Low-Complexity DFT-Based Channel Estimation with Leakage Nulling for OFDM Systems

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***Abstract*—In this letter, a low-complexity but near-optimal DFT-based channel estimator with leakage nulling is proposed for OFDM systems using virtual subcarriers. The proposed estimator is composed of a time-domain (TD) index set estimation considering the leakage effect followed by a low-complexity TD post-processing to suppress the leakage. The performance and complexity of the proposed channel estimator are analyzed and verified by computer simulation. Simulation results show that the proposed estimator outperforms conventional estimators and provides near-optimal performance while keeping the low complexity comparable to the simple DFT-based channel estimator.**

***Index Terms*—Channel estimation, Leakage nulling, OFDM.**

I. Introduction

Although the linear minimum mean square error (LMMSE) estimator [1] is optimal in the sense of the mean square error (MSE) performance, the discrete Fourier transform (DFT)-based estimator has been more preferred due to the comparable complexity-performance trade-off so that it has been widely used in practice for orthogonal frequency division multiplexing (OFDM) systems [2]. However, such a DFT-based estimator suffers from nonnegligible performance degradation due to the dispersive leakage caused by virtual subcarriers which are commonly used in practical OFDM systems [3][4]. This problem is caused from the broken orthogonality of the DFT matrix and is commonly called the Gibbs phenomenon [5], which results in the corruption of the channel impulse response (CIR) [6]. Because the accuracy of a channel estimation becomes more critical as the operating signal-to-noise ratio (SNR) and the required data rate increase, it is required to achieve more precise channel estimation considering the leakage effect while maintaining the computational complexity for OFDM systems such as the 3rd generation partnership project (3GPP) long term evolution (LTE) [7].

In literature, DFT-based channel estimators considering the leakage effect can be classified as an iteration-based estimator, such as in [3], or an extrapolation-based estimator, such as in [4]. The iteration-based estimator gradually eliminates the leakage to recover the undistorted CIR but the required complexity for satisfactory performance is quite high [3]. In the extrapolation-based estimator, the leakage is suppressed by eliminating the channel frequency response discontinuity at the virtual subcarriers via extrapolation within an affordable complexity [4]. However, although wireless channels are typically very sparse so that there is a chance to further reduce the complexity [8], the extrapolation-based estimator adopts a frequency-domain (FD) post-processing so that such a time-domain (TD) complexity reduction employing the channel sparsity nature is not available. Also, any TD reduced-complexity estimator, such as the most significant tap (MST) selection-based estimator [8], needs to consider the leakage effect because severe performance degradation may occur from the false tap selection due to the distorted CIRs.

In this letter, a low-complexity DFT-based channel estimator for OFDM systems with leakage nulling is proposed, in which MSTs are selected and a regularization-based TD post-processing is performed. Thus, it is expected that the proposed estimator can effectively reduce the complexity while maintaining the channel estimation accuracy.

Notation: $\|\cdot\|$ denotes the norm of a vector, $\mathrm{diag}(\mathbf{a})$ denotes the diagonal matrix with $\mathbf{a}$ on its diagonal, and $\mathrm{diag}^{-1}(\mathbf{a})$ denotes its inverse. Also, $|\boldsymbol{\Omega}|$ and $\langle\mathbf{A}\rangle$ stand for the cardinality of a set $\boldsymbol{\Omega}$ and the number of non-zero elements in a matrix $\mathbf{A}$, respectively, and $\mathbf{A}_{\mathrm{X,Y}}$ and $[\mathbf{A}]_{m,n}$ respectively denote the submatrix of $\mathbf{A}$ with row index set $\boldsymbol{\Omega}_{\mathrm{X}}$ and column index set $\boldsymbol{\Omega}_{\mathrm{Y}}$ and the (m,n)th element of $\mathbf{A}$.

