Performance Analysis and Optimization of Best-M Feedback for OFDMA Systems

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***Abstract*—In this paper, the best-M feedback, where each user reports the channel quality indicators (CQIs) on its M best resource blocks (RBs), for orthogonal frequency division multiple access (OFDMA) systems is analyzed and optimized. First, the closed-form expression of the average sum-rate is derived as a function of M and the number of bits for a signal to noise ratio (SNR) quantization B. Then, M and B are jointly optimized to minimize the feedback overhead such that the desired system performance can be achieved. Numerical results confirm that the proposed analysis is quite well matched to the exact result so that our work is useful to optimize M and B instead of time-consuming computer simulations, and that the required feedback overhead to achieve a given performance decreases as the number of users and the average SNR increase.**

***Index Terms*—Orthogonal frequency division multiple access (OFDMA), best-M feedback, channel quality indicator (CQI).**

I. Introduction

In orthogonal frequency division multiple access (OFDMA) systems, the spectral efficiency can be enhanced through adaptive subcarrier, power and bit allocations [1]-[3], however, to maximize downlink performance, a base station (BS) should have perfect channel state information (CSI) of all users, which is not a practical assumption. For a practical system, channel quality indicator (CQI) feedback schemes [4]-[5] were proposed, where each user reports the quantized value of a channel amplitude or a signal to noise ratio (SNR) on each resource block (RB). Although such use of CQI requires only several bits per RB, it is still far beyond a practical range in typical OFDMA systems to let all users report CQIs on all RBs. To solve this problem, the best-M feedback method, where each user reports the CQIs on its M best RBs among total N RBs ($M < N$), was proposed in [6]. Following works [7]-[10] have shown that it can reduce the feedback overhead remarkably without significant performance degradation, and it has been already adopted in 3GPP long term evolution (LTE) standard [11], named as *user equipment selected sub-band CQI* feedback type.

The system performance and the feedback overhead of a best-M scheme are largely dependent on both M and the number of bits for a SNR quantization, B. For too large M and B, the performance close to that of the full channel information feedback can be achieved, while the total feedback overhead is prohibitive. On the other hand, for quite small M and B, although the overhead is rapidly reduced, the average sum-rate becomes severely degraded. Therefore, it is important to jointly optimize M and B minimizing the total feedback overhead required to achieve a desired system performance. When the BS and the users share the predefined quantization codebooks for multiple B and the BS adaptively controls M and B according to the system and channel environment, the system performance can be significantly enhanced. Nevertheless, most of the relevant works have concentrated on the proving the efficiency of the best-M feedback through the computer simulations. In [10], the cell throughput analysis and the optimization of M were proposed, however, it was assumed that all users have the same average SNR and the BS performs the throughput maximization scheduling, which is rather impractical for the cellular systems, and the optimization of B was not taken into account.

In this paper, we consider the OFDMA systems where the users are uniformly distributed in a cell and the BS performs the SNR based proportional fair scheduling (PFS) [12]-[13] with the best-M feedback. First, the average sum-rate is derived as a function of M and B. Then, by using the analytical results, M and B are jointly optimized to minimize the total feedback overhead required to achieve a desired system performance. Finally, numerical results are provided not only to validate the correctness of the proposed analysis and optimization but also to show that the minimum feedback overhead per user to achieve a given fraction of the average sum-rate of the full feedback method decreases as the number of users K and the average SNR increase.

