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RESEARCH ARTICLE

Partial feedback schemes for MIMO–OFDMA systems using random beamforming: analysis and optimization

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ABSTRACT

In this paper, partial feedback schemes for a multiple input multiple output orthogonal frequency division multiple access system using a random beamforming are analyzed and optimized. For partial feedback schemes, the partial channel quality indicator feedback schemes and the partial channel quality rank indicator feedback schemes are considered. For these schemes, we first derive the effective downlink spectral efficiencies by considering the required uplink resource for feedback together over block fading channels. Then, by using the analysis, the amount of feedback overhead per user is optimized to maximize the effective downlink spectral efficiency according to the system and channel parameters. From the analysis and numerical examples, we show that the partial channel quality rank indicator feedback scheme provides better performance than the partial channel quality indicator feedback scheme unless the channel rapidly varies owing to its feedback efficiency, and the proposed adaptive control of the feedback overhead can improve the performance of practical systems, such as the 3rd generation partnership project (3GPP) long term evolution, in practical scenarios. Copyright © 2012 John Wiley & Sons, Ltd.

KEYWORDS
OFDMA; MIMO; MIMO–OFDMA; partial feedback information

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1. INTRODUCTION

Multi-user multiple input multiple output (MIMO) orthogonal frequency division multiple access (OFDMA) is a multiple access technique combining OFDMA with MIMO techniques. Since OFDMA decomposes a frequency-selective fading channel into a number of orthogonal narrowband frequency-flat fading channels, we can enhance the spectral efficiency through adaptive subcarrier, power, and bit allocation by combining MIMO techniques, such as optimal dirty paper coding (DPC) [1,2] or sub-optimal zero-forcing beamforming (ZF-BF) [3,4] or ZF-DPC [5,6]. However, these MIMO techniques assume perfect channel state information (CSI) at the transmitter, which is not a practical assumption.

As alternative approaches, limited feedback schemes for MIMO broadcast channels have been widely studied in literature [7–15]. In [8–10], the quantized CSI feedback scheme was proposed, where each user reports the quantized channel information to its base station (BS), and the BS selects the users and the corresponding precoding matrix. In [11–13], the channel quality indicator (CQI) feedback scheme was proposed, where pilots are transmitted on randomly chosen spatial beams, and each user reports the signal to noise ratio (SNR) of each beam as a CQI. It was shown in [10,12] that as the number of users, K increases, the optimal sum-rate[†] growth of $\log\log K$ can be obtained by using an orthogonal random BF (R-BF) with CQI feedback as well as by using a ZF-BF with quantized CSI feedback. To further reduce the feedback overhead, the channel quality rank indicator (CQRI) feedback scheme was proposed in [14,15], where only SNR ranks for the user selection are reported from all users in the first feedback and the SNR information of the selected users for the adaptive transmission or rate control is reported in the second feedback. However, although such use of quantized

[†]Here, the sum-rate denotes the sum of all users' achievable data rates.

CSI, CQI or CQRI requires only small number of bits per subchannel, it is still far beyond a practical range in typical multi-user MIMO–OFDMA systems to let all users report such information on all subchannels.

To solve this problem, partial feedback approaches have been proposed [16–26], which can be classified into two types: the fixed partial feedback type [16–20] where each user reports channel information on a predetermined number of its best subchannels (having the highest SNRs or largest capacity), and the selective partial feedback type [20–26] where each user reports channel information on all subchannels whose quality (SNR or capacity) exceeds a predetermined threshold. A kind of fixed partial CQI feedback scheme is already adopted in the 3GPP long term evolution (LTE) standard [17], named as *user equipment selected sub-band CQI* feedback type, and the fixed partial CQRI feedback scheme also has been extensively used within the European Wireless World Initiative New Radio (WINNER) projects [18]. These partial feedback schemes have been considered as promising feedback overhead reduction strategies for practical cellular systems.

Note that for a given partial feedback scheme, the amount of feedback affects not only the required uplink resource for feedback but also the downlink performance. As we reduce the feedback amount, although less uplink resource is required, the downlink performance degrades. On the other hand, as we increase the feedback amount, the downlink performance improves, whereas the required uplink resource increases. Thus, it is very important to optimize the feedback amount to maximize the overall system performance considering both the downlink resource for the data transmission and the uplink resource for the downlink channel information feedback. In most previous work, however, the system performance was evaluated just in terms of achievable downlink throughput or downlink sum-rate at a certain feedback overhead, and the overall system performance optimization considering the uplink resource for feedback was not considered. In [22], the feedback overhead optimization to achieve a near-optimal sum capacity growth rate was considered. However, it cannot give an insight on how to optimize the feedback overhead to maximize the overall system performance.

As a reasonable criterion for the system performance, we suggest the effective downlink spectral efficiency (EDSE) defined as *the average number of bits transmitted to users per the entire resources used for data transmission (in downlink) and the downlink channel information feedback (in uplink)*, which is different from the conventional downlink spectral efficiency (DSE) defined as *the average number of bits transmitted to users per the downlink resource used for data transmission*. By employing the EDSE instead of the DSE, we can obtain the optimal feedback overhead, which maximizes the overall system performance including the uplink resource used for the channel information feedback together. Note that the EDSE of a partial feedback scheme significantly depends on the system and channel parameters, such as the number of users, the user distribution, and the time and frequency selectivity of the channel, and so on. For example, in order to maximize the EDSE, we need to allow each user to send feedback information on less (more) subchannels as the number of users increases (decreases). Thus, as we adaptively control the feedback overhead according to the system and channel parameters, the enhanced overall spectral efficiency in practical networks can be achieved.

In this paper, we consider the optimization problem to maximize the EDSE in multi-user MIMO–OFDMA systems with partial feedback information. As a MIMO technique, we focus on the orthogonal R-BF with CQI feedback, which is simple and much less affected by quantization error than the precoding with quantized CSI feedback. As the partial feedback schemes, the partial CQI feedback (PCQI) schemes and the partial CQRI (PCQRI) feedback schemes are considered. For these fixed and selective PCQI and PCQRI feedback schemes (F-PCQI, S-PCQI, F-PCQRI, S-PCQRI), we first derive and compare the closed-form expressions on the DSE in an i.i.d. block-fading channel, which is the first main contribution of this paper. Then, by using the analysis, a practically implementable feedback optimization scheme is proposed to maximize the EDSE, which is the second main contribution of this paper, and is verified to prove the performance gain when applied to the 3GPP LTE system in realistic channel environment with practical scenarios.

The remainder of this paper is organized as follows. In Section 2, the multi-user MIMO–OFDMA system model is introduced, and the partial feedback schemes are described in detail. The downlink performance and its asymptotic behavior of each feedback scheme are respectively derived in closed forms in Section 3, and the feedback overhead optimization to maximize the overall system performance is considered in Section 4. Finally, concluding remark is given in Section 5.

A note on notation and abbreviations: $\{f(y)\}^{-1}$ denotes the inverse function of $y = f(x)$, and $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ denotes the binomial coefficient. $\mathbf{S}^{-1}$ is the inverse of a square matrix $\mathbf{S}$, $[\mathbf{M}]_{ij}$ is the element of the ith row and the jth column of matrix $\mathbf{M}$, and $|\mathbf{M}|$ is the cardinality of matrix $\mathbf{M}$. $\mathbf{M}^T$ and $\mathbf{M}^*$ denote the transpose and conjugate transpose, respectively. Also, $\mathbf{I}_M$ is the $M \times M$ identity matrix, and $\mathbb{Z}^{M \times N}$, $\mathbb{R}^{M \times N}$ and $\mathbb{C}^{M \times N}$ denote the set consisting all $M \times N$ matrices whose elements are integer, real number, and complex number, respectively.

2. SYSTEM MODEL

Consider a multi-user MIMO–OFDMA system shown in Figure 1, where a BS has N_t antennas and K users have N_r antennas. Let N_s and N_b be the number of subcarriers and the number of sub-bands [27] (consisting of contiguous $N_c = N_s/N_b$ subcarriers) in a MIMO–OFDMA symbol, respectively. For each sub-band, the BS constructs $N_a(= \min(N_t, N_r))$ (pseudo) random beams so that a MIMO–OFDMA symbol is composed of $N_a N_b (= N)$

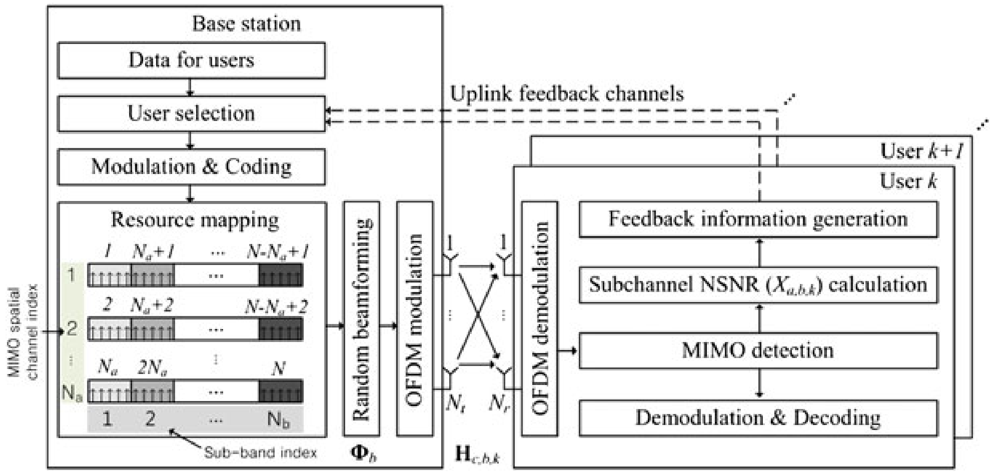

Figure 1. The multi-user MIMO–OFDMA system model.

subchannels. Note that a subchannel can be interpreted as a scheduling unit in spatial and frequency domains so that at most, N users can be supported per scheduling interval $T_s = TN_m$, where T is a MIMO–OFDMA symbol duration and N_m is the number of MIMO–OFDMA symbols per scheduling interval. Dropping the time index for simplicity, the kth user's received signal vector on the cth subcarrier of the bth sub-band in a certain MIMO–OFDMA symbol, $\mathbf{y}_{c,b,k} \in \mathbb{C}^{N_r \times 1}$, can be expressed as

$$\mathbf{y}_{c,b,k} = \sqrt{P_b}\left(\frac{D_b}{D_k}\right)^{\eta/2} \mathbf{H}_{c,b,k}\Phi_b \mathbf{s}_{c,b} + \mathbf{n}_{c,b,k} \quad \text{for } c=1,...,N_c,\ b=1,...,N_b,\ k=1,...,K, \tag{1}$$

where $\mathbf{s}_{c,b} \in \mathbb{C}^{N_a \times 1}$ is the data symbol vector with $E[\mathbf{s}_{c,b}\mathbf{s}^*_{c,b}] = \frac{1}{N_a}\mathbf{I}_{N_a}$, and $\mathbf{n}_{c,b,k} \in \mathbb{C}^{N_r \times 1}$ is the additive white circularly symmetric complex Gaussian noise vector with covariance matrix $N_o\mathbf{I}_{N_r}$. Also, D_b and D_k respectively denote the cell radius and the distance between the BS and the kth user, η is the path-loss exponent, and P_b is the average received signal energy at the cell boundary. The long-term average SNR of the kth user is given by $\rho_k = \frac{P_b}{N_o N_a}\left(\frac{D_b}{D_k}\right)^{\eta} = \rho_b \left(\frac{D_b}{D_k}\right)^{\eta}$, where $\rho_b = P_b/(N_o N_a)$ is the long-term average SNR at the cell boundary. In addition, $\mathbf{H}_{c,b,k} \in \mathbb{C}^{N_r \times N_t}$ is the MIMO fading channel matrix composed of i.i.d. complex Gaussian random variables with zero mean and unit variance, and $\Phi_b \in \mathbb{C}^{N_t \times N_a}$ is the (pseudo) R-BF matrix on the bth sub-band, which is generated according to an isotropic distribution [12]. The R-BF matrices are constructed to be the same for different subcarriers on a sub-band and to be independent for different sub-bands. Since Φ_b consists of N_a random orthonormal vectors, that is, $\Phi_b = [\phi_{b,1}, ..., \phi_{b,N_a}]$ where, if $a_1 = a_2$, $\phi^*_{b,a_1}\phi_{b,a_2} = 1$, otherwise, $\phi^*_{b,a_1}\phi_{b,a_2} = 0$ for $a_1, a_2 = 1, ..., N_a$, the effective fading channel $\overline{\mathbf{H}}_{c,b,k} (= \mathbf{H}_{c,b,k}\Phi_b) \in \mathbb{C}^{N_r \times N_a}$ also consists of i.i.d. complex Gaussian random variables with zero mean and unit variance.

