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Performance of DS/SSMA systems using TCM under impulsive noise environment

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Abstract

In this paper, we investigate the effects of impulsive noise on the DS/SSMA systems using TCM. We obtain the probability of bit error bound of the systems, considering both impulsive noise and Rician fading unavoidable in mobile communication environments. It turns out that we can achieve some coding gain by using TCM under impulsive noise environment at moderate to high signal to noise ratio (SNR). It is observed that the bit error probability is dominated by the background noise variance when the SNR is low and by the tail noise variance when the SNR is high. © 1998 Published by Elsevier Science B.V. All rights reserved.

Keywords: Impulsive noise; Trellis coded modulation; DS/SSMA

1. Introduction

As is evident, the demand for mobile communication is increasing rapidly: recently, code division multiple access (CDMA) has become an interesting research area due to its applications in mobile communication.

To get better communication systems, a number of modulation and coding techniques have been developed. In [9], it has been shown that optimally designed rate $n/(n+1)$ trellis codes mapped into the conventional 2^{n+1} point signal sets (e.g., the 2^{n+1}-ary phase shift keying (PSK) and amplitude shift keying (ASK) signal constellations) can provide some coding gain without bandwidth expansion. Following the work, many researchers have studied various aspects of the trellis coded modulation (TCM). For example, the performance of the direct sequence spread spectrum multiple access (DS/SSMA) systems using TCM has been studied in [2]. Most of the work on TCM has been accomplished under the additive white Gaussian noise (AWGN) assumption [2,9,10].

It is well known that sometimes the Gaussian noise assumption cannot be entirely justified [5]. The non-Gaussian nature of atmospheric noise,

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impulses caused by turning-on some electrical devices, etc. are among the typical examples. Therefore, it is worthwhile to study the effects of impulsive noise on communication systems. In this paper, we will investigate the effects of impulsive noise on the DS/SSMA system using TCM. We will obtain the upper bound on the probability of bit error for the M-ary PSK trellis coded DS/SSMA systems under the impulsive noise fading environment. We will then compare the bound with that obtained for the uncoded DS/SSMA system for various channel states.

2. System model

The transmitter model for the trellis-coded DS/SSMA is illustrated in Fig. 1. The information bits, k_1 and $k_2 = n - k_1$, select a coset of the 2^n-ary PSK signal constellation through the $k_1/(k_1+1)$ convolutional code. The selected signal multiplied by the DS sequence is modulated by the carrier and transmitted.

Let the kth user's complex baseband coded signal be

$$x^k(t) = \sum_{p=-\infty}^{\infty} x_p^k P_T(t - pT), \quad (1)$$

where T is the symbol period, P_T is a rectangular pulse with duration T, and x_p^k is the complex baseband symbol during the pth symbol period determined by the coset selection. Similarly, the pseudo noise (PN) chip signal is defined as

$$a^k(t) = \sum_{m=-\infty}^{\infty} a_m^k \Psi(t - mT_c), \quad (2)$$

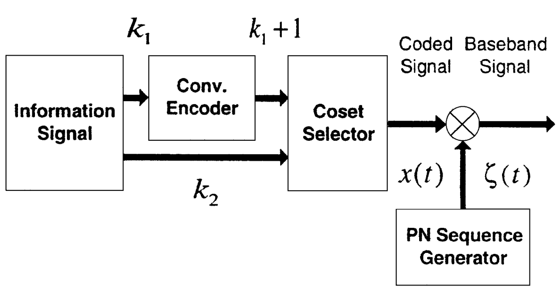

Fig. 1. The transmitter system model for the trellis-coded DS/SSMA.

where a_m^k is the mth chip, and Ψ is the chip waveform with duration T_c. Then, the transmitted signal modulated by a carrier with angular frequency $\omega_c = 2\pi f_c$ is

$$s^k(t) = \sqrt{\frac{2E_s}{T}} \mathrm{Re}\{\zeta^k(t)\exp[\mathrm{j}(w_c t + \psi^k)]\}, \quad (3)$$

where $\zeta^k(t) = a^k(t)x^k(t)$ and ψ^k is the random phase of the kth carrier. Then, the received signal can be written as

$$r(t) = \sqrt{\frac{2E_s}{T}} \mathrm{Re}\left\{\sum_{k=1}^{K} \rho^k(t)\zeta^k(t-\tau^k)\exp[\mathrm{j}(w_c t + \beta^k)]\right\} + \eta(t), \quad (4)$$

where $\rho^k(t)$ is the fading process, τ^k is the random delay at the receiver, and $\beta^k = \psi^k - w_c\tau^k$ is the random phase at the receiver.