II. Generalized Framework for OFDM Channel Estimation

Consider an OFDM system with N subcarriers in which U subcarriers with index set $\boldsymbol{\Omega}_{\mathrm{U}}$ are actually used, i.e., $\boldsymbol{\Omega}_{\mathrm{U}}\subset\boldsymbol{\Omega}_{\mathrm{N}}=\{0,1,\ldots,N-1\}$. Among $\boldsymbol{\Omega}_{\mathrm{U}}$, P subcarriers with index set $\boldsymbol{\Omega}_{\mathrm{P}}\subset\boldsymbol{\Omega}_{\mathrm{U}}$ are used for pilot subcarriers. Here, $V=(N–U)P/U$ subcarriers with index set $\boldsymbol{\Omega}_{\mathrm{V}}\subset\boldsymbol{\Omega}_{\mathrm{N}}\backslash\boldsymbol{\Omega}_{\mathrm{U}}$ can be considered as artificial pilot subcarriers. Also, a length-G cyclic prefix (CP) with index set $\boldsymbol{\Omega}_{\mathrm{G}}=\{0,1,\ldots,G-1\}$ is used and similarly as in [1], it is assumed that G is larger than the maximum delay spread $\tau_{\max}$ which is much larger than the maximum number of paths, L, i.e., $L\ll\tau_{\max}<G$. Also, P and $\boldsymbol{\Omega}_{\mathrm{P}}$ are assumed to be well designed for successful channel estimation as in [9]. Let $\boldsymbol{\Omega}_{\tau}$ be the index set of the nonzero CIR taps. Then, the $G\times 1$ CIR vector $\mathbf{h}$ can be written as $\mathbf{h}=[h(0)\,h(1)\,\cdots\,h(G-1)]^T$ with $G\times G$ covariance matrix $\mathbf{R}\triangleq\mathbb{E}\{\mathbf{h}\mathbf{h}^H\}$, where $h(n)$ is the complex gain at the nth tap

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and nonzero only when $n\in\mathbf{\Omega}_\tau$. Then, after the CP removal, the received vector in the TD can be written as

$$\mathbf{y}=\mathbf{x}\otimes\mathbf{h}+\mathbf{n}, \tag{1}$$

where $\mathbf{x}$ is the $N\times1$ transmitted OFDM symbol vector in the TD before the CP insertion, $\mathbf{n}$ is the $N\times1$ independent identically distributed (i.i.d) complex white Gaussian noise vector in the TD with mean zero and covariance matrix $\sigma_n^2\mathbf{I}_N$, and $\otimes$ denotes the circular convolution. Here, the time and frequency synchronizations are assumed to be perfect by applying good synchronization schemes such as in [10].

Let $\mathbf{\Omega}_\mathrm{F}$ and $\mathbf{\Omega}_\mathrm{T}$ respectively be the selected FD and TD index sets of the DFT-based channel estimator. Also, $\mathbf{F}$ denotes the $N\times N$ unitary DFT matrix with $[\mathbf{F}]_{m,n}\triangleq\exp(-j2\pi mn/N)$. Then, the estimated CIR $\hat{\mathbf{h}}$ and channel frequency response $\hat{\mathbf{g}}$ can be respectively described by least square (LS) estimation, FD index selection, FD post-processing, TD index selection, and/or TD post-processing as

$$\hat{\mathbf{h}} = \frac{U}{NP}\mathbf{P}(\mathbf{F}_\mathrm{F,T})^H\mathbf{K}\mathbf{Q}\mathbf{F}_\mathrm{P,N}\mathbf{y}, \tag{2}$$

$$\hat{\mathbf{g}} = \mathbf{F}_\mathrm{U,T}\hat{\mathbf{h}} = \frac{U}{NP}\mathbf{\Phi}\left(\mathbf{F}_\mathrm{P,G}\mathbf{h}+\mathbf{Q}\mathbf{F}_\mathrm{P,N}\mathbf{n}\right), \tag{3}$$

where $\mathbf{K}$ and $\mathbf{P}$ respectively denote the $|\mathbf{\Omega}_\mathrm{F}|\times P$ FD post-processing matrix and the $|\mathbf{\Omega}_\mathrm{T}|\times|\mathbf{\Omega}_\mathrm{T}|$ TD post-processing matrix, $\mathbf{Q}\triangleq\mathrm{diag}^{-1}(\mathbf{F}_\mathrm{P,N}\mathbf{x})$, and $\mathbf{\Phi}\triangleq\mathbf{F}_\mathrm{U,T}\mathbf{P}(\mathbf{F}_\mathrm{F,T})^H\mathbf{K}$.