After OFDMA demodulation and MIMO detection, the desired symbols are decoded. Furthermore, the SNR of each subchannel, which is defined as the average SNR of all subcarriers within the subchannel at the corresponding spatial stream and time slot, is measured before the feedback information is generated. Note that the SNR of each subchannel depends on the MIMO scheme, and we assume that all users exploit the ZF receiver for the MIMO detection so that it is sufficient for each user to estimate only its effective fading channel $\overline{\mathbf{H}}_{c,b,k}$[‡]. We assume that each user has perfect knowledge on its effective fading channel so that the SNR of the subchannel at the ath stream on the bth sub-band of the kth user in the tth time slot is given by [28]

$$Z_{a,b,k}(t) = \frac{\rho_k}{N_c N_m}\sum_{c=1}^{N_c}\sum_{m=1}^{N_m} \left(\left[(\overline{\mathbf{H}}^*_{c,b,k}((t-1)N_m+m) \times \overline{\mathbf{H}}_{c,b,k}((t-1)N_m+m))^{-1}\right]_{aa}\right)^{-1}, \tag{2}$$

‡Here, we assume that pilot symbols for the bth sub-band are precoded using the same R-BF matrix Φ_b and then transmitted along with the data symbols in the bth sub-band.

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and the normalized SNR (NSNR), defined as the instantaneous SNR divided by the long-term average SNR, is given by

$$X_{a,b,k}(t) = \frac{1}{\rho_k} Z_{a,b,k}(t). \tag{3}$$

Here, we define $\left\{Y^N_{n,k}(t)\right\}^N_{n=1}$ and $\left\{I^N_{n,k}(t)\right\}^N_{n=1}$ respectively as the increasing order statistic of $\{X_{a,b,k}(t)\}^{Na,N_b}_{a=1,b=1}$ [29] and the corresponding subchannel index. Using the NSNR information, each user generates the feedback information according to its feedback scheme and the allowed amount of feedback overhead.

Let $\Omega \in$ {F-PCQI, S-PCQI, F-PCQRI, S-PCQRI} be the partial feedback scheme and β be the allowed amount of feedback overhead $(0<\beta\leq 1)$, defined as the average number of reported subchannels per user normalized by the total number of subchannels. In the PCQI schemes ($\Omega \in$ {F-PCQI, S-PCQI}), each user periodically reports its best subchannel indices and the corresponding instantaneous SNRs as well as the minimum SNR among all subchannels as shown in Figure 2. The feedback information of the kth user can be defined as

$$\mathbf{F}^{\Omega}_k(t;\beta) = \left\{\mathbf{u}^{\Omega}_k(t;\beta), \mathbf{g}^{\Omega}_k(t;\beta)\right\}, \quad \Omega \in \{\text{F-PCQI,S-PCQI}\} \tag{4}$$

where $\mathbf{u}^{\Omega}_k(t;\beta) = \left[u^{\Omega}_{1,k}(t)\ u^{\Omega}_{2,k}(t) \ldots u^{\Omega}_{A^{\Omega}_k(t),k}(t)\right]^T \in \mathbb{Z}^{A^{\Omega}_k(t)\times 1}$ and $\mathbf{g}^{\Omega}_k(t;\beta) = \left[g^{\Omega}_{1,k}(t)\ g^{\Omega}_{2,k}(t) \ldots g^{\Omega}_{A^{\Omega}_k(t),k}(t), g^{\min}_k(t)\right]^T \in \mathbb{R}^{(A^{\Omega}_k(t)+1)\times 1}$, respectively, denote the selected subchannel index vector and the corresponding SNR vector of the kth user. Here, $A^{\Omega}_k(t)$ is the number of reported subchannels from the kth user in the tth slot, given as $A^{\Omega}_k(t) = \beta N$ in the fixed case and a random variable with $E[A^{\Omega}_k(t)] = \beta N$ in the selective case. Also, $u^{\Omega}_{n,k}(t)$ and $g^{\Omega}_{n,k}(t)$ respectively denote the nth preferred subchannel index and the corresponding SNR of the kth user, which are respectively given as

$$u^{\Omega}_{n,k}(t) = I^N_{N-n+1,k}(t), \tag{5}$$

$$g^{\Omega}_{n,k}(t) = \rho_k Y^N_{N-n+1,k}(t), \tag{6}$$

and

$$g^{\min}_k(t) = \rho_k Y^N_{1,k}(t), \quad \Omega \in \{\text{F-PCQI,S-PCQI}\}. \tag{7}$$

Based on the feedback information, the BS performs the normalized SNR based proportional fair scheduling (NSNR-PFS) to accommodate both efficiency and fairness [30,31] for the non-empty subchannel (feedback information from at least one user). Also, all the empty subchannels are allocated to the user having the highest minimum NSNR. Here, we assume that each user reports its long-term average SNR, ρ_k, to the BS in a period much longer than that of the CQI reporting, which is already implemented in most practical cellular systems. The selected user matrix for the $(t+1)$th time slot, $\mathbf{k}^{\Omega}(t+1) \in \mathbb{Z}^{N_a\times N_b}$, is given as follows:

$$\mathbf{k}^{\Omega}(t+1) = \left[k^{\Omega}_{a,b}(t+1)\right]^{N_a,N_b}_{a=1,b=1}, \quad \Omega \in \{\text{F-PCQI,S-PCQI}\}, \tag{8}$$

where $k^{\Omega}_{a,b}(t)$ denotes the user index selected for the subchannel at the ath stream on the bth sub-band in the tth slot, which is given as

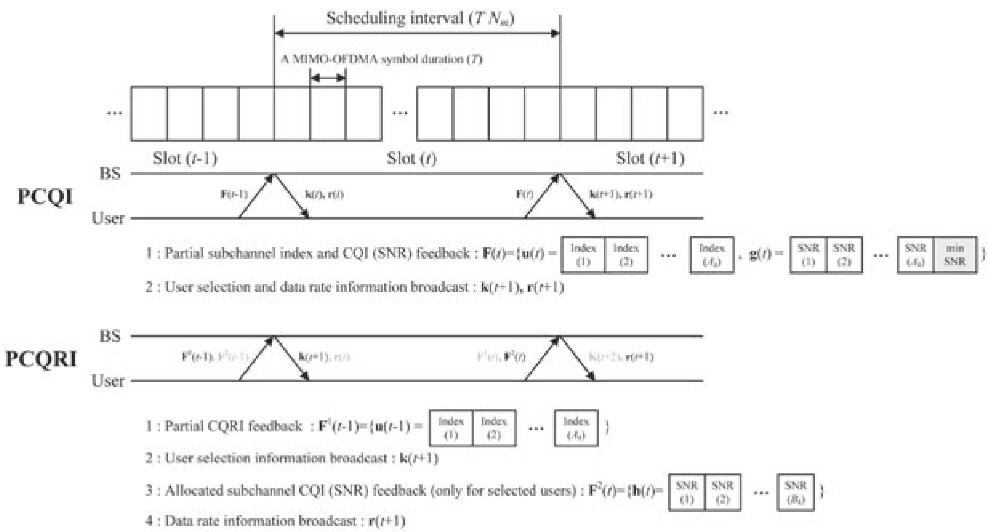

Figure 2. The frame structures of the PCQI and the PCQRI feedback schemes.

$$k^{\Omega}_{a,b}(t+1)=\begin{cases}\underset{k\in\{k|(a,b)\in\mathbf{u}^{\Omega}_{k}(t)\}}{\arg\max} X_{a,b,k}(t), & \text{if } (a,b)\in\mathcal{N}^{\Omega}(t),\\ \underset{k\in\{1,\ldots,K\}}{\arg\max}\ Y^{N}_{1,k}(t) & \text{, otherwise,}\end{cases} \quad (9)$$

where $\mathcal{N}^{\Omega}(t)=\{u^{\Omega}_{n,k}(t)|n=1,...,A^{\Omega}_{k}(t),k=1,...,K\}$ is the index set of non-empty subchannels. The selected NSNR, $x^{\Omega}_{a,b}(t+1)$, at the ath stream on the bth sub-band for the $(t+1)$th slot is given as

$$x^{\Omega}_{a,b}(t+1)=\begin{cases}X_{a,b,k^{\Omega}_{a,b}(t+1)}(t) & \text{, if } (a,b)\in\mathcal{N}^{\Omega}(t),\\ Y^{N}_{1,k^{\Omega}_{a,b}(t+1)}(t) & \text{, otherwise,}\end{cases} \quad (10)$$

and let the selected NSNR matrix be denoted as

$$\mathbf{x}^{\Omega}(t+1)=\left[x^{\Omega}_{a,b}(t+1)\right]^{N_a,N_b}_{a=1,b=1},\quad \Omega\in\{\text{F-PCQI,S-PCQI}\}. \quad (11)$$

Note that the data rate is assigned based on the channel quality in the feedback information, but the actual achievable rate depends on the channel quality at the time of transmission, and an efficient hybrid-ARQ (HARQ) protocol is typically used to track the channel quality gap. In this paper, an efficient HARQ protocol is assumed for simplicity so that the achievable data rate matrix at the $(t+1)$th time slot is given as $\mathbf{r}^{\Omega}(t+1)=\left[r^{\Omega}_{a,b}(t+1)\right]^{N_a,N_b}_{a=1,b=1}$, where

$$r^{\Omega}_{a,b}(t+1)=\begin{cases}\log_2(1+\rho_{k^{\Omega}_{a,b}(t+1)}X_{a,b,k^{\Omega}_{a,b}(t+1)}(t+1)) & \text{, if } (a,b)\in\mathcal{N}^{\Omega}(t),\\ \log_2(1+\rho_{k^{\Omega}_{a,b}(t+1)}Y^{N}_{1,k^{\Omega}_{a,b}(t+1)}(t+1)) & \text{otherwise.}\end{cases} \quad (12)$$

After the user selection and data rate control on each subchannel, the BS broadcasts the results $\mathbf{k}^{\Omega}(t+1)$ and $\mathbf{r}^{\Omega}(t+1)$ to the users along with the data transmission.