In general, impulsive noise can be modeled as a random variable whose pdf has a tail heavier than that of a Gaussian pdf [4]. In this paper, the impulsive noise $\eta(t)$ is modeled as a Gaussian mixture noise with the ε-contaminated pdf [4]

$$f(x) = (1-\varepsilon)f_{\mathrm{B}}(x) + \varepsilon f_{\mathrm{T}}(x), \quad (5)$$

where ε is called the contamination ratio. In Eq. (5), we assume that the background noise pdf f_{B} and the tail noise pdf f_{T} are Gaussian with equal mean and variances σ_{B}^2 and $\sigma_{\mathrm{T}}^2 = M\sigma_{\mathrm{B}}^2$, respectively: we use the notation $\alpha(x;\varepsilon, m_f, \sigma_{\mathrm{B}}^2, \sigma_{\mathrm{T}}^2)$ to denote the ε-contaminated pdf (5), with m_f the common mean of f_{B} and f_{T}. Typically, ε is a small positive constant determining the frequency that noise of a large absolute value occurs. The parameter M is a positive constant ranging typically from several to several tens, and determines how heavy the tail of the pdf is.

We now define the signal to noise ratio as

$$\mathrm{SNR} = \frac{E_s}{2E\{\eta^2\}} = \frac{E_s}{2(1+(M-1)\varepsilon)\sigma_{\mathrm{B}}^2} \quad (6)$$

and the signal to background noise ratio (SBR) as

$$\mathrm{SBR} = \frac{E_s}{2\sigma_{\mathrm{B}}^2}. \quad (7)$$

The characteristics of the impulsive noise are different for different values of ε and M even when the SNR is fixed. This implies that the values of ε and M should be specified to investigate the effects of the impulsive noise. If ε is very small ($(M-1)\varepsilon \ll 1$), the SBR can be a good approximation to the SNR. It is easy to see that the SBR is useful when we want to know the effects of the contamination ratio on the performance of the system [6].

3. Performance analysis

Define a complex coded sequence of length n by $X=(X_1, X_2, \ldots, X_n)$, where $X_p=\sqrt{E_S}x_p$. Let $Y=(Y_1, Y_2, \ldots, Y_n)$ be the corresponding complex receiver output sequence, and $Z=(Z_1, Z_2, \ldots, Z_n)$ be the complex inter-user interference sequence. Then, after the demodulation through the coherent in-phase (I) and quadrature (Q) demodulators, the output signal sequence sampled at time p is [2]

$$Y_p = \rho_p X_p + Z_p + \eta_p, \tag{8}$$

where $\eta_p = \eta_{pi} + \mathrm{j}\eta_{pq}$ is a complex impulsive noise and η_{pi} and η_{pq} are i.i.d. random variables whose pdfs are $\alpha(x;\varepsilon,0,\sigma_{\mathrm{B}}^2,\sigma_{\mathrm{T}}^2)$, and ρ_p is the fading envelope with the pdf [3]

$$f_\rho(x) = 2x(1+K)\exp[-\{K+x^2(1+K)\}] \times I_0(2x\sqrt{K(K+1)}). \tag{9}$$

In Eq. (9), I_0 is the zeroth-order modified Bessel function of the first kind, and K is the Rician parameter defined as the ratio of the energy of the direct component to that of the diffused multipath components. The work in [2] has been accomplished under AWGN environment assumption: on the contrary, we consider the impulsive noise environment in this paper.

