_\epsilon^{\psi,\text{lb}}(D, E|\rho_s) \le C_\epsilon^{\psi}(D,E|\rho_s)$, is given by

$$C_\epsilon^{\psi,\text{lb}}(D,E|\rho_s) = d\rho_s\lambda_n \max_{\nu_p\in(0,1/2)} \max_{(P,\mathbf{d},r,q)\in\mathbf{U}_\epsilon^{\psi,\text{sub}}(D,E,|d)} \frac{\nu_p}{\tau_p} r\tau(1-\epsilon), \quad (32)$$

where, for some constant $0 < c_2 < \infty$,

$$\mathbf{U}_\epsilon^{\psi,\text{sub}}(D,E|\mathbf{d}) = \left\{(P,\mathbf{d},r,q)\in\mathcal{M}^\psi(d)\times\mathcal{R}^\psi\times\mathcal{Q}^\psi \,|\, r \le \log_2\left(1+\beta^{\text{lb}}(D,E|\nu_p,P,N,q)\right)\right\}, \quad (33)$$

$$\beta^{\text{lb}}(D,E|\nu_p,P,N,q) = f\left(\frac{\log\left(\frac{q\min\left(D,\frac{E}{P}\right)}{N}\frac{(1-2\nu_p-\epsilon(1-\nu_p))}{\tau(1-\nu_p)(1-\epsilon)}\right)}{\frac{\nu^{\frac{\alpha}{2}}N_0}{N^\alpha P}+\frac{\nu_p\lambda_s c_2 K_\alpha\nu}{N}}\right), \quad (34)$$

and $f(x) = \min(x, x^{\alpha/2})$ for $x > 0$. □

In the sequel, the asymptotic growth rates of $C_\epsilon^{\psi,\text{ub}}(D,E|\rho_s)$ and $C_\epsilon^{\psi,\text{lb}}(D,E|\rho_s)$ are derived to determine that of the DE-RATR. For analytical convenience, it is assumed that τ is chosen well to make $\min(D/(\tau L(r)), E/(P\tau L(r)))$ be a natural number. Since $\log_2(1+x)\le x$ for $x>0$, an upper bound the DE-RATR in Lemma 1 can be rewritten as

$$\begin{aligned}C_\epsilon^{\psi}(D,E|\rho_s) &\le d\lambda_s \max_{\nu_p\in(0,1]} \max_{r\in(0,\beta^{\text{ub}}(D,E|\nu_p,P,N,q)]} \max_{P\in\left(0,\frac{E}{\tau}\right]} \max_{N\in\{1,\dots,N_{\max}(P)\}} \max_{q\in(0,1]} R^{\text{ub}}(D,E|\nu_p,P,\mathbf{d},r,q)\\ &= d\lambda_s \max_{\nu_p\in(0,1]} \min\left(\max_{r\in(0,\beta^{\text{ub}}(D,E|\nu_p,P,N,q)]} \max_{P\in\left(0,\frac{E}{\tau}\right]} \max_{N\in\{1,\dots,N_{\max}(P)\}} \times\max_{q\in(0,1]}\frac{e^{A(P,N,r,q,\nu_p)}}{1-\epsilon}, \frac{b\nu_p}{\tau_p}\right),\end{aligned} \quad (35)$$

where

$$A(P,N,r,q,\nu_p) = \log q + \log r - c_3\frac{\nu_p\lambda_s(2^r-1)^{2/\alpha}}{N} \quad (36)$$

and $c_3 = c_1 K_\alpha d^2$.

Let q^*, N^*, P^*, r^*, and ν_p^* be, respectively the values of q, N, P, r, and ν_p to maximize $R^{\text{ub}}(D,E|\nu_p,P,\mathbf{d},r,q)$. Since $q\in(0,1]$ and $N_{\max}(P) = \min(D/(\tau L(r)), E/(P\tau L(r)), dc_N\sqrt{\lambda_n/\log\lambda_n})$, it is straightforward from (36) that $q^* = 1$ and $N^* = N_{\max}(P)$ so that $A(P,N^*,r,q^*,\nu_p)$ is given as

$$A(P,N^*,r,q^*,\nu_p) = \log r - \frac{\nu_p\lambda_s(2^r-1)^{2/\alpha}c_3}{\min\left(\frac{D}{\tau L(r)}, \frac{E}{P\tau L(r)}, dc_N\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right)}. \quad (37)$$

