LETTER

A Partial IR Hybrid ARQ Scheme Using Rate-Compatible Punctured LDPC Codes in an HSDPA System

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SUMMARY In this paper, an efficient partial incremental redundancy (P-IR) scheme is proposed for an H-ARQ using block type low density parity check (B-LDPC) codes. The performance of the proposed P-IR scheme is evaluated in an HSDPA system using IEEE 802.16e B-LDPC codes. Simulation results show that the proposed H-ARQ using IEEE 802.16e B-LDPC codes outperforms the H-ARQ using 3GPP turbo codes.

key words: *low-density parity check codes, hybrid ARQ, partial IR, turbo codes, HSDPA*

1. Introduction

For high speed packet data communication, hybrid-automatic repeat request (H-ARQ) techniques are essential to resolve the problems caused from deep fading and interference which result in a low signal to interference plus noise ratio (SINR) at a receiver. Commonly, an H-ARQ scheme is used in conjunction with an adaptive modulation and coding (AMC) scheme to cope with short-term fading or abrupt interference which is not fully handled by the AMC scheme alone.

Retransmission strategies of the H-ARQ can be classified into two categories. One is Chase combining, where each retransmission is identical to the original transmission, and the other is incremental redundancy (IR) combining where each retransmission consists of new redundancy bits from a channel encoder. Obviously, IR combining has both diversity and coding gain while Chase combining has only diversity gain. Among two kinds of IR strategies, one is a full IR (F-IR) scheme where only new redundant bits are transmitted, and the other is a partial IR (P-IR) scheme where both systematic bits and new redundant bits are transmitted. Although the F-IR scheme has more coding gain, the P-IR scheme allows us to decode each packet without combining previous packets. The P-IR scheme is very useful when a transmitter fails to obtain any ACK/NACK information. In this case, the transmitter can do its best by transmitting another packet using P-IR.

For IR combining, a rate-compatible punctured code is required such as rate-compatible punctured convolutional (RCPC) codes [1] or rate-compatible punctured turbo (RCPT) codes [2]. Among them, a family of RCPT codes is adopted in the 3rd generation wireless standard, for an example, high speed downlink packet access (HSDPA) [3]. However, low density parity check (LDPC) codes have emerged recently with more powerful merits over turbo codes, such as better performance at moderate to high code rates and lower decoding complexity. In addition, structured LDPC codes allow fast encoding and low memory requirement and such LDPC codes have been adopted as an optional forward error correction (FEC) code in existing standards [4], [5]. To develop rate compatible punctured LDPC codes, several good puncturing techniques have been proposed [6], [7]. Recently in [8], rate compatible punctured B-LDPC (RCP-BLDPC) codes were proposed by the authors, and they can support a wide range of puncturing. Also, it was shown that their performance is better than that of RCPT codes in the HSDPA. However, H-ARQ schemes using those LDPC codes have not yet been considered in any commercial standards. Although F-IR techniques for an H-ARQ can be straightforwardly solved by transmitting coded bits in the reverse order of puncturing, there should be careful consideration for a P-IR strategy because each packet should have a similar and good self-decoding performance as well as good combining decoding performance.

In this paper, we propose an efficient H-ARQ scheme with P-IR using RCP-BLDPC codes. Based on the puncturing algorithm in [8], a parity bit partitioning method is proposed, so that each packet can have both similar and good self-decoding performance and good combined performance. In addition, the performances of the proposed H-ARQ scheme are evaluated with and without adaptive modulation (AM) and compared with those of the existing H-ARQ scheme using the turbo codes in [3].

2. The Proposed Parity Partitioning Method

Recently, several studies about practical rate-compatible punctured LDPC codes (RCP-LDPC codes) have been reported [6]–[8]. In [6], the puncturing algorithm is only trying to maximize the recovery reliability of the puncturing node. Later, in [7], the puncturing algorithm in [6] was improved by trying to maximize the minimum reliability among those provided from all check nodes. Thus, it is more effective when the amount of puncturing is quite large, which means that H-ARQ schemes using the RCP-LDPC codes in [7] can support wider code rates than those in [6].