In this letter, a slowly time-varying channel is assumed so that $\mathbf{K}$ and $\mathbf{P}$ need to be computed once in a long period and $\mathbf{Q}$ can be pre-computed so that the corresponding complexity is negligible. Thus, computing (2) and (3) requires $\frac{N}{3}\log_2 N$ complex multiplications for the N-point fast Fourier transform (FFT) operation [5] ($\mathbf{F}_\mathrm{P,N}\mathbf{y}$), P for the LS estimation ($\mathbf{Q}\mathbf{F}_\mathrm{P,N}\mathbf{y}$), $\langle\mathbf{K}\rangle$ for the FD post-processing matrix multiplication ($\mathbf{K}\mathbf{Q}\mathbf{F}_\mathrm{P,N}\mathbf{y}$), $\frac{NP}{3U}\log_2(\frac{NP}{U})$ for the $\frac{NP}{U}$-point inverse FFT (IFFT) operation [5] ($\frac{U}{NP}(\mathbf{F}_\mathrm{F,T})^H\mathbf{K}\mathbf{Q}\mathbf{F}_\mathrm{P,N}\mathbf{y}$), $\langle\mathbf{P}\rangle$ for the TD post-processing matrix multiplication ($\frac{U}{NP}\mathbf{P}(\mathbf{F}_\mathrm{F,T})^H\mathbf{K}\mathbf{Q}\mathbf{F}_\mathrm{P,N}\mathbf{y}$), and $\frac{N}{3}\log_2 N$ for the N-point FFT operation ($\mathbf{F}_\mathrm{U,T}\hat{\mathbf{h}}$) so that the complexity (the number of complex multiplications) of a generalized DFT-based channel estimator can be expressed as

$$C=\frac{2N}{3}\log_2 N+P+\frac{NP}{3U}\log_2\frac{NP}{U}+\langle\mathbf{K}\rangle+\langle\mathbf{P}\rangle\,. \tag{4}$$

In Table I, $|\mathbf{\Omega}_\mathrm{F}|$, $|\mathbf{\Omega}_\mathrm{T}|$, $\langle\mathbf{K}\rangle$, $\langle\mathbf{P}\rangle$, $\mathbf{K}$, and $\mathbf{P}$ of the DFT-based channel estimators in [1][4] are summarized, where $\rho=\left(\frac{1}{P}\|\mathbf{F}_\mathrm{P,N}\mathbf{x}\|^2\right)/(N\sigma_n^2)$ denotes the average SNR of the pilot symbols. Here, $\mathbf{\Omega}_\mathrm{F}=\mathbf{\Omega}_\mathrm{P}$ for an ordinary estimator but $\mathbf{\Omega}_\mathrm{F}=\mathbf{\Omega}_\mathrm{P}\bigcup\mathbf{\Omega}_\mathrm{V}$ in the conventional extrapolation-based estimator. Also, $\mathbf{\Omega}_\mathrm{T}=\mathbf{\Omega}_\mathrm{G}$ for an ordinary estimator but $\mathbf{\Omega}_\mathrm{T}$ is comprised of MSTs in the estimator using TD complexity reduction. The simple estimator [1] performs the noise reduction only using $\mathbf{\Omega}_\mathrm{F}=\mathbf{\Omega}_\mathrm{P}$, $\mathbf{\Omega}_\mathrm{T}=\mathbf{\Omega}_\mathrm{G}$, $\mathbf{K}=\mathbf{I}_P$, and $\mathbf{P}=\mathbf{I}_G$ and the conventional estimator [4] extrapolates the channel frequency response at $\mathbf{\Omega}_\mathrm{V}$ from the channel frequency response at $\mathbf{\Omega}_\mathrm{P}$ by using the FD post-processing with the extrapolation matrix $\mathbf{K}$ to recover the orthogonality in the DFT matrix.