Then, in order not to reduce the denominator in the second term in (37), $P^* = E/(\tau L(r)\min(D/(\tau L(r)), dc_N\sqrt{\lambda_n/\log\lambda_n})$ so that $A(P^*,N^*,r,q^*,\nu_p)$ is given as

$$A(P^*,N^*,r,q^*,\nu_p) = \log r - \frac{\nu_p\lambda_s(2^r-1)^{2/\alpha}c_3}{\min\left(\frac{D}{\tau L(r)}, dc_N\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right)}. \quad (38)$$

Thus, if $\beta^{\text{ub}}(D,E|\nu_p,P^*,N^*,q^*) > 0$, $r^*\in(0,\beta^{\text{ub}}(D,E|\nu_p,P^*,N^*,q^*)]$ exists so that $C_\epsilon^{\psi,\text{ub}}(D,E|\rho_s)$ is obtained by substituting (38) into (35) as

$$C_\epsilon^{\psi,\text{ub}}(D,E|\rho_s) = d\max_{\nu_p\in(0,1]} \min\left(\frac{\lambda_s r^*}{1-\epsilon}\exp\left(-\frac{\nu_p\lambda_s\left(2^{r^*}-1\right)^{2/\alpha}c_3}{\min\left(\frac{Dr^*}{b}, dc_N\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right)}\right), \lambda_s b\frac{\nu_p}{\tau_p}\right). \quad (39)$$

By substituting P^*, N^*, and q^* into (31) and setting $L(r) = 1$, $\beta^{\text{ub}}(D,E|\nu_p,P^*,N^*,q^*)$ is given as

$$\beta^{\text{ub}}(D,E|\nu_p,P^*,N^*,q^*) = \min\left(\frac{\min\left(\frac{D}{\tau}, dc_N\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right)^{\alpha-1}E}{d^\alpha N_0\tau}c_4, \left(\frac{\min\left(\frac{D}{\tau}, dc_N\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right)}{\nu_p\lambda_s c_1 K_\alpha d^2}c_4\right)^{\frac{\alpha}{2}}\right), \quad (40)$$

where $c_4 = 1-\log(1+\log(1-\epsilon)/\min(D/\tau, dc_N\sqrt{\lambda_n/\log\lambda_n}))$. Thus, r^* exists and has a constant value if and only if $E = \Omega(\min(D,\sqrt{\lambda_n/\log\lambda_n})^{-\alpha+1})$ and $\nu_p = O((1/\lambda_s)\min(D, \sqrt{\lambda_n/\log\lambda_n}))$. Note that, the first term in the minimum function in (39) is a decreasing function of ν_p while the other term is an increasing function of ν_p. Thus, ν_p^* is easily obtained to make both terms be the same as

$$\nu_p^* = \begin{cases}\Theta\left(\min\left(\frac{1}{\lambda_s}\sqrt{\frac{\lambda_n}{\log\lambda_n}}, 1\right)\right), & \text{if } D = \Omega\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\\ \Theta\left(\min\left(\frac{D}{\lambda_s}, 1\right)\right), & \text{if } D = O\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right).\end{cases} \quad (41)$$

Since $C_\epsilon^{\text{PMRI}}(D,E|\rho_s) \le C_\epsilon^{\text{PMRI,ub}}(D,E|\rho_s)$ and $\lambda_s = \rho_s\lambda_n$, an upper bound of $C_\epsilon^{\text{PMRI}}(D,E|\rho_s)$ is obtained as

$$C_\epsilon^{\text{PMRI}}(D,E|\rho_s) \le C_\epsilon^{\text{PMRI,ub}}(D,E|\rho_s) = \begin{cases} O\left(\min\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}, \rho_s\lambda_n\right)\right),\\ \quad \text{if } D = \Omega\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right),\\ O\left(\min(D,\rho_s\lambda_n)\right),\\ \quad \text{if } D = \Omega(1),\ D = o\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega(D^{-\alpha+1}),\\ 0, \text{if } D = o(1) \text{ or } E = o\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right)\\ \quad \text{or } E = o(D^{-\alpha+1}).\end{cases} \quad (42)$$