On the other hand, in the PCQRI schemes ($\Omega\in\{\text{F-PCQRI, S-PCQRI}\}$), the feedback is divided into two steps, that is, $\mathbf{F}^{\Omega,1}_{k}(t-1;\beta)$ and $\mathbf{F}^{\Omega,2}_{k}(t)$. As the first feedback for the user selection at the BS, each user reports the partial subchannel indices in the descending order of SNRs as shown in Figure 2, that is,

$$\begin{aligned}\mathbf{F}^{\Omega,1}_{k}(t;\beta)=&\mathbf{u}^{\Omega}_{k}(t;\beta)\\ =&[u^{\Omega}_{1,k}(t)u^{\Omega}_{2,k}(t)\ldots u^{\Omega}_{A^{\Omega}_{k}(t),k}(t)]^{T},\\ &\Omega\in\{\text{F-PCQRI, S-PCQRI}\},\end{aligned} \quad (13)$$

where $u^{\Omega}_{n,k}(t)=I^{N}_{N-n+1,k}(t)$ for $n=1,...,A^{\Omega}_{k}$, $\Omega\in\{\text{F-PCQRI, S-PCQRI}\}$. The SNR rank of the (a,b)th subchannel at the kth user, $\mu_{a,b,k}(t)$, is obtained at the BS as

$$\mu_{a,b,k}(t)=\begin{cases}N-n+1, & \text{if } u^{\Omega}_{n,k}(t)=(a,b),\\ 0, & \text{otherwise.}\end{cases} \quad (14)$$

Based on the first feedback information, the BS allocates each non-empty subchannel to the user having the highest rank. If more than one users tie, the BS randomly selects one among them. For empty subchannels, a random user selection is used. The selected user at the (a,b)th subchannel, $k^{\Omega}_{a,b}(t+1)$, is given as

$$k^{\Omega}_{a,b}(t+1)=\begin{cases}\underset{k\in\{1,\ldots,K\}}{\arg\max}\ \mu_{a,b,k}(t-1), & \text{if } (a,b)\in\mathcal{N}^{\Omega}(t),\\ \text{randomly choose one among}\{1,\ldots,K\}, & \text{otherwise.}\end{cases} \quad (15)$$

After the user selection, the BS broadcasts the user selection matrix, $\mathbf{k}^{\Omega}(t+1)$, in the tth slot. Let $\mathbf{v}^{\Omega}_{k}(t)=\left\{v^{\Omega}_{n,k}(t)\right\}^{B^{\Omega}_{k}(t)}_{n=1}=\{(a,b)|k^{\Omega}_{a,b}(t+1)=k\}$ and $B^{\Omega}_{k}(t)=|\mathbf{v}^{\Omega}_{k}(t)|$ be the set of selected subchannels of the kth user and its cardinality, respectively. Each selected user $k\in S^{\Omega}_{K}(t)=\{k|B^{\Omega}_{k}(t)>0\}$ reports the SNR of the selected subchannels as the second feedback for the rate control as

$$\begin{aligned}\mathbf{F}^{\Omega,2}_{k}(t)=&\mathbf{h}^{\Omega}_{k}(\mathbf{v}^{\Omega}_{k}(t))\\ =&[h^{\Omega}_{1,k}(t)h_{2,k}(t)\ldots h_{B^{\Omega}_{k}(t),k}(t)]^{T},\\ &\Omega\in\{\text{F-PCQRI, S-PCQRI}\},\end{aligned} \quad (16)$$

where $h^{\Omega}_{n,k}(t)=\rho_k X_{v^{\Omega}_{n,k}(t),k}(t)$. The selected SNR is directly determined from the second feedback, that is,

$$x^{\Omega}_{a,b}(t+1)=X_{a,b,k^{\Omega}_{a,b}(t+1)}(t),\quad \Omega\in\{\text{F-PCQRI, S-PCQRI}\}. \quad (17)$$

After the rate control based on $\mathbf{x}^{\Omega}(t+1)$, the BS broadcasts the results along with the data transmission. For the

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Table I. The average number of feedback bits per scheduling interval.

	Average number of feedback bits
F-PCQI	$F^{F\text{–}PCQI}(\beta = \frac{N_p}{N}\|N, K) = \left[(N_q + \lceil \log_2(N) \rceil)\, N_p + N_q\right] K$ (SNRs with indices on partial (N_p) subchannels and minimum SNR of all (K) users)
S-PCQI	$F^{S\text{–}PCQI}(\beta = e^{-\alpha}\|N, K) = [(N_q + \lceil \log_2(N) \rceil)\, Ne^{-\alpha} + N_q]K$ (SNRs with indices on partial (average $Ne^{-\alpha}$) subchannels and minimum SNR of all (K) users)
F-PCQRI	$F^{F\text{–}PCQRI}(\beta = \frac{N_p}{N}\|N, K) = \lceil \log_2(N) \rceil\, N_p K + NN_q$ (Indices of partial (N_p) subchannels of all (K) users and SNRs of the selected users)
S-PCQRI	$F^{S\text{–}PCQRI}(\beta = e^{-\alpha}\|N, K) = \lceil \log_2(N) \rceil Ne^{-\alpha} K + NN_q$ (Indices of partial (average $Ne^{-\alpha}$) subchannels of all (K) users and SNRs of the selected users)

PCQRI schemes, we also assume that the (a, b)th element of the achievable rate matrix can be written as $r^{\Omega}_{a,b}(t+1) = \log_2(1 + \rho_{k^{\Omega}_{a,b}(t+1)} X_{a,b,k^{\Omega}_{a,b}(t+1)}(t+1))$.

Note that the achievable data rate on a certain slot, $\mathbf{r}^{\Omega}(s)$, depends on the channel parameters such as the joint distributions of the NSNRs on all subchannels during the time slots from feedback reporting to actual transmission, which will be referred to as a *scheduling period*, and the long-term average SNRs of all users, as well as the system parameters such as the number of subchannels and the number of users. Let $\mathbf{p} = [N, K, f_{\underline{X},\underline{\rho}}]$ denote the system and channel parameter vector, where $f_{\underline{X},\underline{\rho}}$ denotes the joint pdf of $\underline{X}$ (the NSNRs on all subchannels during one scheduling period) and $\underline{\rho}$ (the long-term average SNRs of all users). The DSE can be obtained as

$$R^{\Omega}_{\mathrm{DL}}(\beta|\mathbf{p}) = E_{\underline{X},\underline{\rho}}\left[\frac{1}{N_b}\sum_{a=1}^{N_a}\sum_{b=1}^{N_b}\left[\mathbf{r}^{\Omega}(t)\right]_{a,b}\right] = E_{\underline{X},\underline{\rho}}\left[\frac{1}{N_b}\sum_{a=1}^{N_a}\sum_{b=1}^{N_b}\log_2(1+\rho_{k^{\Omega}_{a,b}(t)}X_{a,b,k^{\Omega}_{a,b}}(t))\right] \quad \text{[bps/Hz]}. \tag{18}$$

Now, consider the EDSE. The EDSE of the scheme Ω with overhead β at given $\mathbf{p}$ is given by

$$S^{\Omega}(\beta|\mathbf{p}) = \frac{R^{\Omega}_{\mathrm{DL}}(\beta|\mathbf{p})T_{\mathrm{DL}}W_{\mathrm{DL}}}{T_{\mathrm{DL}}W_{\mathrm{DL}} + T^{\Omega}_{\mathrm{UL}}(\beta|\mathbf{p})W^{\Omega}_{\mathrm{UL}}(\beta|\mathbf{p})} \quad \text{[bps/Hz]}, \tag{19}$$

where $T_{\mathrm{DL}} = T_s (= TN_m)$ [s] is the downlink time duration for data transmission, and $T^{\Omega}_{\mathrm{UL}}(\beta|\mathbf{p})$ [s] is the uplink time duration for channel information feedback. In addition, $W^{\mathrm{DL}} = \frac{N_s}{T}$ [Hz] is the downlink bandwidth for data transmission, and $W^{\Omega}_{\mathrm{UL}}(\beta|\mathbf{p})$ [Hz] is the uplink bandwidth used for feedback. In this paper, the feedback information is assumed to be conveyed by a fixed modulation order, M_{UL}, and channel coding rate, C_{UL}, through the uplink control channel, for example, *PUCCH* in LTE [17]. When the number of total feedback bits for a given feedback scheme Ω with β is $F^{\Omega}(\beta|N, K)$, the required uplink resource is given by

$$T^{\Omega}_{\mathrm{UL}}(\beta|\mathbf{p})W^{\Omega}_{\mathrm{UL}}(\beta|\mathbf{p}) = \frac{F^{\Omega}(\beta|N, K)}{M_{\mathrm{UL}}C_{\mathrm{UL}}} \quad [\mathrm{s} \times \mathrm{Hz}]. \tag{20}$$

For a partial feedback scheme Ω, the optimal amount of feedback overhead at given $\mathbf{p}$, $\beta^{\Omega}_{opt}(\mathbf{p})$, is obtained as

$$\beta^{\Omega}_{\mathrm{opt}}(\mathbf{p}) = \underset{0<\beta\le 1}{\arg\max}\, S^{\Omega}(\beta|\mathbf{p}). \tag{21}$$

In Table I, the (average) number of feedback bits per scheduling interval of the partial feedback schemes ($F^{\Omega}(\beta|N, K)$) are summarized for a given feedback ratio[§]. Here, N_q bits are used for representing SNR[¶] and $\lceil \log_2(N) \rceil$ denotes the number of bits to indicate a subchannel index. Although the selective schemes are known to have performance gain over the fixed schemes [20], it might require additional feedback overhead for user identification, which is ignored in this paper, since each user

[§]Here, we assume that each user sends feedback information once in every scheduling interval for simplicity. Also, it is straightforward to modify (20) according to the case where we set each user's feedback period proportional to its coherence time.

[¶]In this paper, we ignore the SNR quantization effect on the system performance for simplicity.

reports a variable number of CQIs or CQRIs. As will be shown in Section 3, the fixed and selective schemes provide the same downlink performance unless N is quite small. Hence, for typical values of N, a fixed partial feedback scheme is preferred. Also, the PCQRI schemes are more efficient than the PCQI schemes in terms of the feedback overhead because each user reports only the indices and only selected users report SNRs. Hence, when the channel varies slowly, a PCQRI scheme may be preferred than a PCQI scheme. However, as the time selectivity of the channel increases, it is expected that the PCQRI schemes suffer from additional delay caused by the two-step approach.

3. DOWNLINK SPECTRAL EFFICIENCY OF PARTIAL FEEDBACK SCHEMES

In this section, the DSEs of the four partial feedback schemes are derived[‖]. In order to make the mathematical analysis tractable, we assume the block fading channel [32,33], where the channel state remains quasi-static within a fading block, but becomes independent across a different fading block. The main results in this paper do not deviate much from those under a practical channel model as long as the system parameters are appropriately chosen, which will be verified in Section 4. The detailed assumptions are as follows.

(A1) The zero-delay and error-free uplink feedback channel is assumed so that the channel variation between feedback reporting and actual transmission is ignored, that is, $x^{\Omega}_{a,b}(t) = X_{a,b,k^{\Omega}_{a,b}(t)}(t)$.

(A2) Frequency flat fading channel per sub-band is assumed, that is, $\overline{\mathbf{H}}_{c_1,b,k} = \overline{\mathbf{H}}_{c_2,b,k}$ for $c_1 \neq c_2$ and $c_1, c_2 = 1, \cdots, N_c$.