Now, we assume [7] that the inter-user interference Z_p is a Gaussian random variable independent of η_p. Then

$$\bar{\eta}_p = Z_p + \eta_p \tag{10}$$

is also a complex impulsive noise with an ε-contaminated pdf $\alpha(x;\varepsilon,0,\bar{\sigma}_{\mathrm{B}}^2,\bar{\sigma}_{\mathrm{T}}^2)$ by the results in [6,7], where $\bar{\sigma}_{\mathrm{B}}^2=\sigma_{\mathrm{B}}^2+\sigma_{\mathrm{I}}^2, \bar{\sigma}_{\mathrm{T}}^2=\sigma_{\mathrm{T}}^2+\sigma_{\mathrm{I}}^2, \sigma_{\mathrm{I}}^2 = [(Q-1)E_s]/3N$, Q is the number of users, and N is the length of a PN sequence. From now on, we drop the bars of $\bar{\eta}_p$, $\bar{\sigma}_{\mathrm{B}}^2$ and $\bar{\sigma}_{\mathrm{T}}^2$ for convenience.

We assume that the estimation of fading parameters is perfect and the channel state information is known by using a pilot tone. Then the pairwise error probability given $\rho=(\rho_1,\rho_2,\ldots,\rho_n)$ can be obtained as

$$\begin{aligned} P(x\to\tilde{x}|\rho) &= \Pr\left\{\sum_{p\in v}(\|Y_p-\rho_p X_p\|^2 - \|Y_p-\rho_p\tilde{X}_p\|^2)\geqslant 0\right\} \\ &= \Pr\left\{\sum_{p\in v}(-E_s\rho_p^2\|x_p-\tilde{x}_p\|^2 - 2\sqrt{E_s}\rho_p\mathrm{Re}\{\eta_p(x_p-\tilde{x}_p)^*\})\geqslant 0\right\}, \end{aligned} \tag{11}$$

where $\tilde{X}_p$ is the pth symbol of a coded sequence $\tilde{X}\neq X$, v is the set of p such that $X_p\neq\tilde{X}_p$, and x and $\tilde{x}$ are the normalized versions of X and $\tilde{X}$, respectively.

Now, let us define $\bar{d}^2=\sum_{p\in v}\rho_p^2\|d_p\|^2$ and $v=\sum_{p\in v}v_p = -\sum_{p\in v}(d_{pi}\rho_p\eta_{pi}+d_{pq}\rho_p\eta_{pq})$, where the pth distance $d_p=x_p-\tilde{x}_p=d_{pi}+\mathrm{j}d_{pq}$. Then we can rewrite Eq. (11) as

$$P(x\to\tilde{x}|\rho) = \Pr\left\{-E_s\bar{d}^2+2\sqrt{E_s}\sum_{p\in v}v_p\geqslant 0\right\}, \tag{12}$$

from which we have [6]

$$P(x\to\tilde{x}|\rho) \leqslant \Pr\left\{-E_s\bar{d}^2+2\sqrt{E_s}\sum_{p\in v}\bar{v}_p\geqslant 0\right\}, \tag{13}$$

where $\bar{v}_p$ is a random variable with pdf $f_{\bar{v}_p}(x)=\alpha(x;\varepsilon,0,\rho_p^2\|d_p\|^2\sigma_{\mathrm{B}}^2,\rho_p^2\|d_p\|^2\sigma_{\mathrm{T}}^2)$. Next, if we let $\bar{v}$ be a random variable with the pdf $f_{\bar{v}}(x)=\alpha(x;\varepsilon,0,\bar{d}^2\sigma_{\mathrm{B}}^2,\bar{d}^2\sigma_{\mathrm{T}}^2)$, we have [6] from Eq. (13),

$$P(x\to\tilde{x}|\rho) \leqslant \Pr\{-E_s\bar{d}^2+2\sqrt{E_s}\bar{v}\geqslant 0\}. \tag{14}$$

Since the pdf of $\hat{v}=-E_s\bar{d}^2+2\sqrt{E_s}\bar{v}$ is easily obtained to be $f_{\hat{v}}(x)=\alpha(x;\varepsilon,-E_s\bar{d}^2,4E_s\sigma_{\mathrm{B}}^2\bar{d}^2,$