TABLE I
PARAMETERS FOR DFT-BASED CHANNEL ESTIMATORS

Estimator	$(\lvert\mathbf{\Omega}_\mathrm{F}\rvert, \lvert\mathbf{\Omega}_\mathrm{T}\rvert)$ $(\langle\mathbf{K}\rangle,\langle\mathbf{P}\rangle)$	**FD and TD post-processing matrices** $(\mathbf{K},\mathbf{P})$
Simple [1]	(P,G) $(0,0)$	$(\mathbf{I}_P,\mathbf{I}_G)$
Conv. [4]	$(\frac{NP}{U},G)$ $(VP,0)$	$\left(\begin{bmatrix}\mathbf{0}_{V/2\times P}\\ \mathbf{I}_P\\ \mathbf{0}_{V/2\times P}\end{bmatrix}+\begin{bmatrix}\mathbf{I}_{V/2} & \mathbf{0}_{V/2\times V/2}\\ \mathbf{0}_{P\times V/2} & \mathbf{0}_{P\times V/2}\\ \mathbf{0}_{V/2\times V/2} & \mathbf{I}_{V/2}\end{bmatrix}\cdot\right.$ $\left.[\mathbf{F}_\mathrm{V,G}(\mathbf{F}_\mathrm{P,G})^H(\mathbf{F}_\mathrm{P,G}(\mathbf{F}_\mathrm{P,G})^H+\frac{G}{\rho}\mathbf{I}_P)^{-1}],\mathbf{I}_G\right)$
Proposed	(P,L) $(0,L^2)$	$(\mathbf{I}_P,\,\frac{NP}{U}[(\mathbf{F}_{\tau,\tau})^H\mathbf{F}_{\tau,\tau}+\frac{L}{\rho}\mathbf{I}_L]^{-1})$

III. PROPOSED OFDM CHANNEL ESTIMATION

For more accurate channel estimation with low complexity, the proposed estimator first performs the TD index set estimation from the $G\times1$ CIR estimate $\hat{\mathbf{h}}=1/P(\mathbf{F}_\mathrm{P,G})^H\mathbf{Q}\mathbf{F}_\mathrm{P,N}\mathbf{y}$ and then the TD post-processing with the leakage nulling matrix $\mathbf{P}$ to suppress the leakage.

A. Threshold setting and TD index set estimation

Let $\mathbf{L}=(\mathbf{F}_\mathrm{P,G})^H\mathbf{F}_\mathrm{P,G}-P\mathbf{I}_G$ be the $G\times G$ leakage matrix with $[\mathbf{L}]_{m,n}\triangleq\exp\left(-\frac{j\pi(m-n)}{NP/U}\right)\frac{\sin(\pi U(m-n)/N)}{\sin(\pi U(m-n)/(NP))}$ [6]. Then, with virtual subcarriers (i.e., $V\neq0$ and $N\neq U$), the $G\times1$ CIR estimate is obtained as

$$\begin{aligned}\hat{\mathbf{h}} &= \frac{1}{P}(\mathbf{F}_\mathrm{P,G})^H\mathbf{Q}\mathbf{F}_\mathrm{P,N}\mathbf{y}\\ &= \mathbf{h}+\mathbf{l}+\mathbf{w},\end{aligned} \tag{5}$$

where $\mathbf{l}=\frac{1}{P}\mathbf{L}\mathbf{h}$ denotes the $G\times1$ leakage vector with $G\times G$ covariance matrix $\mathbf{R}_\mathbf{ll}\triangleq\mathbb{E}\{\mathbf{l}\mathbf{l}^H\}=\frac{1}{P^2}\mathbf{L}\mathbf{R}\mathbf{L}^H$ and $\mathbf{w}=\frac{1}{P}(\mathbf{F}_\mathrm{P,G})^H\mathbf{Q}\mathbf{F}_\mathrm{P,N}\mathbf{n}$ denotes the $G\times1$ i.i.d complex white Gaussian noise vector in the TD with mean zero and covariance matrix $\frac{1}{\rho P}\mathbf{I}_\mathrm{G}$. Also, the nth element of $\hat{\mathbf{h}}$, $\hat{h}(n)$, is zero mean complex Gaussian random variable with variance $[\mathbf{R}]_{n,n}+[\mathbf{R}_\mathbf{ll}]_{n,n}+\frac{1}{\rho P}$ if $n\in\mathbf{\Omega}_\tau$ or $[\mathbf{R}_\mathbf{ll}]_{n,n}+\frac{1}{\rho P}$ if $n\notin\mathbf{\Omega}_\tau$.