(A3) The NSNRs of all subchannels and all users are independent and identically distributed.

(A4) Users are independent and identically distributed in a cell with uniform distribution per unit area so that $f_{D_k}(l) = \frac{2l}{D_b^2}$ for $0 \leq l \leq D_b$, and $f_{D_k}(l) = 0$, otherwise.

(A5) The number of subchannels, N, is sufficiently large.

[‖]Our analysis of the PCQI schemes can be considered as an extension of the work in [12,22] with R-BF and NSNR-PFS.

From the above i.i.d. block fading channel assumption, the joint pdf of the NSNRs on all subchannels during a scheduling period and the long-term average SNRs of all users can be simplified as $f_{\underline{X},\underline{\rho}}(\underline{X},\underline{\rho}) = \prod_{a=1}^{N_a}\prod_{b=1}^{N_b}\prod_{k=1}^{K} f_X(X_{a,b,k}) f_\rho(\rho_k)$. Here, considering a typical cellular system, we assume $N_t \geq N_r (= N_a)$ so that, $f_X(x) = e^{-x}$ [28]. From (18) and the above assumptions, the DSE of the partial feedback scheme Ω can be obtained by

$$
\begin{aligned}
R^{\Omega}_{\mathrm{DL}}(\beta|\mathbf{p}') &= \lim_{N\to\infty} E_{\underline{X},\underline{\rho}}\left[\frac{1}{N_b}\sum_{a=1}^{N_a}\sum_{b=1}^{N_b} \log_2(1+\rho_{k^{\Omega}_{a,b}(t)} x^{\Omega}_{a,b}(t))\right] \\
&= \lim_{N\to\infty} E_{x^{\Omega},\rho}\left[N_a \log_2(1+\rho x^{\Omega})\right] \\
&= E_\rho\left[\lim_{N\to\infty} N_a E_{x^{\Omega}}\left[\log_2(1+\rho x^{\Omega})\right]\right] \\
&= \int_0^{D_b}\left(\lim_{N\to\infty} E_{x^{\Omega}}\left[\log_2\left(1+\rho_b\left(\frac{D_b}{l}\right)^{\eta} x^{\Omega}\right)\right]\right) \\
&\quad \times \frac{2N_a l}{D_b^2}\mathrm{d}l \quad [\text{bps/Hz}],
\end{aligned} \tag{22}
$$

where $\mathbf{p}' = [K, f_{X^{\Omega}}, f_\rho]$ denotes the simplified system and channel parameter vector after the i.i.d. block fading channel assumption and $f_{x^{\Omega}}(x)$ is the pdf of the selected SNR at a certain subchannel for the scheme Ω. The following Definition and Lemmas are useful for deriving DSE.

Definition 1. *Let EQ_1 and EQ_2 respectively denote*

$$EQ_1 : \int_0^{\infty} \log_2(1+c_1 y)e^{-c_2 y}\mathrm{d}y = \frac{e^{c_2/c_1}}{c_2\log(2)} E_1\left(\frac{c_2}{c_1}\right), \tag{23}$$

$$EQ_2 : \int_{\alpha}^{\infty} \log_2(1+c_1 y)e^{-c_2 y}\mathrm{d}y = \frac{1}{c_2\log(2)}\left(e^{-c_2\alpha}\log(1+c_1\alpha) + e^{\frac{c_2}{c_1}} E_1\left(\frac{c_2}{c_1}(1+c_1\alpha)\right)\right), \tag{24}$$

where $E_1(x) = \int_x^{\infty} e^{-t}t^{-1}\mathrm{d}t$ is the first order exponential integral function [34].

Lemma 1. *Since $f_X(x) = e^{-x}$, the pdf and cdf of $Y^N_{n,k}$* [29] *are respectively given as*

$$
\begin{aligned}
f_{Y^N_{n,k}}(y) &= n\binom{N}{n}\sum_{m=0}^{n-1}\binom{n-1}{m} \\
&\quad \times (-1)^m e^{-(N-n+m+1)y},
\end{aligned} \tag{25}
$$

$$F_{Y_{n,k}^N}(y) = n\binom{N}{n}\sum_{m=0}^{n-1} \frac{\binom{n-1}{m}(-1)^m\left(1-e^{-(N-n+m+1)y}\right)}{N-n+m+1}. \tag{26}$$

Lemma 2. *Let* $R(n|N,\rho) = E\left[\log_2(1+\rho Y_{n,k}^N)\right]$, *which can be obtained as*

$$\begin{aligned} R(n|N,\rho) &= \frac{n\binom{N}{n}}{\log(2)}\sum_{m=0}^{n-1}\frac{\binom{n-1}{m}(-1)^m e^{\frac{N-n+m+1}{\rho}}}{(N-n+m+1)} \\ &\quad\times E_1\left(\frac{N-n+m+1}{\rho}\right), \\ &\simeq \log_2\left(1-\rho\log\left(1-\frac{n}{N+1}\right)\right), \\ &\quad \textit{when } N \gg 1 \end{aligned} \tag{27}$$

Proof. (27) is straightforward from Lemma 1 and EQ_1. Also, consider i.i.d. random variables $\{G_{a,b,k} = \log_2(1+\rho X_{a,b,k})\}_{a=1,b=1}^{N_a,N_b}$ and their order statistics $\{G_{n,k}^N\}_{n=1}^N$. In [29], it is shown that $E[G_{n,k}^N] \simeq \{F_{G_{a,b,k}}(\frac{n}{N+1})\}^{-1}$ for sufficiently large N. Since $F_{X_{a,b,k}}(x) = 1-e^{-x}$, $F_{G_{a,b,k}}(y) = 1-e^{-(2^y-1)/\rho}$. Thus, $R(n|N,\rho) \simeq \left\{F_{G_{a,b,k}}\left(\frac{n}{N+1}\right)\right\}^{-1} = \log_2\left(1-\rho\log\left(1-\frac{n}{N+1}\right)\right)$. □

3.1. Subchannel spectral efficiency at given ρ

In this subsection, $R_S^\Omega(\beta|N,K,\rho) \triangleq E_{x^\Omega}\left[\log_2(1+\rho x^\Omega)\right]$ and $R_S^\Omega(\beta|K,\rho) \triangleq \lim_{N\to\infty} R_S^\Omega(\beta|N,K,\rho)$ are derived. Let $p_C(r|\beta,K)$ denotes the probability that r CQIs (or CQRIs) are reported for a specific subchannel, which is given as

$$p_C(r|\beta,K) = \binom{K}{r}\beta^r(1-\beta)^{K-r}. \tag{28}$$

Then, the subchannel spectral efficiency is given as

$$R_S^\Omega(\beta|N,K,\rho) = \sum_{r=0}^{K} p_C(r|\beta,K)E_{x_r^\Omega}\left[\log_2(1+\rho x_r^\Omega)\right], \tag{29}$$

where x_r^Ω is the selected NSNR at a certain subchannel with r reported CQ(R)Is in scheme Ω.

Theorem 1. $R_S^\Omega(\beta|N,K,\rho)$ *is obtained as (29), where*

$$E_{x_r^\Omega}\left[\log_2(1+\rho x_r^\Omega)\right] = \begin{cases} \frac{a_1}{\log(2)}\sum_{\boldsymbol{n}\in\mathcal{N}}\sum_{\boldsymbol{m}\in\mathcal{M}(\boldsymbol{n})} a_2a_3\sum_{\boldsymbol{l}\in\mathcal{L}}\frac{a_4}{a_5}e^{\frac{a_5}{\rho}}E_1\left(\frac{a_5}{\rho}\right), & r>0, \Omega=\textit{F-PCQI}, \\ \frac{K}{\log(2)}\sum_{m=0}^{K-1}\frac{\binom{K-1}{m}(-1)^m e^{N(m+1)/\rho}}{m+1}E_1\left(\frac{N(m+1)}{\rho}\right), & r=0, \Omega=\textit{F-PCQI}, \\ \frac{r}{\log(2)}\sum_{m=0}^{r-1}\frac{\binom{r-1}{m}(-1)^m e^{(m+1)\alpha}}{m+1}\left(e^{-(m+1)\alpha}\log(1+\rho\alpha)+e^{\frac{(m+1)}{\rho}}E_1\left(\frac{(m+1)}{\rho}(1+\rho\alpha)\right)\right), & r>0, \Omega=\textit{S-PCQI}, \\ \frac{K}{\log(2)}\sum_{m=0}^{K-1}\frac{\binom{K-1}{m}(-1)^m e^{N(m+1)/\rho}}{m+1}E_1\left(\frac{N(m+1)}{\rho}\right), & r=0, \Omega=\textit{S-PCQI}, \\ \sum_{n=N-N_p+1}^{N}\sum_{m=1}^{r}\binom{r}{m}(\frac{1}{N_p})^m(1-\frac{N-n+1}{N_p})^{r-m}R(n|N,\rho), & r>0, \Omega=\textit{F-PCQRI}, \\ \frac{1}{(N-N_p)}\sum_{n=1}^{N-N_p}R(n|N,\rho), & r=0, \Omega=\textit{F-PCQRI}, \\ \sum_{n=1}^{N}\sum_{m=1}^{r}c_1^m c_2^{r-m}\frac{n\binom{N}{n}}{\log(2)\left(1-\mathcal{B}_{F_X(\alpha)}(n,N-n+1)\right)}\sum_{m=0}^{n-1}\frac{\binom{n-1}{m}(-1)^m}{(N-n+m+1)} & \\ \cdot\left(e^{-(N-n+m+1)\alpha}\log(1+\rho\alpha)+e^{\frac{N-n+m+1}{\rho}}E_1\left(\frac{N-n+m+1}{\rho}(1+\rho\alpha)\right)\right), & r>0, \Omega=\textit{S-PCQRI}, \\ \frac{1}{\log(2)(1-e^{-\alpha})}\left[-e^{-\alpha}\log(1+\rho\alpha)+e^{\frac{1}{\rho}}\left(E_1\left(\frac{1}{\rho}\right)-E_1\left(\alpha+\frac{1}{\rho}\right)\right)\right], & r=0, \Omega=\textit{S-PCQRI}. \end{cases} \tag{30}$$

Here, $\boldsymbol{n} = (n_0,\ldots,n_{r-1})^T, \boldsymbol{m} = (m_0,\ldots,m_{r-1})^T$, *and* $\boldsymbol{l} = (l_1,\ldots,l_{r-1})^T$. *Also,* $\mathcal{N} \subset \mathbb{Z}^{r\times1}, \mathcal{M}(\boldsymbol{n}) \subset \mathbb{Z}^{r\times1}$, *and* $\mathcal{L} \subset \mathbb{Z}^{(r-1)\times1}$ *respectively denote the set of all vectors whose elements are integers between* $N-N_p+1$ *and* N, *the set consisting of all vectors whose* i*th element* m_i *is an integer between* 0 *and* n_i, *and the set consisting of all vectors whose elements are either 0 or 1.*