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However, these puncturing patterns are non-deterministic, so that a large amount of memory is required to prepare several bit-wise transmission orders at both the transmitter and receiver, especially when there is a large number of retransmissions and the codeword lengths are not small. In [8], a generalized formula for generating puncturing patterns was proposed for block-type low-density parity-check (B-LDPC) codes, which allows a generalized and simple structured retransmission method for the H-ARQ. The puncturing formula in [8] distributes punctured bits uniformly in the permutated zigzag edge connections to obtain a good performance.

Let us recall the B-LDPC code and the puncturing algorithm in [8]. The B-LDPC code is defined by an m by n parity check matrix, where n is the length of the codeword and m is the number of parity check bits in the codeword. An m by n parity check matrix $\mathbf{H}$ of a B-LDPC code can be defined using the sub-matrices:

$$\mathbf{H} = \begin{bmatrix} \mathbf{P}^{a_{1,1}} & \mathbf{P}^{a_{1,2}} & \dots & \mathbf{P}^{a_{1,n_b}} \\ \mathbf{P}^{a_{2,1}} & \mathbf{P}^{a_{2,2}} & \dots & \mathbf{P}^{a_{2,n_b}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}^{a_{m_b,1}} & \mathbf{P}^{a_{m_b,1}} & \dots & \mathbf{P}^{a_{m_b,n_b}} \end{bmatrix}, \quad (1)$$

where $\mathbf{P}^{a_{i,j}}$ denotes a $z \times z$ sub-matrix (denoted as a block in the sequel), which is either an $a_{i,j}$ times circularly-shifted version of a $z \times z$ identity matrix or a zero matrix. Here, $\mathbf{H}$ can be partitioned as $\mathbf{H} = \left[(\mathbf{H}_s)_{zm_b \times zk_b} | (\mathbf{H}_p)_{zm_b \times zm_b}\right]$, where $\mathbf{H}_s$ ($\mathbf{H}_p$) corresponds to the systematic (parity) bits, $n = zn_b$, $m = zm_b$, and $k_b = n_b - m_b$. In addition, the dual-diagonal parity structure, $\mathbf{H}_p$, is further partitioned as

$$\mathbf{H}_p = [\mathbf{H}_o | \mathbf{H}_d] = \left[\begin{array}{c|cccccc} \mathbf{P}^{b_1} & \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{b_2} & \mathbf{I} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \mathbf{0} & \mathbf{P}^{b_3} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{P}^{p} & \vdots & \vdots & \dots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \dots & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{P}^{b_{m_b-1}} & \mathbf{I} \\ \mathbf{P}^{q} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{P}^{b_{m_b}} \end{array}\right], \quad (2)$$

where the matrix $\mathbf{H}_o$ has three non-zero blocks and $\mathbf{H}_d$ is the $zm_b \times z(m_b - 1)$ matrix with dual-diagonal blocks. Here, the block at the ith row and the jth column of $\mathbf{H}_d$ is equal to a $z \times z$ identity matrix when $i = j$, is equal to $\mathbf{P}^{b_i}$ when $i = j + 1$, and is a zero matrix elsewhere. Note that the row block location of $\mathbf{P}^p$ is the lth row block of $\mathbf{H}_o$. Note that the LDPC codes used in IEEE 802.16e [5] are special cases of the B-LDPC code with $b_2 = b_3 = \ldots = b_{m_b} = 0$. In this case, $b_1 = q$ and $p = 0$ from (3). Here, q is selected to be prime with the block size z.

Let us define $p(i)$ as the location of ith puncturing bit, $0 < p(i) < N$. If we assume that the puncturing does not go further when the number of remaining blocks is an odd number in the puncturing block selection†, the location of the ith puncturing bit is given as follows:

$$p(i) = K_t(i)z + c_z(mod(i, z)), \quad (3)$$

$$c_z(mod(i, z)) = \begin{cases} u'_z(mod(i, z)), & \text{if } K_t(i) \le l, \\ u''_z(mod(i, z)), & \text{otherwise,} \end{cases} \quad (4)$$

where

$$K_t(i) = 2^{t-1}(2u_{m_b/2^t}(\text{mod}(\lfloor i/z \rfloor, m_b/2^t))+1), \quad (5)$$

$$u'_z(i) = mod(b_1 u_z(i), z), \quad (6)$$

$$u''_z(i) = mod((z - q)u_z(i), z). \quad (7)$$

Here, the uniform selection sequence, $\mathbf{u_n}$, is defined as

$$\mathbf{u_1} = \{0\}, \quad (8)$$

$$\mathbf{u_{2k}} = \{u_k(0), u_k(0) + k, u_k(1), u_k(1) + k, \ldots, u_k(k - 1), u_k(k - 1) + k\}, \quad (9)$$