When there is no virtual subcarriers (i.e., $V=0$ and $N=U$), (5) can be rewritten as $\hat{\mathbf{h}}=\mathbf{h}+\mathbf{w}$. Let $\mathbf{\Omega}_n=\{0,1,\ldots,n-1\}$ and $\mathcal{P}(\mathbf{\Omega}_n)$ be the power set of $\mathbf{\Omega}_n$. Also, $f_n^\gamma:\mathbb{C}^n\rightarrow\mathcal{P}(\mathbf{\Omega}_n)$ denotes the function that maps any vector $\mathbf{a}\in\mathbb{C}^n$ to the corresponding set $\mathbf{\Omega}_\mathbf{a}\in\mathcal{P}(\mathbf{\Omega}_n)$, in which the indices of the elements whose absolute values exceed γ are included. Then, the TD index set for the conventional MST selection-based estimator [8] can be represented as $\mathbf{\Omega}_\mathrm{T}=f_G^\gamma(\hat{\mathbf{h}})$ for a given threshold γ.

However, the accuracy of the MST selection with virtual subcarriers is severely degraded due to the distortion caused by the leakage. Also, the leakage remains in the selected MST so that an error floor occurs unless a proper processing for the leakage is performed. To overcome the above problems, the proposed MST selection scheme is composed of the two steps as in Fig. 1: an initial index set estimation with the initial threshold γ_i to reduce the number of candidates ($|\mathbf{\Omega}_\mathrm{C}|\ll|\mathbf{\Omega}_\mathrm{G}|$) followed by a recursive MST selection with a successive leakage cancellation to determine the TD index set $\mathbf{\Omega}_\mathrm{T}$. In step 1, the initial threshold is designed to satisfy the target probability of miss detection P_MD under the assumption that $|\mathbf{\Omega}_\tau|=L$, $[\mathbf{R}]_{n,n}=\frac{1}{L}$ for $n\in\mathbf{\Omega}_\tau$, and $[\mathbf{R}_\mathbf{ll}]_{n,n}=$

Initialization step: $\mathbf{\Omega}_{\mathrm{T}} \leftarrow \phi$
First step (candidate index set estimation): $\mathbf{\Omega}_{\mathrm{C}} \leftarrow f_{G}^{\gamma_{\mathrm{i}}}(\hat{\mathbf{h}})$
Second step (recursion): **while**
$k \leftarrow \arg\max_{n \in \mathbf{\Omega}_{\mathrm{C}}} \left|\hat{h}(n)\right|$
If $\left|\hat{h}(k)\right| > \gamma_{\mathrm{r}}$, $\mathbf{\Omega}_{\mathrm{C}} \leftarrow \mathbf{\Omega}_{\mathrm{C}} \backslash \{k\}$, $\mathbf{\Omega}_{\mathrm{T}} \leftarrow \mathbf{\Omega}_{\mathrm{T}} \bigcup \{k\}$, and $\hat{h}(j) \leftarrow \hat{h}(j) - \frac{1}{P}\hat{h}(k)[\mathbf{L}]_{j,k}$ for $j \in \mathbf{\Omega}_{\mathrm{C}} \backslash \{k\}$
else break
end while

Fig. 1. The proposed MST selection with a successive leakage cancellation.

$\frac{1}{P^2G^2} tr\left(\mathbf{L}\mathbf{L}^H\right)$ for $n \in \mathbf{\Omega}_{\mathrm{G}}$. Because P_{MD} can be approximated as $P_{\mathrm{MD}} \approx 1 - \exp\left(-\frac{L\gamma_{\mathrm{i}}^2}{1/L + tr(\mathbf{L}\mathbf{L}^H)/(P^2G^2) + 1/(\rho P)}\right)$ similarly as shown in [11] under these assumptions, the initial threshold is obtained as

$$\gamma_{\mathrm{i}} = \sqrt{\frac{1}{L}\left(\frac{1}{L} + \frac{1}{P^2G^2} tr(\mathbf{L}\mathbf{L}^H) + \frac{1}{\rho P}\right) \ln\left(\frac{1}{1 - P_{\mathrm{MD}}}\right)}. \quad (6)$$