Furthermore, $a_1 = \frac{r}{N_p^r}$, $a_2 = (-1)^{\sum_{i=0}^{r-1} m_i} n_0 \binom{N}{n_0}\binom{n_0-1}{m_0}$, $a_3 = \prod_{i=1}^{r-1} \frac{n_i \binom{N}{n_i}\binom{n_i-1}{m_i}}{N-n_i+m_i+1}$, $a_4 = (-1)^{\sum_{i=1}^{r-1} l_i}$, $a_5 = \sum_{i=0}^{r-1} l_i (N-n_i+m_i+1)$, $l_0 = 1$, $c_1 = \frac{1-\mathcal{B}_{F_X(\alpha)}(n,N-n+1)}{Ne^{-\alpha}}$, *and* $c_2 = \sum_{l=1}^{n-1} \frac{1-\mathcal{B}_{F_X(\alpha)}(l,N-l+1)}{Ne^{-\alpha}}$. *The sketch for the proof is given in Appendix A.*

Theorem 2. *As* $N \to \infty$, $R_S^{\Omega}(\beta|K,\rho) = \lim_{N\to\infty} R_S^{\Omega}(\beta|N,K,\rho)$ *is obtained as*

$$
R_S^{\Omega}(\beta|K,\rho) = \begin{cases} \left(1-(1-\beta)^K\right)\log_2(1-\rho\log(\beta)) \\ \quad + \sum_{r=1}^{K} \frac{r\binom{K}{r}\beta^{r-1}(1-\beta)^{K-r}}{\log(2)} \sum_{m=0}^{r-1} \frac{\binom{r-1}{m}(-\beta^{-1})^m e^{\frac{m+1}{\rho}}}{m+1} E_1\left((m+1)\left(\frac{1}{\rho}-\log(\beta)\right)\right), & \Omega = \{\text{F-PCQI, S-PCQI}\}, \\ \left(1-(1-\beta)^{K-1}\right)\log_2(1-\rho\log(\beta)) + \frac{K}{\log(2)} \sum_{m=0}^{K-1} \frac{\binom{K-1}{m}(-1)^m e^{\frac{m+1}{\rho}}}{m+1} \\ \quad \cdot E_1\left((m+1)\left(\frac{1}{\rho}-\log(\beta)\right)\right) + \frac{(1-\beta)^{K-1}e^{\frac{1}{\rho}}}{\log(2)}\left(E_1\left(\frac{1}{\rho}\right) - E_1\left(\frac{1}{\rho}-\log(\beta)\right)\right), & \Omega = \{\text{F-PCQRI, S-PCQRI}\}. \end{cases} \tag{31}
$$

The proof of Theorem 2 is given in Appendix B.

Figure 3 compares $R_S^{\Omega}(\beta|N,K,\rho)$ and $R_S^{\Omega}(\beta|K,\rho)$ of each partial feedback scheme (both from analysis and Monte-Carlo simulation under the i.i.d. block fading assumption), as a function of β when $N = 32$ and 128, respectively. Here, we set $K = 10$, $\rho = 10$ dB, $N_s = 1024$, and $N_t = N_r = N_a = 2$. From the results, we can verify that the analytic results given in Theorem 1 are quite well matched with those obtained from Monte-Carlo simulation. Here, $R_S^{\Omega}(\beta|N,K,\rho)$ (or $R_S^{\Omega}(\beta|K,\rho)$) increases as β increases owing to the increased multiuser diversity (in both PCQI and PCQRI schemes) and the reduced probability of occurring empty subchannels (in PCQI schemes only), which explains the high growth rate at small β and the larger growth rate of the PCQI schemes than the PCQRI schemes. Also, the asymptotic analysis given in Theorem 2 does not deviate much from the exact analysis even for quite small N and is almost the same when N is as small as 128. Thus, the asymptotic result in Theorem 2 can be used without any loss for practical cellular systems.

3.2. DSE of the partial feedback schemes

As shown in (22), the DSE, $R_{\mathrm{DL}}^{\Omega}(\beta|\mathbf{p}')$, can be obtained as

$$
R_{\mathrm{DL}}^{\Omega}(\beta|\mathbf{p}') = \int_0^{D_b} R_S^{\Omega}\left(\beta|K,\rho_b\left(\frac{D_b}{l}\right)^{\eta}\right) \times \frac{2N_a l}{D_b^2}\mathrm{d}l \quad \text{[bps/Hz]}, \tag{32}
$$

which is given by the following Definition 2 and Theorem 3.

Definition 2. *Let* $G_1(c_3,c_4)$, $G_2(c_3,c_4)$, *and* $G_3(c_3,c_4,c_5)$ *respectively denote*

$$
G_1(c_3,c_4) = \int_0^1 \log\left(1+c_3u^{-c_4}\right)\mathrm{d}u, \tag{33}
$$

$$
G_2(c_3,c_4,c_5) = \int_0^1 e^{c_3u^{c_5}} E_1\left(c_3u^{c_5}-c_4\right)\mathrm{d}u, \tag{34}
$$

$$
\begin{aligned} G_3(c_3,c_4) &= \int_0^1 e^{c_3u^{c_4}} E_1\left(c_3u^{c_4}\right)\mathrm{d}u \\ &= \sum_{s=0}^{\infty} \frac{c_3^s}{s!}\left[\left(\frac{u^{c_4s+1}}{c_4s+1}\right)\left(E_1\left(c_3u^{c_4}\right) - \left(c_3u_4^c\right)^{-\frac{c_4s+1}{c_4}}\Gamma\left(\frac{c_4s+1}{c_4}, c_3u^{c_4}\right)\right)\right]_{u=0}^{u=1} \\ &= \sum_{s=0}^{\infty} \frac{c_3^s E_1(c_3) - c_3^{-\frac{1}{c_4}}\left(\Gamma\left(\frac{c_4s+2}{c_4}, c_3\right) - \Gamma\left(\frac{c_4s+2}{c_4}, 0\right)\right)}{s!\,(c_4s+1)}, \end{aligned} \tag{35}
$$

where $\Gamma(a,b) = \int_b^{\infty} u^{a-1}e^{-u}\mathrm{d}u$ *is the upper incomplete Gamma function* [34]. *When* $c_4 = 2$, $G_1(c_3,c_4) = 2\sqrt{c_3}\arctan\left(\frac{1}{\sqrt{c_3}}\right) + \log(1+c_3)$, *and* $G_2(c_3,c_4,c_5)$ *can be obtained by using numerical integration.*

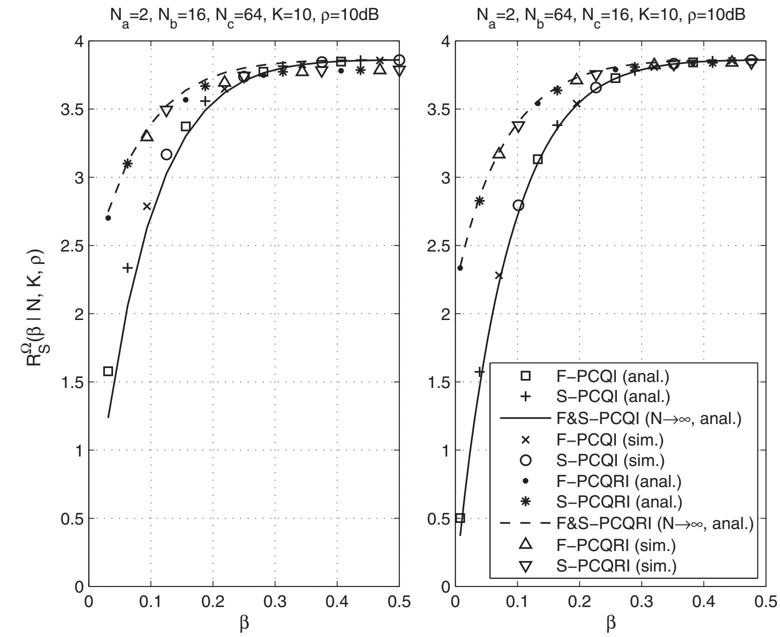

Figure 3. $R_S^{\Omega}(\beta|N,K,\rho)$ and $R_S^{\Omega}(\beta|K,\rho)$ when (left) $N=32$ and (right) 128, $K=10$, $\rho=10$ dB.

Theorem 3. *The DSE of the partial feedback scheme* Ω *is given as*

$$R^{\Omega}_{\mathrm{DL}}(\beta|\mathbf{p}') = \begin{cases} \frac{N_a(1-(1-\beta)^K)}{\log(2)} G_1\left(-\rho_b\log(\beta), \frac{\eta}{2}\right) + \frac{N_a}{\log(2)} \sum_{r=1}^{K} r\binom{K}{r}\beta^{r-1}(1-\beta)^{K-r} \sum_{m=0}^{r-1}\binom{r-1}{m}\frac{(-\beta^{-1})^m}{m+1} G_2\left(\frac{m+1}{\rho_b}, (m+1)\log(\beta), \frac{\eta}{2}\right) & \\ + \frac{N_a(1-\beta)K}{\log(2)} \sum_{m=0}^{K-1}\frac{\binom{K-1}{m}(-1)^m}{m+1} G_3\left(\frac{N(m+1)}{\rho_b}, \frac{\eta}{2}\right), & \Omega = \{\text{F-PCQI, S-PCQI}\}, \\ \frac{N_a(1-(1-\beta)^{K-1})}{\log(2)} G_1\left(-\rho_b\log(\beta), \frac{\eta}{2}\right) + \frac{N_a K}{\log(2)} \sum_{m=0}^{K-1}\frac{\binom{K-1}{m}(-1)^m}{m+1} G_2\left(\frac{m+1}{\rho_b}, (m+1)\log(\beta), \frac{\eta}{2}\right) & \\ + \frac{N_a(1-\beta)^{K-1}}{\log(2)} \left(G_3\left(\frac{1}{\rho_b}, \frac{\eta}{2}\right) - G_2\left(\frac{1}{\rho_b}, \log(\beta), \frac{\eta}{2}\right)\right), & \Omega = \{\text{F-PCQRI, S-PCQRI}\}. \end{cases} \quad (36)$$

The proof of Theorem 3 is given in Appendix C.

4. FEEDBACK OVERHEAD OPTIMIZATION

Let us recall the definition of EDSE in (19) with the DSE in (36), which is given as

$$S^{\Omega}(\beta|\mathbf{p}') = \frac{R^{\Omega}_{\mathrm{DL}}(\beta|\mathbf{p}')N_m N_s}{N_m N_s + \frac{F^{\Omega}(\beta|N,K)}{M_{\mathrm{UL}}C_{\mathrm{UL}}}} \quad [\text{bps/Hz}]. \quad (37)$$

As can be seen in Figure 3, the growth rate of the DSE is quite high around $\beta = 0$, but monotonically decreases as β increases (like a double logarithm). Also, from Table I, the growing slope of the feedback overhead, $F^{\Omega}(\beta|N,K)$, according to β is a constant. Hence, when β is smaller than a certain threshold, the EDSE increases with β owing to the rapid growth rate of the DSE. On the other hand, when β is larger than the threshold, the EDSE decreases with β owing to the reduced growth rate of the DSE. Thus, the EDSE is a unimodal function of β and has a global optimum at $\beta = \beta^{\Omega}_{\mathrm{opt}}$. For each partial feedback scheme, we can easily find $\beta^{\Omega}_{\mathrm{opt}}$ from (36) by a simple numerical algorithm such as bisection search method [35], which can be easily applied in a practical system to adaptively control the feedback overhead using (36) and (37).