On the other hand, consider the case where $(P, N, q) = (P^\dagger, N^\dagger, q^\dagger)$ and $\nu_p = \nu_p^\dagger$, given by

$$(P^\dagger, N^\dagger, q^\dagger) = \begin{cases} \left(\frac{E}{z_{1,1}\sqrt{\frac{\lambda_n}{\log\lambda_n}}}, z_{1,2}\sqrt{\frac{\lambda_n}{\log\lambda_n}}, z_{1,3}\right), \\ \quad \text{if } D = \Omega\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right), \\ \left(\frac{E}{z_{2,1}D}, \frac{z_{2,2}D}{\tau}, z_{2,3}\right), \\ \quad \text{if } D = \Omega(1),\ D = o\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega(D^{-\alpha+1}), \end{cases} \tag{43}$$

$$\nu_p^\dagger = \begin{cases} \min\left(\frac{z_{1,4}}{\lambda_s}\sqrt{\frac{\lambda_n}{\log\lambda_n}}, z_{1,5}\right), \\ \quad \text{if } D = \Omega\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right), \\ \min\left(\frac{z_{2,4}D}{\lambda_s}, z_{2,5}\right), \\ \quad \text{if } D = \Omega(1),\ D = o\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega(D^{-\alpha+1}), \end{cases} \tag{44}$$

where $0 < z_{k,l} < \infty$, $k = 1, 2$ and $l = 1, \ldots, 5$, are selected in order to satisfy the delay and energy constraints. Note that the existence for such constants is straightforward because, as long as $z_{k,1}z_{k,3}/z_{k,2} > \tau(1-\nu_p)(1-\epsilon)/(1-2\nu_p-\epsilon(1-\nu_p))$, $\beta^{\text{lb}}(D, E|\nu_p^\dagger, P^\dagger, N^\dagger, q^\dagger) > 0$ from (34), which implies that $\mathbf{U}_\epsilon^{\psi,\text{sub}}(D, E|d) \neq \phi$. By substituting (43) and (44) into (32) and (33) in Lemma 2, a lower bound of the DE-RATR is obtained as

$$C_\epsilon^{\text{PMRI}}(D, E|\rho_s) \geq C_\epsilon^{\text{PMRI,lb}}(D, E|\rho_s) = \begin{cases} \Omega\left(\min\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}, \rho_s\lambda_n\right)\right), \\ \quad \text{if } D = \Omega\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega\left(\left(\frac{\lambda_n}{\log\lambda_n}\right)^{-\frac{\alpha-1}{2}}\right), \\ \Omega\left(\min(D, \rho_s\lambda_n)\right), \\ \quad \text{if } D = \Omega(1),\ D = o\left(\sqrt{\frac{\lambda_n}{\log\lambda_n}}\right),\ E = \Omega(D^{-\alpha+1}), \end{cases} \tag{45}$$

which, combined with (42), concludes the proof of Theorem 1 if Lemmas 1 and 2 are proved.

For simplicity, $S_{i,j,k}(P, d_{i,j}, r, q)$, $S_{i,j}(P, d_{i,j}, r, q)$, $S_i(P, \mathbf{d}_i, r, q)$, $T_{i,j,k}(P, d_{i,j}, r, q)$, $T_{i,j}(P, d_{i,j}, r, q)$, $T_i(P, \mathbf{d}_i, r, q)$, and $Q_{i,j}$ of the typical e2e link are, respectively abbreviated as $S_{j,k}(P, d_j, r, q)$, $S_j(P, d_j, r, q)$, $S(P, \mathbf{d}, r, q)$, $T_{j,k}(P, d_j, r, q)$, $T_j(P, d_j, r, q)$, $T(P, \mathbf{d}, r, q)$, and Q_j. Furthermore, they are, respectively abbreviated as $S_{j,k}$, S_j, S, $T_{j,k}$, T_j, and T whenever it does not cause any confusion. Let $M = \min(\hat{D}, \hat{E})$ and μ be the average of the serving time for an M-packet at a queue in each p2p link. Then, the proofs for the lemmas are as follows.

Proof of Lemma 1: An upper bound on the DE-RATR can be obtained by using $R^{\text{ub}}(D, E|\nu_p, P, \mathbf{d}, r, q) \geq R(D, E|\nu_p, P, \mathbf{d}, r, q)$ and $\mathbf{U}_\epsilon^{\psi,\text{sup}}(D, E|d) \supset \mathbf{U}_\epsilon^{\psi}(D, E|d)$ instead of $R(D, E|\nu_p, P, \mathbf{d}, r, q)$ and $\mathbf{U}_\epsilon^{\psi}(D, E|d)$ in (14). From (8), $R^{\text{ub}}(D, E|\nu_p, P, \mathbf{d}, r, q)$ can be given by