$4E_s\sigma_T^2\bar{d}^2$), we easily have from Eq. (14)

$$P(x \to \tilde{x}|\rho) \leqslant \Pr\{\hat{v} \geqslant 0\}$$
$$= (1-\varepsilon)\int_0^\infty \phi(x; -E_s\bar{d}^2, 4E_s\sigma_B^2\bar{d}^2)\,dx + \varepsilon\int_0^\infty \phi(x; -E_s\bar{d}^2, 4E_s\sigma_T^2\bar{d}^2)\,dx, \quad (15)$$

where $\phi(x; m_\phi, \sigma^2)$ is the Gaussian pdf with mean m_ϕ and variance σ^2. Applying Chernoff bound to Eq. (15), we obtain

$$P(x \to \tilde{x}|\rho) \leqslant (1-\varepsilon)D_B^{\bar{d}^2}(\lambda_1) + \varepsilon D_T^{\bar{d}^2}(\lambda_2), \quad (16)$$

where $D_B(\lambda_1) = \exp[-E_s\lambda_1 + 2E_s\sigma_B^2\lambda_1^2]$, $D_T(\lambda_2) = \exp[-E_s\lambda_2 + 2E_s\sigma_T^2\lambda_2^2]$, and λ_1 and λ_2 are Chernoff parameters. The optimum values of λ_1 and λ_2 and the minimum bounds D_B and D_T can be obtained to be $\lambda_{1,\mathrm{opt}} = 1/4\sigma_B^2$, $\lambda_{2,\mathrm{opt}} = 1/4\sigma_T^2$, $D_B = \exp[-E_s/8\sigma_B^2]$, and $D_T = \exp[-E_s/8\sigma_T^2]$. Thus, substituting the values into Eq. (16), we get

$$P(x \to \tilde{x}|\rho) \leqslant (1-\varepsilon)D_B^{\bar{d}^2} + \varepsilon D_T^{\bar{d}^2}. \quad (17)$$

The unconditional pairwise error probability bound is then obtained by taking the expectation of Eq. (17) over ρ as

$$P(x \to \tilde{x}) \leqslant (1-\varepsilon)E\{D_B^{\bar{d}^2}\} + \varepsilon E\{D_T^{\bar{d}^2}\}. \quad (18)$$

Then, by the definition of the transfer function [1], the bit error probability bound is obtained as

$$P_b \leqslant (1-\varepsilon)\cdot\frac{1}{2n}\frac{\partial \bar{T}(D,I)}{\partial I}\bigg|_{D=D_B, I=1} + \varepsilon\cdot\frac{1}{2n}\frac{\partial \bar{T}(D,I)}{\partial I}\bigg|_{D=D_T, I=1}, \quad (19)$$

where $\bar{T}(D,I)$ is the transfer function of the super state diagram whose branch label gains are modified based on the discussion in [1], and the factor $\frac{1}{2}$ is inserted based on the discussion on the error bound in [8]. It should be noted that for $\varepsilon = 0$, the impulsive noise becomes Gaussian noise and Eq. (19) becomes the same result as that shown in [1].

Now, let us explain the implication of Eq. (19). At low SNR, the bit error probability is high. If the bit error probability is much higher than ε, the first term in the right-hand side of Eq. (19) will dominate since the second term cannot be larger than ε. At high SNR, the bit error probability is low. If the bit error probability is much lower than ε, the second term will dominate.

4. Numerical results

In the following result, we assume that the length N of the PN sequence is 1024 and the number Q of users is 50. The bit error probabilities of the 2-state $\frac{1}{2}$ rate 4-PSK trellis coded DS/SSMA and uncoded BPSK DS/SSMA systems for $M = 5, 20$ are shown in Fig. 2 when no fading is present. We can see that the 2-state $\frac{1}{2}$ rate 4-PSK trellis coded DS/SSMA system has roughly 2 dB gain over the uncoded BPSK DS/SSMA system when the bit error probability is 10^{-5}. We can also see that the bit error probability increases as M increases for fixed ε. In Fig. 3, the bit error probabilities of the 2-state $\frac{1}{2}$ rate 4-PSK trellis coded DS/SSMA system are shown for various M when $K = 0, 50$ and $\varepsilon = 0.01$. We can see that the bit error probability increases as M increases as in the no-fading case. In addition, it is easily seen that the performance depends more on the value of M as the fading becomes less severe (K gets larger). The bit error probabilities for various ε when $M = 10$ and $K = 0, 50$ are shown in Fig. 4. Note that the SNR is a function of ε. Thus, we plot the bit error probability versus the SBR so that the effect of ε on the system can be seen clearly.