Figures 4 and 5 respectively depict $\beta^{\Omega}_{\mathrm{opt}}$ and $S(\beta^{\Omega}_{\mathrm{opt}}|\mathbf{p}')$, $\Omega \in$ {F-PCQI, F-PCQRI}, under the i.i.d. block fading assumption, as a function of K with different N_m and ρ_b. The results are obtained from a bisection search using (36) and (37) as well as the exhaustive search using the Monte-Carlo simulations. We set $N_s = 1024$, $N_t = N_r = N_a = 2$, $N_b = 64$, $N_c = 16$, $\eta = 4$, $M_{\mathrm{UL}} = 2$, $C_{\mathrm{UL}} = 0.5$, and $N_q = 5$ bits. First, we can see that $\beta^{\Omega}_{\mathrm{opt}}$ and $S(\beta^{\Omega}_{\mathrm{opt}}|\mathbf{p}')$ obtained from (36) and (37) are quite well matched to those

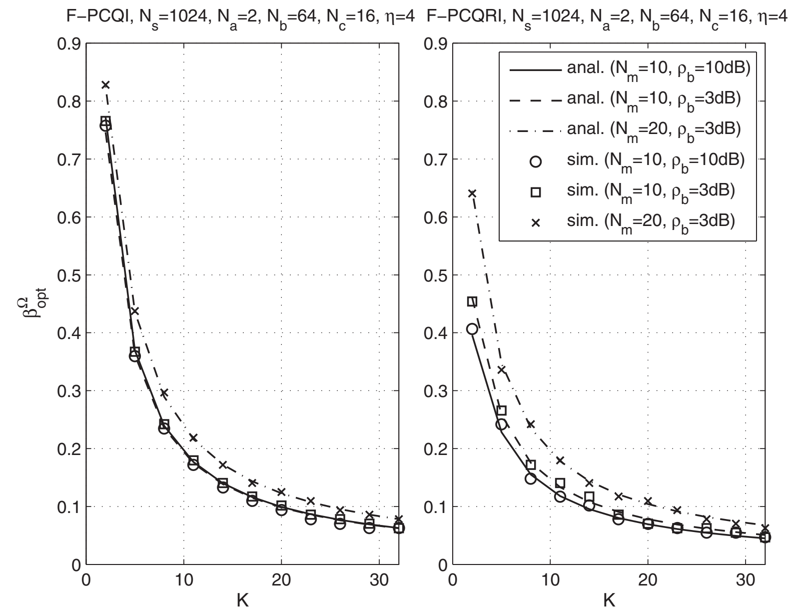

Figure 4. $\beta_{\text{opt}}^{\Omega}$, $\Omega \in$ {F-PCQI, F-PCQRI}, as a function of K with different N_m and ρ_b.

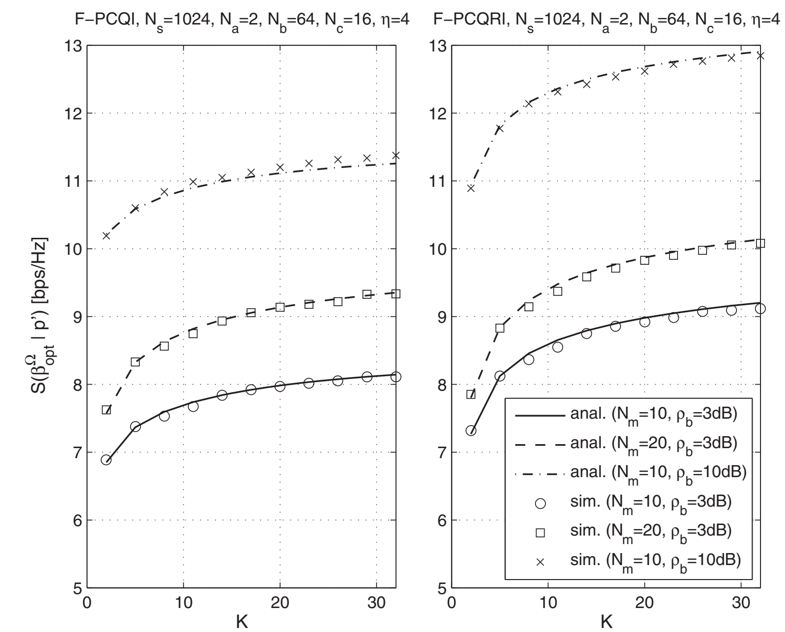

Figure 5. $S(\beta_{\text{opt}}^{\Omega}|\boldsymbol{p}')$, $\Omega \in$ {F-PCQI, F-PCQRI}, as a function of K with different N_m and ρ_b.

obtained from the simulation. Also, it is shown that the optimal feedback overhead depends mainly on K and is given as $O\left(\frac{1}{K}\right)$. In addition, we can see that $\beta_{\text{opt}}^{\Omega}$ increases as N_m increases (the feedback cost becomes cheaper)** or ρ_b decreases (more multiuser diversity is required)††. Hence, it is shown to be helpful to adaptively control the feedback overhead to maximize the overall system performance at given system and channel parameters.

Figure 6 depicts $S(\beta_{\text{opt}}^{\Omega}|\mathbf{p}')$, $\Omega \in$ {F-PCQI, F-PCQRI}, under the assumptions A2) $\sim$ A5) only, by considering the

**Or, uplink feedback rate increases.

††Reduced BS transmit power or larger pathloss exponent.

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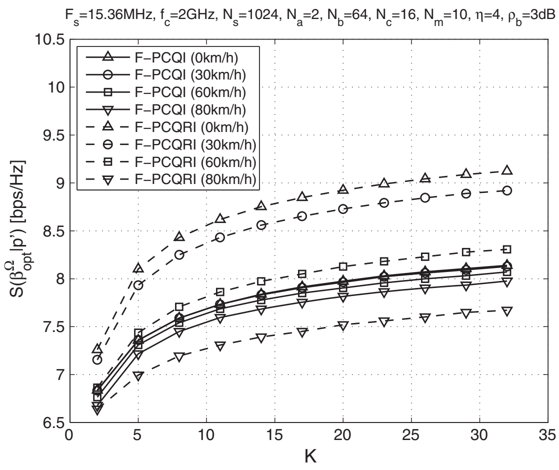

Figure 6. $S(\beta_{\text{opt}}^{\Omega}|\boldsymbol{p}')$, $\Omega \in$ {F-PCQI, F-PCQRI}, as a function of K when v =0, 30, 60, and 80 km/h.

effect of the feedback delay, τ^{Ω} [s], as a function of K when all users have the same mobility of $v = 0$, 30, 60, and 80 km/h. Here, we assume that each element of $\overline{\mathbf{H}}_{c,b,k}$ has a time-correlation coefficient of $J_0(2\pi f_d(v)\tau^{\Omega})$ [36], where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind, and $f_d(v) = \frac{vf_c}{c}$ [Hz] is the maximum Doppler shift from v with the carrier frequency f_c and $c = 3 \times 10^8$ m/s. Note that, for the F-PCQI and F-PCQRI schemes, the feedback delay τ^{Ω} are respectively given as 1 and 2 scheduling intervals, that is, $\tau^{F-PCQI} = \frac{N_s N_m}{F_s}$ [s] and $\tau^{F\text{–PCQRI}} = \frac{2N_s N_m}{F_s}$ [s], where F_s is the sampling frequency. Here, we set $f_c = 2$ GHz, $F_s = 15.36$ MHz, $N_s = 1024$, $N_t = N_r = N_a = 2$, $N_b = 64$, $N_c = 16$, $N_m = 10$, $\eta = 4$, $\rho_b = 3$ dB, $M_{\text{UL}} = 2$, $C_{\text{UL}} = 0.5$, and $N_q = 5$ bits. From the results, it is shown that, as the mobility increases, the performance of both partial feedback schemes get degraded since the NSNR gap between the user selection and the actual data transmission increases. Also, we can verify that, unless the mobility is high, the PCQRI schemes are better than the PCQI schemes owing to the enhanced feedback efficiency. However, when the mobility further increases, the PCQRI schemes become worse since the two-step approach with the longer feedback delay is much affected by the channel variation.

Based on the above results, we consider an adaptive feedback overhead control scheme where each group of neighboring cells share information and jointly control the feedback overhead in a long-term manner (say, once in a few hours) because neighboring BSs are very likely to have similar system and channel parameters, and the average user demands, such as the average number of active users, tend not to change rapidly. In the exemplary scheme, each BS in a group periodically (say, once in a few hours) reports the average number of active users and the average SNR at the cell boundary (the minimum average SNR among users) to the group chief BS through the back-haul links. By using the reported information, $\mathbf{p}'$ is parametrically estimated, the EDSE for a candidate β is obtained using (36) and (37), and the optimal feedback overhead $\beta_{\text{opt}}^{\Omega}$ is calculated and shared using (21) and a bisection search. Note that the required information exchange through the back-haul link and the computational complexity for the optimization are negligible.

Figures 7 and 8 depict the EDSE obtained from Monte-Carlo simulation with the LTE system parameters [17] under practical channel models such as ITU-R pedestrian B and vehicular A channel models, respectively. Here, the locations of BSs and users are generated from Poisson point precesses [37] with the densities of λ_{BS} and $\lambda_{\text{user}} = \lambda_{\text{BS}} \times [2 : 3 : 32]$, respectively, and the BS transmission power is set to yield 3 dB average SNR at the distance of the average cell radius. To prove the performance gain of the proposed work when applied to practical systems, the EDSE using $\beta_{\text{opt}}^{\Omega}$ obtained from the analytic results with the i.i.d. block fading assumption is compared with those using fixed values of β ($\beta = 0.01, 0.3$) as well as the maximized EDSE over all β. We set $f_c = 2$ GHz, $F_s = 15.36$ MHz, $W_{\text{DL}} = 10$ MHz, $N_s = 1024$, $N_t = N_r = N_a = 2$, $N_b = 50$, $N_c = 12$, $\eta = 4$, $M_{\text{UL}} = 2$, $C_{\text{UL}} = 0.5$, and $N_q = 5$ bits. The shadowing effect is also considered (a log-normal distribution with zero mean and 7 dB standard deviation). One subframe (1 ms) in LTE frame structure, which consists of two slots (seven MIMO–OFDMA symbol per slot) with the normal cyclic prefix length, is assumed to be a scheduling interval, that is, $N_m = 14$, and we set 1 and 2 subframes as the feedback delays for the F-PCQI and F-PCQRI schemes, respectively. We use independent random unitary beams in each sub-band with frequency-selective fading so that the correlation effect among the post SNRs after ZF receiver is also included. Also, we assume that each element in $\overline{\mathbf{H}}_{c,b,k}$

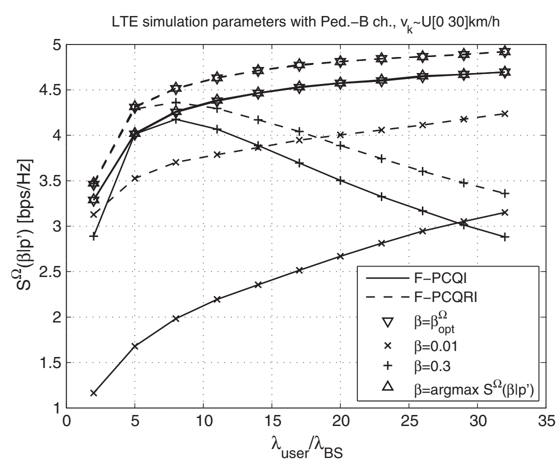

Figure 7. $S^{\Omega}(\beta|\boldsymbol{p}')$ as a function of $\lambda_{\text{user}}/\lambda_{\text{BS}}$ under ITU-R Pedestrian B channel model with LTE system parameters when β =0.01, 0.3, and $\beta_{\text{opt}}^{\Omega}$, $\Omega \in$ {F-PCQI, F-PCQRI}.