$$\begin{aligned} &R(D, E|\nu_p, P, \mathbf{d}, r, q) \\ &\quad \leq \frac{b}{\tau}\frac{\Pr\left\{\bar{W}\left(\min(S,\hat{E})\right) \leq \hat{D}, S \leq \hat{E}\right\}}{E\left[\min\left(\bar{W}\left(\min(S,\hat{E})\right), \hat{D}\right)\right]} \\ &\quad \leq \frac{b}{\tau}\frac{\Pr\left\{S \leq \hat{E}\,\middle|\,\bar{W}\left(\min(S,\hat{E})\right) \leq \hat{D}\right\}}{E\left[\bar{W}\left(\min(S,\hat{E})\right)\right]} \\ &\quad \leq \frac{b}{\tau}\frac{q}{E\left[\min(S,\hat{E})\right]} \\ &\quad \leq \frac{b}{\tau}\frac{q}{E[S]}\frac{1}{\Pr\{S \leq \hat{E}\}} \\ &\quad \leq b\frac{q}{\tau}\frac{p\left(P, \frac{d}{N}, r, q\right)}{L(r)}\frac{1}{1-\epsilon}, \end{aligned} \tag{46}$$

where the first inequality can be obtained from (8) because $1 - P_{\text{out}}(D, E|P, \mathbf{d}, r, q) \leq \Pr\{\bar{W}(S) \leq \hat{D}, S \leq \hat{E}\} = \Pr\{\bar{W}(\min(S, \hat{E})) \leq \hat{D}, S \leq \hat{E}\}$ and τ_p/ν_p denotes the average packet arrival interval at the source so that it should be no less than the average serving time at a queue in each p2p link, i.e.,

$$\frac{\nu_p}{\tau_p} \leq \frac{1}{\tau E\left[\min\left(\bar{W}\left(\min(S,\hat{E})\right), \hat{D}\right)\right]}, \tag{47}$$

the second inequality holds from Bayes' rule and the fact that $\Pr\{X \leq \hat{D}\}/E[\min(X, \hat{D})] \leq 1/E[X]$ for a random variable X as shown in [26], the third inequality holds since $E[\bar{W}(\min(S, \hat{E}))] = E[\min(S, \hat{E})]/q$ and $\Pr\{S \leq \hat{E}|\bar{W} \times (\min(S, \hat{E})) \leq \hat{D}\} \leq 1$, the fourth inequality holds as like the second inequality, and the last inequality holds since $\Pr\{S \leq \hat{E}\} \geq 1-\epsilon$ for $(P, \mathbf{d}, r, q) \in \mathbf{U}_\epsilon^{\psi}(D, E|d)$ and $E[S(P, \mathbf{d}, r, q)] \geq L(r)/p(P, d/N, r, q)$. Let $N_s(M)$ be a binomial random variable with success probability $p(P, d/N, r, q)$ denoting the number of success transmissions during M trials. Then, we have

$$\begin{aligned} &P_{\text{out}}(D, E|P, \mathbf{d}, r, q) \\ &\quad \geq 1 - \Pr\left\{N_s(M)/L(r) \geq N\right\} \\ &\quad \geq 1 - \left(1 + (e-1)p\left(P, \frac{d}{N}, r, q\right)\right)^M \exp\left(-NL(r)\right) \\ &\quad \geq 1 - \exp\left(Mep\left(P, \frac{d}{N}, r, q\right) - N\right), \end{aligned} \tag{48}$$

where the first inequality holds because the number of successful transmissions of each P-packet within M slots at least should be greater than or equal to the number of hops, the second inequality holds by the Chernoff bound of $\Pr\{N_s(M) \geq NL(r)\} = E[\exp(N_s(M))]\exp(-NL(r))$, the third inequality holds since $1 + x \leq \exp(x)$, $e - 1 < e$, and