$$\mathbf{u_{2k+1}} = \{k, u_k(0), u_k(0) + k + 1, u_k(1), u_k(1) + k + 1, \ldots, u_k(k - 1), u_k(k - 1) + k + 1\}, \quad (10)$$

where k is a natural number and $u_n(i)$, $i = 0, 1, \ldots, n - 1$, denotes the ith element in the sequence $\mathbf{u_n}$. Also, $mod(n, z)$ is the modulo-z operation on n. The parameter t is the step number in the block selection defined in [8]. This puncturing method always constructs a uniform geometry in a permutated zigzag edge connection composed of only variable nodes in the parity part and check nodes in the Tanner graph, regardless of the amount of puncturings. If puncturing nodes construct non-uniform geometry, there exist bit nodes with much lower reliability than others; this degrades the overall decoding performance [8]. Thus, the reliability of a bit node with the minimum reliability can be maximized using the puncturing method.

Now, let us consider a P-IR scheme with T transmissions from a codeword with a length $N = N_s + N_p$, where N_s (N_p) denotes the number of systematic (parity) bits. In this case, N_p parity bits should be divided by the T parity group. For each tth transmission, $1 \le t \le T$, n_t coded bits are transmitted. Also, let us define $\mathbf{P}_k$ as a set of puncturing bits after kth puncturing, that is, $\mathbf{P}_k = \{p(i) | 1 \le i \le k\}$.

In a P-IR strategy, each packet should have similar and good self-decoding performance while maintaining a good performance for the combined packet. The reason why each packet has similar performance is because every packet can be the first packet when the previous packets were lost. In order to ensure it, each packet should have a uniform location of puncturing bits in the zigzag graph. This can be solved by exploiting the puncturing algorithm [8] properly, that is, by dividing all the parity bits into several bit groups which constructs a uniform geometry regardless as to whether those are combined or not.

Then, the bit group $\mathbf{G}_t$ for the P-IR, which is transmitted at the tth transmission, is obtained as

$$\mathbf{G}_t = (\mathbf{T} - \mathbf{P}_{N_p}) \cup (\mathbf{P}_{(t+1)N_p/T} - \mathbf{P}_{tN_p/T}), \quad (11)$$

†Typically, the maximum amount of puncturing is much smaller than in that case.

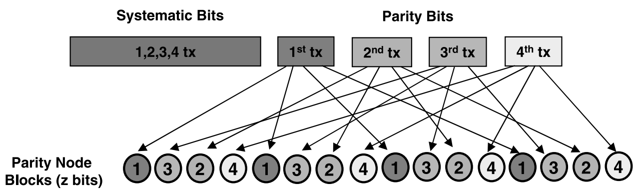

Fig. 1 The proposed parity partitioning method, t=4.

where **T** denotes the set of all variable nodes, and the term $\mathbf{T} - \mathbf{P}_{N_p}$ denotes all systematic bits. Figure 1 shows a schematic view of the proposed parity partitioning method when t is 4. In this way, the parity partitioning method in (11) is an effective way of not only having the combined packet decoded with a good performance, but also of trying to have each packet decoded with a similar and good performance.

3. Performance Evaluation of the Proposed H-ARQ in an HSDPA System

Let us describe simulation environments used for performance evaluation. Turbo codes need to add cyclic redundancy check (CRC) bits for error detecting, while LDPC codes do not have to. Also, turbo coded bits should be interleaved to obtain diversity, while LDPC coded bits do not have to be. The coded bits are mapped onto one of quadrature phase shift keying (QPSK), 16-quadrature amplitude modulation (QAM), and 64-QAM determined by modulation and coding scheme (MCS) feedback information from the receiver. The modulated symbol is spread for code division multiple access (CDMA). At the receiver, perfect channel estimation and SINR estimation are assumed. Here, the maximum numbers of iterations for LDPC and turbo decoding are respectively set to fifty and eight in order to allow similar decoding complexity. At every successful decoding, the receiver feeds back an MCS level to the transmitter corresponding to the current SINR for the next initial transmission. Here, it is assumed that there is no feedback error or no feedback delay. This system structure can be thought of as a simple version of the HSDPA system.