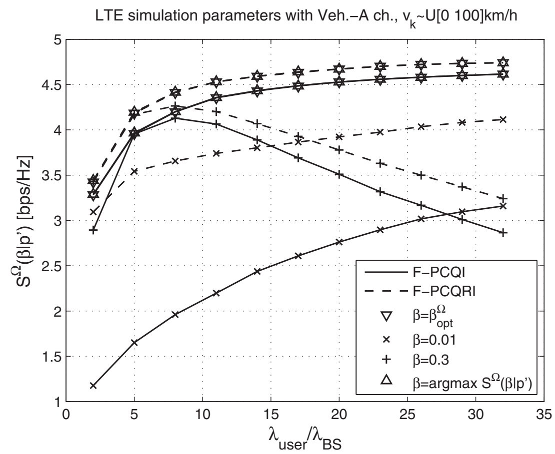

Figure 8. $S^{\Omega}(\beta|\boldsymbol{p}')$ as a function of $\lambda_{\text{user}}/\lambda_{\text{BS}}$ under ITU-R Vehicular A channel model with LTE system parameters when β =0.01, 0.3, and $\beta^{\Omega}_{\text{opt}}$, $\Omega \in$ {F-PCQI, F-PCQRI}.

has a Jakes' Doppler spectrum, and the mobility of each user is distributed uniformly from 0 km/h to $v_{\max}$, where $v_{\max}$ = 30 km/h in pedestrian B channel model, and $v_{\max}$ = 100 km/h in vehicular A channel model, respectively. Note that the channel variation during a scheduling period is reflected in the simulation so that the loss caused by the feedback delay is fully taken into account. From the results, it is shown that the EDSE employing the proposed adaptive feedback overhead control based on the analytic results under the i.i.d. block fading channel model is quite close to the maximum EDSE, and is much better than the EDSE using a fixed β even in realistic system and channel models, which confirms that it is attractive to employ the proposed scheme in practical systems, such as 3GPP LTE, for the efficient use of the feedback channel. In addition, we can verify that the PCQRI scheme shows better performance than the PCQI scheme unless the mobility is not quite high.

5. CONCLUSION

In this paper, we consider MIMO–OFDMA systems using R-BF with two partial feedback schemes: the PCQI and PCQRI schemes. The closed-form expressions on the downlink performance of the two schemes were derived under the i.i.d. block fading channel model. Also, an adaptive feedback overhead control scheme was proposed by optimizing the overhead to maximize the effective spectral efficiency according to the system and channel parameters using the analytical results and practically implementable algorithm.

From numerical examples and simulation results, it was shown that the analytical results are quite well matched to the simulation results so that the optimal overhead can be easily obtained. The optimal overhead can be determined from mainly the number of users $\left(O(\frac{1}{K})\right)$ as well as other system and channel parameters. It was also shown that the PCQRI scheme is better than the PCQI scheme unless the mobility is high. In addition, the proposed adaptive control of the feedback overhead was applied to a practical system and evaluated in realistic channel environment. The simulation results confirmed that it is attractive to employ the proposed scheme in practical systems, such as 3GPP LTE, for efficient use of the feedback channel.

APPENDIX A: SKETCH FOR PROOF OF THEOREM 1

Here, considering the limited space, we only sketch the proof of Theorem 1 without details. First, consider the F-PCQI scheme. Let Y be a random variable denoting the reported NSNR on a subchannel, whose pdf is given by $f_Y(y) = \frac{1}{N_p}\sum_{n=N-N_p+1}^{N} f_{Y^N_{n,k}}(y)$ since the rank of a reported NSNR at a specific subchannel is uniform over $[N-N_p+1, N]$. For $r > 0$, the pdf of x^{Ω}_r is given by $f_{x^{\Omega}_r}(y) = r \left(\frac{1}{N_p}\sum\limits_{n=N-N_p+1}^{N} F_{Y^N_{n,k}}(y)\right)^{r-1}\left(\frac{1}{N_p}\sum\limits_{n=N-N_p+1}^{N} f_{Y^N_{n,k}}(y)\right) = a_1\sum_{\mathbf{n}\in\mathcal{N}}\sum_{\mathbf{m}\in\mathcal{M}(\mathbf{n})} a_2 a_3 \sum_{\mathbf{l}\in\mathcal{L}} a_4\ e^{-a_5 y}$. On the other hand, for $r = 0$, the pdf of x^{Ω}_r is given as $f_{x^{\Omega}_r}(y) = KF^{K-1}_{Y^N_{1,k}}(y) f_{Y^N_{1,k}}(y) = KN\sum\limits_{m=0}^{K-1}\binom{K-1}{m}(-1)^m e^{-N(m+1)y}$. From EQ_1, $E_{x^{\Omega}_r}[\log_2(1+\rho x^{\Omega}_r)]$, Ω = F-PCQI, can be obtained as in (30).

Now, consider the S-PCQI scheme. let X' denote an NSNR greater than α whose cdf and pdf are respectively given as $F_{X'}(y) = 1-e^{\alpha-y}$ and $f_{X'}(y) = e^{\alpha-y}$. Then, for $r > 0$, the pdf of x^{Ω}_r can be written as $f_{x^{\Omega}_r}(y) = rF^{r-1}_{X'}(y) f_{X'}(y) = r\sum\limits_{m=0}^{r-1}\binom{r-1}{m}(-1)^m e^{(m+1)\alpha}e^{-(m+1)y}$. On the other hand, for $r = 0$, the performance of the S-PCQI scheme is same to that of the F-PCQI scheme for a given $\beta = \frac{N_p}{N} = e^{-\alpha}$. From EQ_1, $E_{x^{\Omega}_r}[\log_2(1+\rho x^{\Omega}_r)]$, Ω = S-PCQI, can be obtained as in (30).

Now, consider the F-PCQRI scheme. Let q be the best rank among the $r(> 0)$ reported CQRIs. As each user reports N_p CQRIs, the pmf of q conditioned on r and N_p is given as $Pr(q = n|r, N_p) = \sum\limits_{m=1}^{r}\binom{r}{m}(\frac{1}{N_p})^m(1-\frac{N-n+1}{N_p})^{r-m}$. On the other hand, for $r = 0$, the pdf of x^{Ω}_r can be written as $f_{x^{\Omega}_r}(y) = \frac{1}{N-N_p}\sum_{n=1}^{N-N_p} f_{Y^N_{n,k}}(y)$. From Lemma 2, $E_{x^{\Omega}_r}[\log_2(1+\rho x^{\Omega}_r)]$, Ω = F-PCQRI, can be obtained as in (30).

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Lastly, consider the S-PCQRI scheme. For $r > 0$,

$$E_{Y_{n,k}^N \geq \alpha}[\log_2(1+\rho Y_{n,k}^N)] = \int_{\alpha}^{\infty} \log_2(1+\rho y)\, f_{Y_{n,k}^N}(y|Y_{n,k}^N \geq \alpha)\mathrm{d}y$$
$$= \frac{1}{1-\mathcal{B}_{F_X(\alpha)}(n, N-n+1)} \int_{\alpha}^{\infty} \log_2(1+\rho y)\, f_{Y_{n,k}^N}(y)\mathrm{d}y. \quad \text{(A.1)}$$

By inserting (25) into (A.1), and using EQ_2, (A.1) can be written as

$$E_{Y_{n,k}^N \geq \alpha}[\log_2(1+\rho Y_{n,k}^N)] = \frac{n\binom{N}{n}}{\log(2)\left(1-\mathcal{B}_{F_X(\alpha)}(n,N-n+1)\right)} \sum_{m=0}^{n-1} \frac{\binom{n-1}{m}(-1)^m}{(N-n+m+1)}$$
$$\cdot\left(e^{-(N-n+m+1)\alpha}\log(1+\rho\alpha) + e^{\frac{N-n+m+1}{\rho}} E_1\left(\frac{N-n+m+1}{\rho}(1+\rho\alpha)\right)\right). \quad \text{(A.2)}$$

The pmf of q conditioned on r and α is given as $Pr(q = n|r,\alpha) = \sum_{m=1}^{r}\binom{r}{m}\left(Pr\left(\mu_{a,b,k}=n|X_{a,b,k}\geq\alpha\right)\right)^m \left(Pr(\mu_{a,b,k}<n|X_{a,b,k}\geq\alpha)\right)^{r-m}$, where $Pr(\mu_{a,b,k} = n|X_{a,b,k}\geq\alpha) = \frac{Pr(X_{a,b,k}\geq\alpha|\mu_{a,b,k}=n)Pr(\mu_{a,b,k}=n)}{Pr(X_{a,b,k}\geq\alpha)} = \frac{1-F_{Y_{n,k}^N}(\alpha)}{\left(1-F_{X_{a,b,k}}(\alpha)\right)N} = c_1$, $Pr(\mu_{a,b,k} < n|X_{a,b,k} \geq \alpha) = \sum_{l=1}^{n-1} Pr(\mu_{a,b,k}=l|X_{a,b,k}\geq\alpha) = c_2$. Thus, $Pr(q = n;r,\alpha) = \sum_{m=1}^{r} c_1^m c_2^{r-m}$. On the other hand, for $r = 0$, the cdf and pdf of $x_r^{\Omega}(\leq\alpha)$ can be written as $F_{x_r^{\Omega}}(y) = (1-e^{-y})/(1-e^{-\alpha})$ and $f_{x_r^{\Omega}}(y) = (e^{-y})/(1-e^{-\alpha})$, respectively. From EQ_1 and EQ_2, $E_{x_r^{\Omega}}[\log_2(1+\rho x_r^{\Omega})]$, $\Omega =$ S-PCQRI, can be obtained as in (30).