II. System Model and Best-M Feedback

A. Signal model

Consider a multi-user OFDMA system where a BS and K users exist. An OFDMA symbol consists of N sub-bands. A sub-band is interpreted as a RB in the frequency domain so that at most N users can be supported per scheduling period. Assuming an i.i.d. block fading channel, the kth user's received signal on the nth RB is given as

$$y_{k,n} = \sqrt{PD_k^{-\eta}}h_{k,n}s_n + w_{k,n}, \tag{1}$$

where P is the transmit power, D_k is the distance between the BS and the kth user, and η is the path-loss exponent. The users are distributed within a cell with radius D_B so that the probability density function (PDF) of the D_k is given as $f_{D_k}(l) = 2l/D_B^2$, $0 < l \leq D_B$. Also, $h_{k,n}$ is the fading channel between the BS and the kth user on the nth RB,

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which is an i.i.d. complex Gaussian random variable with zero mean and unit variance, s_n is the data symbol on the nth RB with $E[|s_n|^2] = 1$, and $w_{k,n}$ is the additive white circularly symmetric complex Gaussian noise with variance N_0.

The kth user's long-term average SNR (ASNR) is given by $\rho_k = \rho_B(D_B/D_k)^\eta$, where $\rho_B = PD_B^{-\eta}/N_0$ is the long-term ASNR at the cell boundary, and the normalized SNR (NSNR) on the nth RB is given by $X_{k,n} = |h_{k,n}|^2$ whose cumulative distribution function (CDF) and PDF are respectively given as $F_{X_{k,n}}(x) = 1 - e^{-x}$ and $f_{X_{k,n}}(x) = e^{-x}$. Each user reports each ASNR to the BS in a period much longer than the scheduling period, which is already implemented in most practical systems, so that the feedback information for the user selection and the rate control is generated by using the NSNR on each RB.

B. Quantization

Let $\{G_{k,r}^N\}_{r=1}^N$ and $\{I_{k,r}^N\}_{r=1}^N$ be the increasing order statistics [14] and the corresponding RB indices of $\{X_{k,n}\}_{n=1}^N$, respectively, and $Q(x|B)$ be the quantized value of x for a given NSNR quantization bits, B. We assume that the NSNR is uniformly quantized in dB scale from $\gamma_m^{dB}(B)$ to $\gamma_M^{dB}(B)$ so that the qth output level in linear scale is given as

$$\gamma_q = 10^{0.1(\gamma_m^{dB}(B)+(q-1)\Delta(B))}, \quad q = 1, ..., 2^B, \tag{2}$$

where $\Delta(B) = (\gamma_M^{dB}(B) - \gamma_m^{dB}(B))/2^B$ is the step size. At a given B, the quantization bounds $\gamma_m^{dB}(B)$ and $\gamma_M^{dB}(B)$ are determined to maximize

$$E_{X_{k,n}}[Q(X_{k,n}|B)] = \sum_{q=1}^{2^B} \gamma_q(F_{X_{k,n}}(\gamma_{q+1}) - F_{X_{k,n}}(\gamma_q)), \tag{3}$$

whose joint optimal values can be easily obtained by numerical analysis.

C. Best-M feedback

For the best-M feedback [6]-[10], each user periodically reports the quantized NSNRs on its M best RBs in addition with the corresponding RB indices so that the feedback information of the kth user is given as $\mathbf{F}_k(M,B) = \{\mathbf{G}_k(M,B), \mathbf{I}_k(M)\}$, where $\mathbf{G}_k(M,B) = \{Q(G_{k,N}^N|B), Q(G_{k,N-1}^N|B), ..., Q(G_{k,N-M+1}^N|B)\}$ and $\mathbf{I}_k(M) = \{I_{k,N}^N, I_{k,N-1}^N, ..., I_{k,N-M+1}^N\}$ respectively denote the M best quantized NSNR set and the corresponding RB index set. The number of feedback bits per user per scheduling period is given as

$$F(M,B) = MB + \left\lceil \log_2 \binom{N}{M} \right\rceil \text{ [bits]}, \tag{4}$$

where the second term is the number of bits to indicate the indices of the M best RBs among total N RBs.