$L(r) \geq 1$. Also, an upper bound of $p(P, d/N, r, q)$ can be given by

$$
\begin{aligned}
&p\left(P, \frac{d}{N}, r, q\right) \\
&\quad \leq \exp\left(-\frac{2^r-1}{N^\alpha P} d^\alpha N_0 - \frac{\nu_p \lambda_q (2^r-1)^{\frac{2}{\alpha}}}{N^2} K_\alpha d^2 \frac{\tau(1-\epsilon)}{\tau_p}\right) \\
&\quad \leq \exp\left(-\frac{2^r-1}{N^\alpha P} d^\alpha N_0 - \frac{\nu_p \lambda_s (2^r-1)^{\frac{2}{\alpha}}}{N} c_1 K_\alpha d^2\right) \\
&\quad \leq \exp\left(\min\left(-\frac{2^r-1}{N^\alpha P} d^\alpha N_0, -\frac{\nu_p \lambda_s (2^r-1)^{\frac{2}{\alpha}}}{N} c_1 K_\alpha d^2\right)\right),
\end{aligned} \tag{49}
$$

where the first inequality holds from the fact that the average M-packet serving time μ is bounded by $\mu \geq L(r)\tau/q \geq \tau/q$ and the arrival rate of served M-packets is at least $\nu_p(1-\epsilon)/\tau_p$ for $(P, \mathbf{d}, r, q) \in \mathbf{U}_\epsilon^\psi(D, E|d)$ so that $\rho_p \geq (\nu_p(1-\epsilon)/\tau_p)\mu \geq \nu_p \tau(1-\epsilon)/(\tau_p q)$, the second inequality holds from the fact that there exists a constant $0 < c_1 < \infty$ such that $\lambda_q = \kappa\lambda_n \geq N\lambda_s c_1 \tau_p/((1-\epsilon)\tau)$, because it was shown that a $\Theta(N)$-hop transmission can be configured for each e2e link when $N = O(\sqrt{\lambda_n/\log \lambda_n})$ and $\lambda_s \leq \lambda_n$ [29]. By substituting (49) into (46) and (48), it is easily obtained as

$$
\begin{aligned}
&R(D, E|\nu_p, P, \mathbf{d}, r, q) \\
&\quad \leq \min\left(\frac{rq}{1-\epsilon} \exp\left(-\frac{\nu_p \lambda_s (2^r-1)^{\frac{2}{\alpha}}}{N} c_1 K_\alpha d^2\right), \frac{b\nu_p}{\tau_p}\right),
\end{aligned} \tag{50}
$$

$$
\begin{aligned}
P_{\text{out}}(D, E|P, \mathbf{d}, r, q) \geq 1 - \exp\Bigg(&Me \\
\cdot \exp\Bigg(\min\Bigg(&-\frac{2^r-1}{N^\alpha P} d^\alpha N_0, \\
&-\frac{\nu_p \lambda_s (2^r-1)^{\frac{2}{\alpha}}}{N} c_1 K_\alpha d^2\Bigg)\Bigg) - N\Bigg),
\end{aligned} \tag{51}
$$

so that $\mathbf{U}_\epsilon^{\psi,\text{sup}}(D, E|d) \supset \mathbf{U}_\epsilon^\psi(D, E|d)$. ■

Proof of Lemma 2: For $(P, \mathbf{d}, r, q) \in \mathbf{U}_\epsilon^\psi(D, E|d)$, it is straightforward from (8) that $R(D, E|\nu_p, P, \mathbf{d}, r, q) \geq (\nu_p/\tau_p) r\tau(1-\epsilon)$. Let d_m be the maximum hop distance among all $d_{i,j}$, $\mathbf{d}_{\max} = (d_m, \ldots, d_m)$ and $\sqrt{\nu} = N d_m$. Also, let $T_j^\varpi(P, \sqrt{\nu}/N, r, q)$ denote the number of transmission slots for an M-packet delivery at the jth p2p link without any drop of expired packets. Then, we have

$$
\begin{aligned}
P_{\text{out}}(D, E|P, \mathbf{d}, r, q) &\leq \Pr\{T(P, \mathbf{d}_{\max}, r, q) > M\} \\
&\leq \frac{E[T(P, \mathbf{d}_{\max}, r, q)]}{M} \\
&\leq \frac{1}{M} \sum_{j=1}^{N} E\left[T_j^\varpi(P, d_m, r, q)\right],
\end{aligned} \tag{52}
$$

where the first inequality holds since $T(P, \mathbf{d}, r, q) \geq S(P, \mathbf{d}, r, q)$, the second inequality holds by the Chernoff bound, and the last inequality holds since $T_j^\varpi(P, \sqrt{\nu}/N, r, q) \geq T_j(P, \sqrt{\nu}/N, r, q)$.