Using the parameters in Table 1, H-ARQ performance with B-LDPC codes from IEEE 802.16e [5] and those with turbo codes from [3] are compared, which will be denoted as LDPC H-ARQ and turbo H-ARQ, respectively. The maximum number of retransmissions and the size of packet in the performance evaluation are the same as those in the HSDPA system [3]. At first in Fig. 2, the throughput of the proposed P-IR method is compared with that of the conventional P-IR method [3] which is the same one used in turbo codes. Here, the throughput denotes the number of transmitted bits per second per unit frequency (bit/sec/Hz). From the result, it is shown that the proposed P-IR method has performance gain in a moderate to high SNR region. In Figs. 3 and 4, the performances of LDPC H-ARQ and turbo H-ARQ are compared without AM. The simulation results show that LDPC H-ARQ always outperforms turbo H-ARQ, except when the SNR is extremely low. This is due to the fact that the resultant code rate of the PIR-HARQ becomes 0.6 when the second packet is combined, in which the code rate is sufficiently high. As can be seen in Fig. 4, in a fast fading channel, the performance gap between LDPC H-ARQ and turbo H-ARQ is larger than that in a slow fading channel.

In Fig. 5, the performances of LDPC H-ARQ and that of turbo H-ARQ in [3] are compared with a P-IR with AM. These performances are obtained by selecting the best MCS scheme at each initial transmission. The abscissa denotes the averaged receive SNR and the ordinate denotes the long term averaged throughput of the P-IR with AM. The sim-

Table 1 The simulation parameters.

Center frequency	2 GHz
Bandwidth	3.84 Mcps
Spreading Factor	16
Channel	Frequency flat fading with Jake's model
Mobile velocity	3 km/h, 120 km/h
Etc.	Frequency flat fading and single user
Mother	B-LDPC codes at a rate of 1/2 in [5]

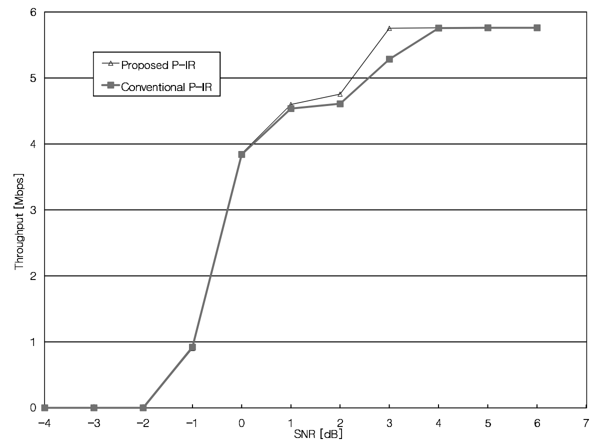

Fig. 2 Throughput comparison of the proposed parity partitioning method in AWGN channel.

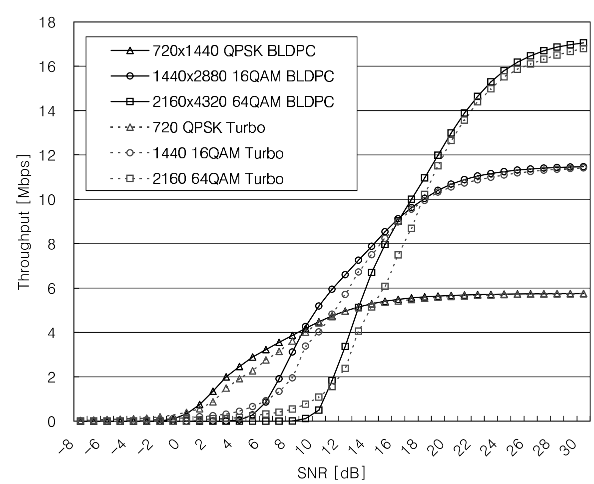

Fig. 3 Throughput comparison of H-ARQ with P-IR in a slow fading channel (ms=3 km/h).