D. Scheduling and performance measure

Using the feedback information, the BS performs the SNR based PFS [12][13] so that the selected user on the nth RB is given as

$$k_n^* = \underset{k=1,...,K}{\arg\max}\, \delta_{\mathbf{I}_k(M)}(n)\, Q(X_{k,n}|B), \tag{5}$$

where $\delta_{\mathbf{x}}(x)$ is the indicator function whose value is 1 if $x \in \mathbf{x}$, and 0, otherwise, and the data rate is given as

$$r_n^* = \log_2\left(1 + Q\left(X_{k_n^*,n}|B\right)\rho_{k_n^*}\right) \text{ [bps/Hz]}. \tag{6}$$

We assume that on the empty RBs where no channel information is reported, no user is selected and no data is transmitted.

III. Sum-Rate Analysis

In this section, we analyze the average sum-rate for given M and B, which is given as

$$R(M,B) = E_{X_{k_n^*,n},\rho_{k_n^*}}[r_n^*] \text{ [bps/Hz]}. \tag{7}$$

The following theorem provides the closed expression of $R(M,B)$.

Theorem 1: For given M and B, the average sum-rate $R(M,B)$ is given as

$$R(M,B) = \sum_{q=1}^{2^B}\left((V(q+1;M))^K - (V(q;M))^K\right)L(q), \tag{8}$$

where

$$\begin{aligned} V(q;M) &= 1 - \tfrac{1}{M}\sum_{r=N-M+1}^{N}\left(1 - \mathcal{B}_{1-e^{-\gamma_{2^B}}}(r, N-r+1)\right) \\ &- \tfrac{1}{M}\sum_{r=N-M+1}^{N}\left(\sum_{p=q}^{2^B-1}\left(\mathcal{B}_{1-e^{-\gamma_{p+1}}}(r, N-r+1) - \mathcal{B}_{1-e^{-\gamma_p}}(r, N-r+1)\right)\right), \end{aligned} \tag{9}$$

and

$$L(q) = \log_2(1+\gamma_q\rho_B) + \tfrac{\eta}{2\log 2}\,{}_2F_1\left(1, \tfrac{2}{\eta}; 1+\tfrac{2}{\eta}; -\tfrac{1}{\gamma_q\rho_B}\right). \tag{10}$$

Here,

$$\mathcal{B}_x(a,b) = \tfrac{1}{B(a,b)}\int_0^x t^{a-1}(1-t)^{b-1}dt \tag{11}$$

is the regularized incomplete beta function and

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt \tag{12}$$

is the beta function. Also,

$${}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty}\tfrac{(a)_n(b)_n}{(c)_n}\tfrac{z^n}{n!} \tag{13}$$

is the hypergeometric function for $|z| < 1$ where $(a)_n = \sum_{m=1}^{n}(a+m-1)$ and $(a)_0 = 1$ [15].

Proof: See Appendix A. ■

Fig. 1 depicts $R(M,B)$ as a function of M for different B, ρ_B and η. The results are obtained from the analysis in Theorem 1 as well as the Monte-Carlo simulations. We set K=10 and $N = 32$. From the results, we can verify that the results from the proposed analysis are quite well matched to those from the Monte-Carlo simulations so that our analysis can be useful not only to estimate the system performance but also to control M and B to obtain a desired average sum-rate.

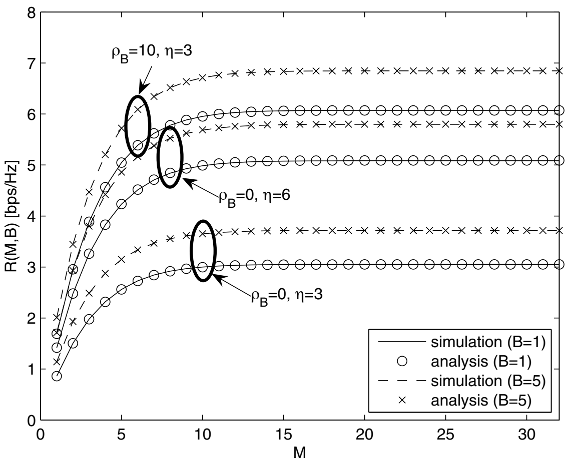

Fig. 1. $R(M, B)$ as a function of M for different B, ρ_B and η.