Consider the case where $L(r) = 1$, i.e., the size of an M-packet is equal to that of a P-packet, and $\tau_p = \tau$. Then, a p2p link can be considered as a Geo/Geo/1 queue with packet departure rate lower-bounded by $qp(P, d_m, r, q)$ and the packet arrival rate upper-bounded by ν_p so that $T_j^\varpi(P, d_m, r, q)$ is a geometric random variable and $E[T_j^\varpi(P, d_m, r, q)] \leq 1/((1-\alpha_m)qp(P, d_m, r, q))$ where $\alpha_m = \nu_p(1 - qp(P, d_m, r, q))/(qp(P, d_m, r, q)(1-\nu_p))$ [28]. Since $qp(P, d_m, r, q) \geq 1-\epsilon$ due to the fact that $(P, \mathbf{d}, r, q) \in \mathbf{U}_\epsilon^\psi(D, E|d)$, $(1 - qp(P, d_m, r, q))/(qp(P, d_m, r, q)) \leq 1/(qp(P, d_m, r, q)) \leq 1/(1-\epsilon)$, which implies that $\alpha_m \leq \nu_p/((1-\nu_p)(1-\epsilon))$. Then, (52) is further upper-bounded as

$$
\begin{aligned}
&P_{\text{out}}(D, E|P, \mathbf{d}, r, q) \\
&\quad \leq \frac{N}{M(1-\alpha_m)qp(P, d_m, r, q)} \\
&\quad \leq \frac{N(1-\nu_p)(1-\epsilon)}{M(1-2\nu_p-\epsilon(1-\nu_p))\, qp(P, d_m, r, q)} \\
&\quad \leq \frac{N(1-\nu_p)(1-\epsilon)}{Mq(1-2\nu_p-\epsilon(1-\nu_p))} \\
&\qquad \cdot \exp\left(\frac{\nu^{\frac{\alpha}{2}}(2^r-1)N_0}{N^\alpha P} + \frac{\nu_p \lambda_q (2^r-1)^{\frac{2}{\alpha}}}{N^2} \frac{K_\alpha \nu}{(1-\epsilon)}\right) \\
&\quad \leq \frac{N(1-\nu_p)(1-\epsilon)}{Mq(1-2\nu_p-\epsilon(1-\nu_p))} \\
&\qquad \cdot \exp\left(\frac{\nu^{\frac{\alpha}{2}}(2^r-1)N_0}{N^\alpha P} + \frac{\nu_p \lambda_s (2^r-1)^{\frac{2}{\alpha}}}{N} c_2 K_\alpha \nu\right) \\
&\quad \leq \frac{N(1-\nu_p)(1-\epsilon)}{Mq(1-2\nu_p-\epsilon(1-\nu_p))} \\
&\qquad \cdot \exp\left(\left(\frac{\nu^{\alpha/2}N_0}{N^\alpha P} + \frac{\nu_p \lambda_s}{N} c_2 K_\alpha \nu\right) \right. \\
&\qquad\qquad \left. \cdot \max\left(2^r-1, (2^r-1)^{\frac{2}{\alpha}}\right)\right),
\end{aligned} \tag{53}
$$

where the second inequality holds since $1-\alpha_m \geq (1-2\nu_p-\epsilon(1-\nu_p))/((1-\nu_p)(1-\epsilon))$, the third inequality holds from the fact that the average serving time is upper bounded by $1/(qp(P, d_m, r, q))$ and the M-packet arrival rate is at most ν_p, i.e., $\rho_p \leq \nu_p/(qp(P, d_m, r, q)) \leq \nu_p/(q(1-\epsilon))$ since $p(P, d_m, r, q) \geq 1-\epsilon$ for $(P, d_m, r, q) \in \mathbf{U}_\epsilon^\psi(D, E|d)$, the fourth inequality holds from the fact that there exists a constant $0 < c_2 < \infty$ such that $\lambda_q = \kappa\lambda_n \geq N\lambda_s c_2(1-\epsilon)$, because it was shown that a $\Theta(N)$-hop transmission can be configured for each e2e link when $N = O(\sqrt{\lambda_n/\log \lambda_n})$ and $\lambda_s \leq \lambda_n$ [29]. Thus, $\mathbf{U}_\epsilon^{\psi,\text{sub}}(D, E|d) \subset \mathbf{U}_\epsilon^\psi(D, E|d)$. ■

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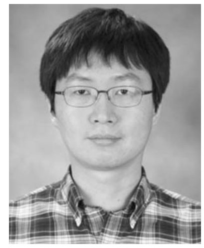

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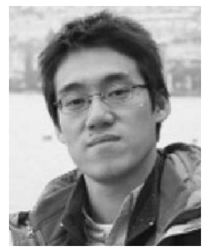

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