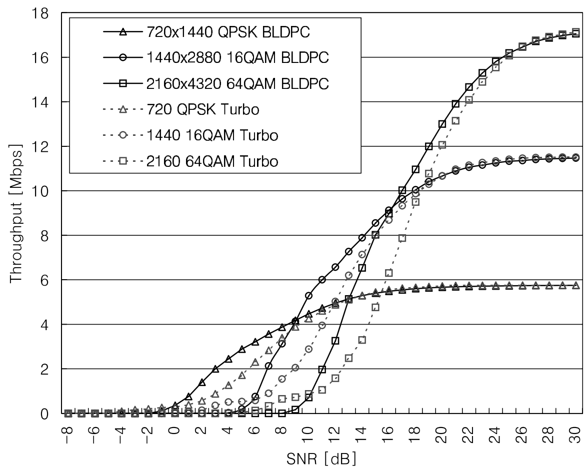

Fig. 4 Throughput comparison of H-ARQ with P-IR in a fast fading channel (ms=120 km/h).

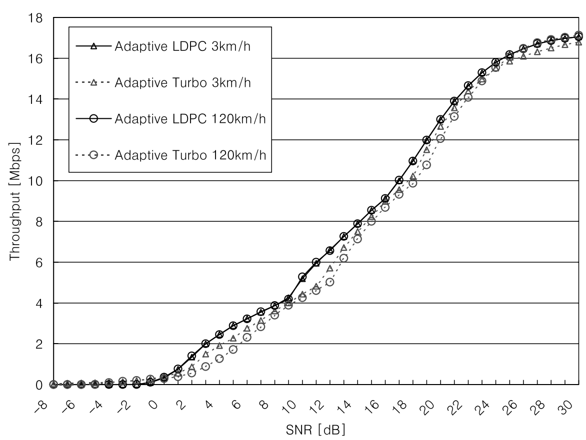

Fig. 5 Throughput comparison of adaptive H-ARQ with P-IR.

ulation results show that adaptive LDPC H-ARQ always outperforms adaptive turbo H-ARQ, except when the SNR is extremely low. Also, note that the performance of the proposed LDPC H-ARQ shows a similar performance regardless of the mobile speed, while that of the turbo H-ARQ shows different performance. Therefore, the H-ARQ scheme in conjunction with AM can be used in an HSDPA system by exploiting 16e B-LDPC codes instead of 3GPP turbo codes.

4. Conclusion

In this paper, we proposed an efficient method of constructing a P-IR H-ARQ packet using B-LDPC codes in IEEE 802.16e. Also, we evaluated its performance and compared it with H-ARQ using turbo codes in an HSDPA system. Simulation results show that the proposed H-ARQ using the B-LDPC codes outperforms H-ARQ using the turbo codes. Therefore, we can exploit the proposed P-IR H-ARQ using B-LDPC codes in the existing or future commercial standards instead of existing H-ARQ using turbo codes.

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References

[1] J. Hagenauer, "Rate-compatible punctured convolutional codes (RCPC codes) and their applications," IEEE Trans. Commun., vol.36, no.4, pp.389–400, April 1998.
[2] D.N. Rowitch and L.B. Milstein, "On the performance of hybrid FEC/ARQ systems using rate compatible punctured turbo (RCPT) codes," IEEE Trans. Commun., vol.48, no.6, pp.948–958, June 2000.
[3] 3GPP, TR 25.848: Physical layer aspects of UTRA High Speed Downlink Packet Access, v4.0.0, March 2001.
[4] 11-04-0889-03-OOOn-tgnsync-proposal-technical specificat-ion.doc.
[5] IEEE P802.16e, IEEE Standard for Local and Metropolitan area networks Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access System, Feb. 2006.
[6] J. Ha, J. Kim, D. Klinc, and S.W. McLaughlin, "Rate-compatible punctured low-density parity-check codes with short block lengths," IEEE Trans. Inf. Theory, vol.52, no.2, pp.728–738, Feb. 2006.
[7] H.Y. Park, J.W. Kang, K.S. Kim, and K.C. Whang, "Efficient puncturing method for rate compatible low-density parity-check codes," IEEE Trans. Wireless Commun., vol.6, no.11, pp.3914–3919, Nov. 2007.
[8] H.Y. Park, K.S. Kim, D.H. Kim, and K.C. Whang, "Structured puncturing for rate-compatible B-LDPC codes with dual-diagonal parity structure," IEEE Trans. Wireless Commun., vol.7, no.10, pp.3692–3696, Oct. 2008.
[9] D. Chase, "Code combining — A maximum likelihood decoding approach for combining an arbitrary number of noisy packets," IEEE Trans. Commun., vol.COM-33, no.5, pp.385–393, May 1985.