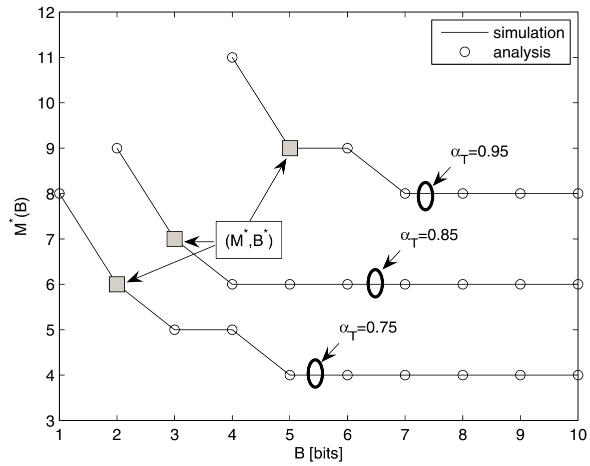

Fig. 2. $M^*(B)$ as a function of B for different α_T.

IV. Feedback Optimization

In this section, we consider the optimization of M and B to minimize the feedback overhead required to achieve a desired fraction of the average sum-rate of the full CQI feedback. Let $\alpha(M, B)$ denote the ratio of the achievable average sum-rate with M and B to that of the full CQI feedback with the sufficiently large quantization bits of B_M, which is given as

$$\alpha(M, B) = \frac{R\,(M, B)}{R\,(N, B_M)}. \tag{14}$$

The optimal M and B for a target ratio α_T $(0 < \alpha_T \leq 1)$ are given as

$$(M^*, B^*) = \underset{1 \leq M \leq N, 1 \leq B \leq B_M}{\arg\min} F\,(M, B)\,, \tag{15}$$

$$\text{subject to } \alpha\,(M, B) \geq \alpha_T. \tag{16}$$

From the result in Theorem 1 by a simple numerical algorithm such as the bisection search method, the optimal M and B can be easily obtained without a complex time-consuming computer simulations. First, for $B = 1, ..., B_M$, find

$$M^*\,(B) = \underset{1 \leq M \leq N}{\arg\min} F\,(M, B) \text{ subject to } \alpha\,(M, B) \geq \alpha_T. \tag{17}$$

Then, we can obtain the joint optimal values as

$$B^* = \underset{1 \leq B \leq B_M}{\arg\min}\; F\,(M^*\,(B)\,, B) \text{ and } M^* = M^*\,(B^*)\,. \tag{18}$$

Fig. 2 depicts $M^*(B)$ as a function of B when $\alpha_T = 0.75$, 0.85, and 0.95, respectively. The jointly optimized (M^*, B^*) are also plotted as the gray rectangles. We set $\rho_B = 0$dB, $\eta = 3$, $B_M = 10$bits, $K = 10$ and $N = 32$. From the results, it is shown that the results from the analysis are exactly the same to those from the Monte-Carlo simulations, and $M^*(B)$ decreases with B. Also, we can verify that, at a given B, both M^* and B^* increase with α_T.

Fig. 3 depicts $F\,(M^*, B^*)$ as a function of α_T for different K, ρ_B, and η. We set $B_M = 10$bits and $N = 32$. Numerical results show that the growth rate of $F\,(M^*, B^*)$ increases with α_T since the growth rate of $R(M, B)$ decreases with both M and B as shown in Fig. 1. Also, it is shown that, at a given α_T, $F\,(M^*, B^*)$ decreases as ρ_B, η, and K increase. Note that increasing η for a given ρ_B make the ASNR of the system increase, and that the expected SNR of the selected user increases with K from the multi-user diversity. Since the growth rate of $\alpha(M, B)$ increases with the ASNR of the system, the required amount of channel information per user decreases with ρ_B, η, and K.