APPENDIX B: PROOF OF THEOREM 2

Let us define an indicator function $I_\alpha(X)$ as, if $X \geq \alpha$, $I_\alpha(X) = 1$, and otherwise, $I_\alpha(X) = 0$. In the S-PCQ(R)I scheme, $\sum_{a=1}^{N_a}\sum_{b=1}^{N_b} I_\alpha(X_{a,b,k})$ denotes the number of reported CQ(R)Is from the kth user. Since $X_{a,b,k}$ is i.i.d., so is $I_\alpha(X_{a,b,k})$. From the weak law of large numbers, for any $\epsilon>0$,

$$Pr\left(\left|\frac{1}{N}\sum_{a=1}^{N_a}\sum_{b=1}^{N_b} I_\alpha(X_{a,b,k}) - \beta\right| < \epsilon\right) > 1-\epsilon, \quad \text{(B.1)}$$

for sufficiently large N. Define $\mathbf{w}_k \in \mathbb{Z}^{N_a\times N_b}$ whose (a,b)th element is $w_{a,b,k} = I_\alpha(X_{a,b,k})$, and the typical set T_ϵ of the set of reported CQ(R)I indicator $\mathbf{W} = \{\mathbf{w}_1\mathbf{w}_2..\mathbf{w}_K\}$, where $\mathbf{w}_k$, $k = 1,\cdots,K$, satisfies (B.1). Let $R_S^{S-PCQ(R)I}(\mathbf{W})$ be the subchannel spectral efficiency at given ρ of the S-PCQ(R)I scheme for a reported CQ(R)I indicator $\mathbf{W}$. For a finite N, the subchannel spectral efficiency of the S-PCQ(R)I scheme is given by

$$R_S^{S\text{–PCQ(R)I}}(\beta|N,K,\rho) = \sum_{\mathbf{W}\in T_\epsilon} Pr(\mathbf{W}\in T_\epsilon) R_S^{S\text{–PCQ(R)I}}(\mathbf{W})$$
$$+ \sum_{\mathbf{W}\in(T_\epsilon)^c} Pr(\mathbf{W}\in(T_\epsilon)^c)$$
$$\times R_S^{S\text{–PCQ(R)I}}(\mathbf{W}). \quad \text{(B.2)}$$

For sufficiently large N, ϵ can be arbitrarily small so that $Pr(\mathbf{W} \in T_\epsilon^{N\times K}) \to 1$. Also, since larger number of CQ(R)Is means better performance, $R_S^{F\text{–PCQ(R)I}}(\beta = \beta-\epsilon|N,K,\rho) \leq R_S^{S\text{–PCQ(R)I}}(\mathbf{W}) \leq R_S^{F\text{–PCQ(R)I}}(\beta + \epsilon|N,K,\rho)$ for $\mathbf{W} \in T_\epsilon$. From Theorem 1, it is easily seen that $R_S^{F\text{–PCQI}}(\beta\pm\epsilon|N,K,\rho) \to R_S^{F\text{–PCQI}}(\beta|N,K,\rho)$ because $p_C(r|\beta\pm\epsilon,K) \to p_C(r|\beta,K)$ and $E_{x_r^{\Omega}}[\log_2(1+\rho x_r^{\Omega})|\beta\pm\epsilon] \to E_{x_r^{\Omega}}[\log_2(1+\rho x_r^{\Omega})|\beta]$. Thus, for sufficiently large N, $R_S^{S\text{–PCQI}}(\beta|N,K,\rho) \to R_S^{F\text{–PCQI}}(\beta|N,K,\rho)$. Similarly, we can show that $R_S^{S\text{–PCQRI}}(\beta|N,K,\rho) \to R_S^{F\text{–PCQRI}}(\beta|N,K,\rho)$, for sufficiently large N.

For the derivation of $R_S^{\Omega}(\beta|K,\rho)$, $\Omega \in$ {F-PCQI, S-PCQI,F-PCQRI, S-PCQRI}, it is sufficient to consider only the S-PCQI and F-PCQRI schemes. For $\Omega =$ S-PCQI, note that, for $r = 0$, $\lim_{N\to\infty} E_{x_r^{\Omega}}[\log_2(1+\rho x_r^{\Omega})] = 0$ as $\lim_{s\to\infty} e^s E_1(s) = \lim_{s\to\infty}\frac{\int_s^\infty e^{-u}u^{-1}du}{e^{-s}} = \lim_{s\to\infty}\frac{-e^{-s}s^{-1}}{-e^{-s}} = \lim_{s\to\infty} s^{-1} = 0$. On the other hand, for $r > 0$, $E_{x_r^{\Omega}}[\log_2(1+\rho x_r^{\Omega})]$ does not depend on N and $\sum_{m=0}^{r-1}\frac{\binom{r-1}{m}(-1)^m}{m+1} = \frac{1}{r}$ so that we obtain $R_S^{\Omega}(\beta|K,\rho)$, $\Omega \in$ {F-PCQI, S-PCQI}, as in (31). Also, for the F-PCQRI case, note that $\sum_{r=1}^{K} p_C(r|\beta,K)\sum_{m=1}^{r}\binom{r}{m}(\frac{1}{N_p})^m(1-$

$\frac{N-n+1}{N_p})^{r-m} = (\frac{n}{N})^K - (\frac{n-1}{N})^K$. We use the approximation in Lemma 2 to obtain

$$\begin{aligned}
&R_S^{F-\text{PCQRI}}(\beta|K,\rho)\\
&= \lim_{N\to\infty} \sum_{n=N-N_p+1}^{N} \log_2\left(1-\rho\log\left(1-\frac{n}{N+1}\right)\right)\\
&\times\left((\frac{n}{N})^K-(\frac{n-1}{N})^K\right)+\frac{(N-N_p)^{K-1}}{N^K}\\
&\times\sum_{n=1}^{N-N_p}\log_2\left(1-\rho\log\left(1-\frac{n}{N+1}\right)\right)\\
&= \int_{1-\beta}^{1} \log_2\left(1-\rho\log(1-u)\right)Ku^{K-1}\mathrm{d}u + (1-\beta)^{K-1}\\
&\times\int_0^{1-\beta}\log_2(1-\rho\log(1-u))\,\mathrm{d}u. \qquad \text{(B.3)}
\end{aligned}$$

By substituting $y = -\rho\log(1-u)$ in (B.3), and using binomial expansion and $\sum_{m=0}^{K-1}\frac{\binom{K-1}{m}(-1)^m\beta^{m+1}}{m+1} = \frac{1-(1-\beta)^K}{K}$, we can easily obtain $R_S^{\Omega}(\beta|K,\rho)$, $\Omega \in$ {F-PCQRI, S-PCQRI}, as in (31).

APPENDIX C: PROOF OF THEOREM 3

From Theorem 2, by substituting $\rho = \rho_b\left(\frac{D_b}{l}\right)^\eta$ in $R_S^{\Omega}(\beta|K,\rho)$, $\Omega \in$ {F-PCQI, S-PCQI}, the DSE of the F-PCQI and S-PCQI schemes can be obtained as

$$\begin{aligned}
&R_{\text{DL}}^{\Omega}(\beta|\mathbf{p}')\\
&= \frac{N_a\left(1-(1-\beta)^K\right)}{\log(2)}\int_0^{D_b}\frac{2l}{D_b^2}\log\left(1-\rho_b\left(\frac{D_b}{l}\right)^\eta\log(\beta)\right)\mathrm{d}l\\
&+\frac{N_a}{\log(2)}\sum_{r=1}^{K}r\binom{K}{r}\beta^{r-1}(1-\beta)^{K-r}\sum_{m=0}^{r-1}\frac{\binom{r-1}{m}(-\beta^{-1})^m}{m+1}\\
&\cdot\int_0^{D_b}\frac{2l}{D_b^2}e^{\frac{m+1}{\rho_b}\left(\frac{l}{D_b}\right)^\eta}\\
&\times E_1\left((m+1)\left(\frac{1}{\rho_b}\left(\frac{l}{D_b}\right)^\eta-\log(\beta)\right)\right)\mathrm{d}l\\
&+\frac{N_a(1-\beta)^K K}{\log(2)}\sum_{m=0}^{K-1}\frac{\binom{K-1}{m}(-1)^m}{m+1}\int_0^{D_b}\frac{2l}{D_b^2}\\
&\times e^{\frac{N(m+1)}{\rho_b}\left(\frac{l}{D_b}\right)^\eta}E_1\left(\frac{N(m+1)}{\rho_b}\left(\frac{l}{D_b}\right)^\eta\right)\mathrm{d}l\\
&= \frac{N_a\left(1-(1-\beta)^K\right)}{\log(2)}\int_0^1\log\left(1-\rho_b u^{-\frac{\eta}{2}}\log(\beta)\right)\mathrm{d}u\\
&+\frac{N_a}{\log(2)}\sum_{r=1}^{K}r\binom{K}{r}\beta^{r-1}(1-\beta)^{K-r}\sum_{m=0}^{r-1}\frac{\binom{r-1}{m}(-\beta^{-1})^m}{m+1}\\
&\cdot\int_0^1 e^{(m+1)\frac{u^{\eta/2}}{\rho_b}}E_1\left((m+1)\left(\frac{u^{\eta/2}}{\rho_b}-\log(\beta)\right)\right)\mathrm{d}u\\
&+\frac{N_a(1-\beta)^K K}{\log(2)}\sum_{m=0}^{K-1}\frac{\binom{K-1}{m}(-1)^m}{m+1}G_3\left(\frac{N(m+1)}{\rho_b},\frac{\eta}{2}\right), \qquad \text{(C.1)}
\end{aligned}$$

$\Omega \in$ {F-PCQI, S-PCQI}. By using $G_1(c_3,c_4)$, $G_2(c_3,c_4,c_5)$, and $G_3(c_3,c_4)$, we can easily obtain $R_{DL}^{\Omega}(\beta|\mathbf{p}')$, $\Omega \in$ {F-PCQI, S-PCQI} as in (36). Similarly, from Theorem 2, the DSE of the F-PCQRI and S-PCQRI schemes can be obtained as

$$\begin{aligned}
&R_{\text{DL}}^{\Omega}(\beta|\mathbf{p}')\\
&= \frac{N_a\left(1-(1-\beta)^{K-1}\right)}{\log(2)}\int_0^{D_b}\frac{2l}{D_b^2}\log\left(1-\rho_b\left(\frac{D_b}{l}\right)^\eta\right.\\
&\left.\times\log(\beta)\right)\mathrm{d}l+\frac{N_a K}{\log(2)}\sum_{m=0}^{K-1}\frac{\binom{K-1}{m}(-1)^m}{m+1}\\
&\times\int_0^{D_b}\frac{2l}{D_b^2}e^{\frac{m+1}{\rho_b}\left(\frac{l}{D_b}\right)^\eta}\\
&\cdot E_1\left((m+1)\left(\frac{1}{\rho_b}\left(\frac{l}{D_b}\right)^\eta-\log(\beta)\right)\right)\mathrm{d}l+\frac{N_a(1-\beta)^{K-1}}{\log(2)}\\
&\times\int_0^{D_b}\frac{2l}{D_b^2}e^{\frac{1}{\rho_b}\left(\frac{l}{D_b}\right)^\eta}\left(E_1\left(\frac{1}{\rho_b}\left(\frac{l}{D_b}\right)^\eta\right)\right.\\
&\left.-E_1\left(\frac{1}{\rho_b}\left(\frac{l}{D_b}\right)^\eta-\log(\beta)\right)\right)\mathrm{d}l\\
&= \frac{N_a\left(1-(1-\beta)^{K-1}\right)}{\log(2)}\int_0^1\log\left(1-\rho_b u^{-\frac{\eta}{2}}\log(\beta)\right)\mathrm{d}u\\
&+\frac{N_a K}{\log(2)}\sum_{m=0}^{K-1}\frac{\binom{K-1}{m}(-1)^m}{m+1}\int_0^1 e^{(m+1)\frac{u^{\eta/2}}{\rho_b}}\\
&\times E_1\left((m+1)\left(\frac{u^{\eta/2}}{\rho_b}-\log(\beta)\right)\right)\mathrm{d}u\\
&+\frac{N_a(1-\beta)^{K-1}}{\log(2)}\int_0^1 e^{\frac{u^{\eta/2}}{\rho_b}}\\
&\times\left(E_1\left(\frac{u^{\eta/2}}{\rho_b}\right)-E_1\left(\frac{u^{\eta/2}}{\rho_b}-\log(\beta)\right)\right)\mathrm{d}u, \qquad \text{(C.2)}
\end{aligned}$$

$\Omega \in$ {F-PCQRI, S-PCQRI}. By using $G_1(c_3,c_4)$, $G_2(c_3,c_4,c_5)$, and $G_3(c_3,c_4)$, we can easily obtain $R_{\text{DL}}^{\Omega}(\beta|\mathbf{p}')$, $\Omega \in$ {F-PCQRI, S-PCQRI} as in (36).

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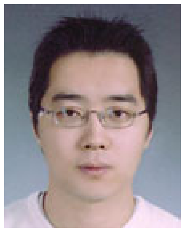

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