In step 2, a successive MST selection and leakage cancellation is done with the recursive threshold γ_{r}. By assuming that the leakage is sufficiently suppressed, the recursive threshold in [8] can be directly used to minimize the MSE as

$$\gamma_{\mathrm{r}} = \sqrt{\frac{\ln\left((G - L)\rho P / L^2\right)}{\rho P - L}}. \quad (7)$$

B. Time-domain post-processing

Similarly as in [4], the regularization-based TD post-processing matrix for a given constant SNR $\bar{\rho}$ is generated from the TD index set $\mathbf{\Omega}_{\mathrm{T}}$ and the $|\mathbf{\Omega}_{\mathrm{T}}| \times |\mathbf{\Omega}_{\mathrm{T}}|$ TD-LMMSE matrix $\mathbf{P}_{\mathrm{o}} = P\mathbf{R}_{\mathrm{T,T}}\left[(\mathbf{F}_{\mathrm{P,T}})^H \mathbf{F}_{\mathrm{P,T}} \mathbf{R}_{\mathrm{T,T}} + 1/\rho \mathbf{I}_{|\mathbf{\Omega}_{\mathrm{T}}|}\right]^{-1}$. By inserting $\mathbf{R}_{\mathrm{T,T}} = \frac{1}{|\mathbf{\Omega}_{\mathrm{T}}|}\mathbf{I}_{|\mathbf{\Omega}_{\mathrm{T}}|}$ and $\rho = \bar{\rho}$ in $\mathbf{P}_{\mathrm{o}}$, $\mathbf{P}$ is obtained as

$$\mathbf{P} = P\left[(\mathbf{F}_{\mathrm{P,T}})^H \mathbf{F}_{\mathrm{P,T}} + |\mathbf{\Omega}_{\mathrm{T}}| / \bar{\rho} \mathbf{I}_{|\mathbf{\Omega}_{\mathrm{T}}|}\right]^{-1}. \quad (8)$$

Also, $\hat{\mathbf{h}}$ and $\hat{\mathbf{g}}$ are obtained from (2) and (3) without any FD post-processing.

C. Complexity analysis

The simple estimator [1] is the simplest because it requires the rank reduction only (i.e., $\langle \mathbf{K} \rangle = 0$ and $\langle \mathbf{P} \rangle = 0$). The proposed estimator needs additional $\langle \mathbf{P} \rangle = |\mathbf{\Omega}_{\mathrm{T}}|^2$ complex multiplications for the TD post-processing, the conventional estimator [4] needs additional $\langle \mathbf{K} \rangle = VP$ complex multiplications for the extrapolation, and the optimal estimator [1] needs $\langle \mathbf{K} \rangle = UP$ complex multiplications for the FD-LMMSE processing, which are summarized in Table II.

D. Performance analysis

Similarly as in [8], the MSE of the proposed estimator under the assumption that $|\mathbf{\Omega}_{\tau}| = L$, $[\mathbf{R}]_{n,n} = \frac{1}{L}$ for $n \in \mathbf{\Omega}_{\tau}$, and

TABLE II
COMPLEXITY COMPARISON AMONG ESTIMATORS

Estimator	Required # of complex multiplications
Simple [1]	$\frac{2N}{3}\log_2 N + P + \frac{NP}{3U}\log_2 \frac{NP}{U}$
Conv. [4]	$\frac{2N}{3}\log_2 N + P + \frac{NP}{3U}\log_2 \frac{NP}{U} + VP$
Optimal [1]	$\frac{N}{3}\log_2 N + P + UP$
Proposed	$\frac{2N}{3}\log_2 N + P + \frac{NP}{3U}\log_2 \frac{NP}{U} + |\mathbf{\Omega}_{\mathrm{T}}|^2$