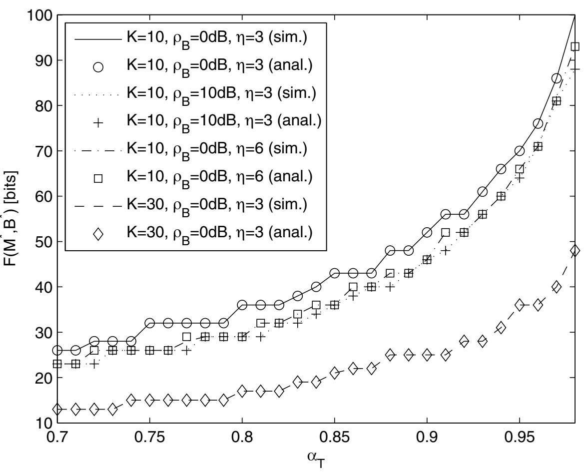

Fig. 3. $F(M^*, B^*)$ as a function of α_T for different K, ρ_B and η.

V. Conclusion

In this paper, we considered the best-M feedback for OFDMA systems. First, we derived the closed-expression of the average sum-rate of the system where users are uniformly distributed in a cell and a BS performs the PFS with the best-M feedback. Then, by using the analysis, the key parameters for the feedback, i.e., the number of reported RBs per user and the SNR quantization bits, were jointly optimized to minimize the total feedback overhead required to achieve a desired average sum-rate. From the numerical results, it was confirmed that the minimum feedback overhead per user to achieve a desired fraction of the average sum-rate of the full feedback method decreases as the number of users and the average SNR of the system increase. In the realistic cellular systems, when the BS and the users share the predefined quantization codebooks for multiple numbers of quantization bits and the BS adaptively controls the feedback overhead with the proposed optimization according to the system and channel environment, the system performance can be significantly enhanced.

Appendix A : Proof of Lemma 1

The average sum-rate is computed as

$$\begin{aligned} R(M,B) &= E_{\rho_{k_n^*}}\left[E_{X_{k_n^*,n}}\left[r_n^*|\rho_{k_n^*}\right]\right] \\ &= E_{\rho_{k_n^*}}\left[\sum_{q=1}^{2^B} Pr\left(Q\left(X_{k_n^*,n}|B\right)=\gamma_q;M\right)\log 2\left(1+\gamma_q\rho_{k_n^*}\right)\right] \\ &\overset{(a)}{=} \sum_{q=1}^{2^B} Pr\left(Q\left(X_{k_n^*,n}|B\right)=\gamma_q;M\right)E_{\rho_k}\left[\log 2\left(1+\gamma_q\rho_k\right)\right], \end{aligned} \tag{A-1}$$

where (a) follows from the independence between the NSNR and ASNR of the selected user. First, we derive $Pr\left(Q\left(X_{k_n^*,n}|B\right)=\gamma_q;M\right)$, which is given as

$$\begin{aligned} &Pr\left(Q\left(X_{k_n^*,n}|B\right)=\gamma_q;M\right) \\ &= \sum_{u=1}^{K} Pr\left(\bar{u}_n=u;M\right)Pr\left(Q\left(X_{k_n^*,n}|B\right)=\gamma_q;u,M\right) \end{aligned} \tag{A-2}$$

where $\bar{u}_n$ is the number of users reporting the channel information on the nth RB, and

$$Pr\left(\bar{u}_n=u;M\right)=\binom{K}{u}\left(\frac{M}{N}\right)^u\left(1-\frac{M}{N}\right)^{K-u}. \tag{A-3}$$