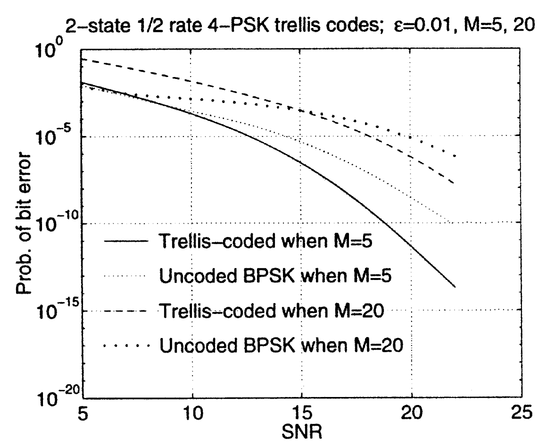

Fig. 2. The bit error probability when $M = 5, 20$ and $\varepsilon = 0.01$.

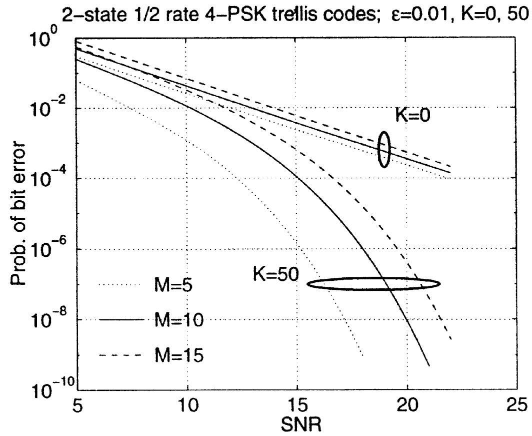

Fig. 3. The bit error probability for various M when $K = 0, 50$ and $\varepsilon = 0.01$.

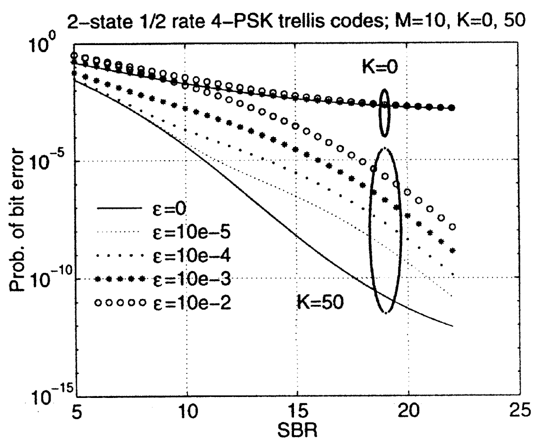

Fig. 4. The bit error probability for various ε when $M = 10$ and $K = 0, 50$.

Since $\varepsilon(M-1) \ll 1$, the value of the SBR is quite close to the value of the SNR. We can see that the bit error probability increases as ε increases for fixed M, and that the bit error probability is dominated by the first term in the right-hand side of Eq. (19) at low SNR and by the second term at high SNR: the transition point is around the SNR which makes the bit error probability ε. In addition, it is easily seen that the performance depends more on the value of ε as the fading becomes less severe (K gets larger).

5. Conclusion

In this paper, we studied the effects of impulsive noise on the performance of 2^{n+1}-PSK trellis coded DS/SSMA systems. We analyzed the system performance and obtained the bit error probability bound under the impulsive noise environment, where we also took into account the effects of Rician fading which is unavoidable in mobile communication systems.

From the results, it is observed that some SNR gain can be obtained by using the TCM scheme in DS/SSMA systems under impulsive noise environment as in the AWGN case except at low SNR. It is also observed that the bit error probability is dominated by background noise at low SNR and by tail noise at high SNR. We also see that the performance of the system gets worse as the impulsiveness of noise increases (M and ε increase) and the degradation becomes severe as the fading becomes less severe (K gets larger).

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