$[\mathbf{R}_{\mathbf{ll}}]_{n,n} = \sigma_{\mathrm{r}}^2$ for $n \in \mathbf{\Omega}_{\mathrm{G}}$ can be obtained by adding $[\mathbf{R}_{\mathbf{ll}}]_{n,n}$ to the variance of CIR estimate as

$$\Gamma_{\mathrm{Prop}} \approx L\left(\frac{1 - P_{\mathrm{MD}}}{\rho P} + P_{\mathrm{MD}}\left(\frac{1}{L} - \frac{\gamma_{\mathrm{i}}^2}{\exp\left(L\gamma_{\mathrm{i}}^2\right) - 1}\right)\right) + (G - L)P_{\mathrm{FA}}\left(\frac{U + \rho NP\gamma_{\mathrm{i}}^2}{\rho NP}\right), \quad (9)$$

where $P_{\mathrm{MD}} = 1 - \exp\left(-\frac{\rho P \gamma_{\mathrm{r}}^2}{1 + \rho P \sigma_{\mathrm{r}}^2 + \rho P / L}\right)$ is the probability of miss detection and $P_{\mathrm{FA}} = \exp\left(-\frac{\rho P \gamma_{\mathrm{r}}^2}{1 + \rho P \sigma_{\mathrm{r}}^2}\right)$ is the probability of false alarm. Here, $\sigma_{\mathrm{r}}^2 \triangleq \frac{1 + 3\delta}{\rho P}$ denotes the approximated residual leakage power after the successive leakage cancellation, where $\delta = \frac{VU}{NP}$. Without the MST selection (i.e., $\gamma_{\mathrm{i}} = 0$ and $\gamma_{\mathrm{r}} = 0$), the second term is dominant in (9) and $G \gg L$ so that the MSE is approximated as $\Gamma_{\mathrm{Prop}} \approx \frac{U}{NP}\frac{G}{\rho}$, which is equal to the MSE lower bound of the conventional estimator [4] or the MSE using $|\mathbf{\Omega}_{\mathrm{T}}| = G$ in (8) when the actual CIR has L uniform taps. Also, with the perfect MST selection (i.e., $P_{\mathrm{MD}} = 0$ and $P_{\mathrm{FA}} = 0$), $\Gamma_{\mathrm{Prop}} \approx \frac{1}{P}\frac{L}{\rho}$, which is quite similar to that of the optimal estimator or the MSE using $|\mathbf{\Omega}_{\mathrm{T}}| = L$ in (8) when the actual CIR has L uniform taps.

To show the impact of the channel estimation on the achievable rate, the achievable rate $R(\rho; \Gamma)$ considering the channel estimation is derived by using the results in [11][12] as a function of the average SNR ρ and the MSE Γ as

$$R(\rho; \Gamma) = \mathbb{E}_{\mathbf{h}}\left[\log_2\left(1 + \frac{\|\mathbf{h}\|^2}{\Gamma + 1/\rho}\right)\right] = \frac{1}{\ln(2)} \sum_{k \in \mathbf{\Omega}_{\tau}} \frac{1}{[\mathbf{R}]_{k,k}} \left(\prod_{i \in \mathbf{\Omega}_{\tau}, i \neq k} \frac{[\mathbf{R}]_{k,k}}{[\mathbf{R}]_{k,k} - [\mathbf{R}]_{i,i}}\right) \times \left[-\exp\left(\frac{\Gamma + 1/\rho}{[\mathbf{R}]_{k,k}}\right)[\mathbf{R}]_{k,k} Ei\left(-\frac{\Gamma + 1/\rho}{[\mathbf{R}]_{k,k}}\right)\right], \quad (10)$$

where $Ei(x) = -\int_{-x}^{\infty} \frac{1}{t}\exp(-t)dt$ denotes the exponential integral function [13].

IV. Simulation Results

For OFDM parameters, $N = 1024$, $U/P = 4$, and $G = 128$ are used and the ITU-R Vehicular A channel model [14] is used. Also, the initial threshold γ_{i} and the recursive threshold γ_{r} are respectively set to satisfy $P_{\mathrm{MD}} = 10^{-3}$ in (6) and (7) and $\bar{\rho}$ in (8) is set to 10^4.