Since the NSNRs of all users are i.i.d.,

$$\begin{aligned} &Pr\left(Q\left(X_{k_n^*,n}|B\right)=\gamma_q;u,M\right) \\ &= \sum_{m=1}^{u}\binom{u}{m}\left(Pr\left(Q\left(X_{k,n}|B\right)=\gamma_q;\delta_{\mathbf{I}_k(M)}(n)=1,M\right)\right)^m \\ &\quad\cdot\left(Pr\left(Q\left(X_{k,n}|B\right)<\gamma_q;\delta_{\mathbf{I}_k(M)}(n)=1,M\right)\right)^{u-m} \\ &= \left(1-\sum_{p=q+1}^{2^B} Pr\left(Q\left(X_{k,n}|B\right)=\gamma_p;\delta_{\mathbf{I}_k(M)}(n)=1,M\right)\right)^u \\ &\quad-\left(1-\sum_{p=q}^{2^B} Pr\left(Q\left(X_{k,n}|B\right)=\gamma_p;\delta_{\mathbf{I}_k(M)}(n)=1,M\right)\right)^u. \end{aligned} \tag{A-4}$$

Let $\mu_{k,n}$ is the channel rank of the kth user on the nth RB which is given as $\mu_{k,n}=r$ when $n=I_{k,r}^N$. Since the rank of the reported NSNR at a given M is uniformly distributed from $N-M+1$ to N,

$$\begin{aligned} &Pr\left(Q\left(X_{k,n}|B\right)=\gamma_p;\delta_{\mathbf{I}_k(M)}(n)=1,M\right) \\ &= \sum_{r=N-M+1}^{N} Pr\left(\mu_{k,n}=r;M\right)Pr\left(Q\left(G_{k,r}^N|B\right)=\gamma_p\right) \\ &= \sum_{r=N-M+1}^{N}\frac{1}{M}Pr\left(\gamma_p\le G_{k,r}^N<\gamma_{q+1};B\right). \end{aligned} \tag{A-5}$$

From the fact that $F_{G_{k,r}^N}(g)=\mathcal{B}_{F_{X_{k,n}}(g)}(r,N-r+1)$ [14],

$$\begin{aligned} &Pr\left(\gamma_p\le G_{k,r}^N<\gamma_{q+1};B\right) \\ &= \begin{cases} \mathcal{B}_{1-e^{-\gamma_{p+1}}}(r,N-r+1)-\mathcal{B}_{1-e^{-\gamma_p}}(r,N-r+1), & p<2^B, \\ 1-\mathcal{B}_{1-e^{-\gamma_p}}(r,N-r+1), & p=2^B. \end{cases} \end{aligned} \tag{A-6}$$

By inserting (A-3) and (A-4) into (A-2) and using the binomial expansion,

$$Pr\left(Q\left(X_{k_n^*,n}|B\right)=\gamma_q;M\right)=\left(V\left(q+1;M\right)\right)^K-\left(V\left(q;M\right)\right)^K. \tag{A-7}$$

Also, since

$$\int_t^\infty \frac{x^{a-1}}{1+x}dx=\frac{t^{a-1}}{1-a}\,{}_2F_1\left(1,1-a;2-a;-\frac{1}{t}\right) \tag{A-8}$$

for $a<1$ [15],

$$\begin{aligned} &E_{\rho_k}\left[\log_2\left(1+\gamma_q\rho_k\right)\right] \\ &= \int_0^{D_B}\log_2\left(1+\gamma_q\frac{Pl^{-\eta}}{N_0}\right)\frac{2l}{D_B^2}dl \\ &= \int_0^1\log_2\left(1+\gamma_q\rho_B t^{-\frac{\eta}{2}}\right)dt \\ &= \log_2\left(1+\gamma_q\rho_B\right)+\frac{\left(\gamma_q\rho_B\right)^{2/\eta}}{\log 2}\int_{\gamma_q\rho_B}^\infty\frac{x^{-2/\eta}}{1+x}dx \\ &= L(q). \end{aligned} \tag{A-9}$$

Hence, from (A-2), (A-7) and (A-9),

$$R(M,B)=\sum_{q=1}^{2^B}\left(\left(V\left(q+1;M\right)\right)^K-\left(V\left(q;M\right)\right)^K\right)L(q), \tag{A-10}$$

which is the desired result.

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