In Fig. 2, the MSE performance of the proposed channel estimator is shown when $\delta = 0.1797$ (typical portion for virtual subcarriers) and $\delta = 0.4922$ (extreme portion for virtual subcarriers). Here, "Simple", "Conv", "Optimal", and "Proposed" respectively denote the simple estimator [1], the extrapolation-based estimator [4], the FD-LMMSE estimator [1], and the proposed estimator. Also, "Sim." and "Ana." stand

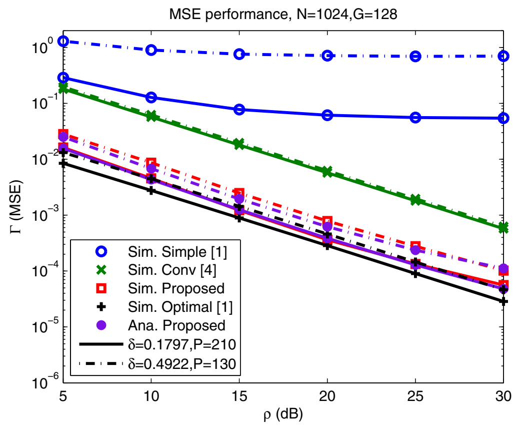

Fig. 2. MSE performance versus SNR ρ.

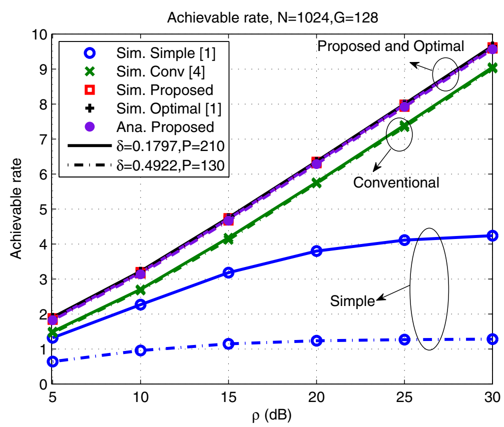

Fig. 3. Achievable rate versus SNR ρ.

for the Monte-Carlo simulation results over 10000-run and the analytic result obtained from (9). From the results, it is shown that the proposed estimator outperforms the conventional estimators regardless of δ and provides near-optimal performance.

In Fig. 3, the achievable rate using the proposed channel estimator is shown by using the same OFDM parameters. Here, "Ana." stands for the analytic result obtained from (10). From the results, it is shown that the proposed channel estimator can provide a meaningful advantage over the conventional ones as well as negligible performance gap from the optimal one in terms of the achievable rate over the entire SNR range of practical systems.

In Fig. 4, the MSE performance as well as the complexity are compared among the proposed and the conventional estimators [1][4] according to δ when $\rho = 20$ dB. Note that the number of complex multiplications required for the optimal estimator is about 10^5 in this case. From the results, it is confirmed that the performance of the proposed estimator is quite close to that of the optimal estimator over a wide range of δ while the complexity of the proposed estimator remains quite close to that of the simple estimator.

V. Conclusion

In this letter, a low-complexity DFT-based channel estimator with leakage nulling was proposed for OFDM systems using virtual subcarriers. The proposed estimator first estimates the MST set by considering the leakage effect and then performs a low-complexity leakage suppression using a regularized TD post-processing. From the results, it is confirmed that the proposed estimator can provide near-optimal performance both in the sense of the MSE and the achievable rate while keeping low complexity similar to the simplest DFT-based channel estimator. Note that the proposed approach can be extended for practical cellular systems using orthogonal frequency division multiple access or single-carrier frequency division multiple access by employing a proper interference cancellation scheme. Thus, it would be fruitful to develop a practical channel estimator suitable for LTE or LTE-advanced systems as the future work.

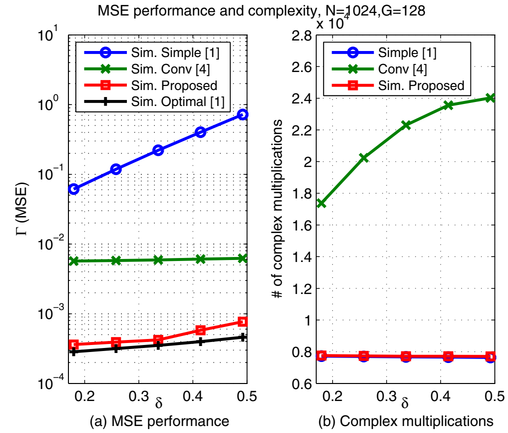

Fig. 4. Performance and complexity tradeoff versus δ.

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