Low Complexity MMSE-SIC Equalizer Employing Time-Domain Recursion for OFDM Systems

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***Abstract*—In this letter, a novel time-domain recursive algorithm is proposed for a minimum-mean-squared error (MMSE) with successive interference cancellation (SIC) scheme. This algorithm allows us to reduce the complexity of classical MMSE-SIC equalizer for suppressing intercarrier interference (ICI) caused by time-varying multipath channels in orthogonal frequency division multiplexing (OFDM) systems, and it can provide almost the same performance as that of the optimal MMSE-SIC equalizer. It is shown in complexity analysis that the proposed scheme shows better complexity reduction than previously reported low-complexity MMSE-SIC schemes when applied to a channel equalizer for an OFDM system.**

***Index Terms*—Equalization, minimum-mean-squared error with successive interference cancellation (MMSE-SIC), orthogonal frequency division multiplexing (OFDM).**

I. Introduction

In orthogonal frequency division multiplexing (OFDM) systems, the time selectivity of wireless channel introduces intercarrier interference (ICI), which degrades system performance [1]–[3]. To compensate the ICI effect, a linear minimum-mean-squared error (MMSE) equalizer can be employed with $\mathcal{O}(N^3)$ complexity, where N denotes the number of subcarriers in an OFDM symbol. However, future wireless OFDM systems may support high mobility, which causes highly time-varying channels where a more powerful equalizer is required. In [2], a linear MMSE equalizer incorporated with successive interference cancellation (SIC) in the frequency domain was proposed with $\mathcal{O}(N^4)$ complexity, which shows good performance even at high normalized Doppler frequency ($f_n = f_m T_s \geq 0.1$), where f_m is the maximum Doppler frequency and T_s is the OFDM symbol interval. This scheme is identical to the time domain Vertical Bell Labs Layered Space-Time (V-BLAST) detection. To reduce the complexity and maintain the performance, we can directly exploit one of various low complexity approaches for V-BLAST signal detection with $\mathcal{O}(N^3)$ complexity, e.g., square-root algorithms in [4]–[6] and fast recursive (FR) algorithms in [7]–[9]. However, those schemes are not suitable for further complexity reduction by exploiting the sparseness of time-domain channel matrix when applied to an OFDM equalizer.

In this letter, a novel time-domain recursive algorithm is proposed to reduce the complexity of the MMSE-SIC scheme for OFDM systems. Even though the ICI effect is shown in a frequency-domain channel matrix after OFDM demodulation, this algorithm exploits the sparseness of a time-domain channel matrix in an OFDM system. The complexity of the proposed algorithm depends not only on N but also on the number of channel taps L. Since L is typically much smaller than N, the proposed scheme has complexity of $(1/2)N^3 + \mathcal{O}(N^2 \log_2 N)$, which is much simpler than previously reported schemes [4]–[9]. Note that there were several low-complexity equalizers that exploited ICI properties in a frequency-domain channel matrix [1], [3]. However, such a scheme is inferior to the MMSE-SIC scheme in performance and suffers from error floor as SNR increases.

Notation: A boldface large and small letter mean a matrix or a vector, respectively. $\mathbf{A}_{m,n}$ denotes the (m,n)th element of $\mathbf{A}$. Also, $\mathbf{A}_{m,:}$ and $\mathbf{A}_{:,n}$ denote the mth row vector and the nth column vector of $\mathbf{A}$, respectively. $\|\cdot\|$ denotes the norm of a vector. $\mathbf{0}_N$ and $\mathbf{I}_N$ represent the $N \times N$ zero and identity matrices, respectively. Also, $(k)_N$ denotes k modulo N, $[\cdot]^T$ and $[\cdot]^H$ stand for transpose and Hermitian, respectively, and $\mathbf{F} = (1/\sqrt{N})[\exp^{-j2\pi(m-1)(n-1)/N}]_{n,m=1,\ldots,N}$ is the $N \times N$ fast Fourier transform (FFT) matrix.

II. System Description

In an OFDM transmitter, the frequency-domain data stream is divided into blocks of length N and modulated by N-point inverse FFT (IFFT). At the receiver, the received blocks are demodulated by N-point FFT. For each block, the input-output relation can be described as

$$\mathbf{y} = \mathbf{F}\mathbf{H}_t\mathbf{F}^H\mathbf{x} + \mathbf{F}\mathbf{w}_t = \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$ and $\mathbf{y} = [y_1, y_2, \ldots, y_N]^T$ are the $N \times 1$ transmitted and received symbol vectors, respectively, and $E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_N$. Also, $\mathbf{w}_t$ and $\mathbf{w}$ denote the vectors of additive complex white Gaussian noise (AWGN) with variance σ^2 in time-domain and frequency-domain, respectively. The matrix $\mathbf{H}_t$ denotes the $N \times N$ time-domain channel matrix, whose (n,l)th element is $h_{n,(n-l)_N}$, $1 \leq n \leq N$, $1 \leq l \leq L$, where $h_{n,l}$ denotes the complex channel gain of the lth tap at the nth time instance. Since $L \ll N$, $\mathbf{H}_t$ is a sparse matrix with NL nonzero elements. The matrix $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_N] = \mathbf{F}\mathbf{H}_t\mathbf{F}^H$ denotes the $N \times N$ frequency-domain channel matrix. In time-selective channels, $\mathbf{H}$ is typically not a diagonal matrix and the off-diagonal elements in $\mathbf{H}$ cause the occurrence of the ICI, which may result in severe performance degradation as f_n increases.

III. MMSE-SIC Equalization

A. Classical MMSE-SIC Scheme

The MMSE-SIC scheme [2] requires N iterations for each OFDM symbol. Each iteration is comprised of three steps for the

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symbol detection: optimal ordering, linear MMSE filtering, and ICI cancellation. The detailed procedure is described as follows. The MMSE filter is given in [2] as

$$\mathbf{G} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_N)^{-1} = \mathbf{H}^H\mathbf{R}^{-1}. \tag{2}$$

Assume that an ordering sequence vector $\mathbf{z}$ is defined as $\mathbf{z} = [z_1, z_2, \ldots, z_N]$, where z_k denotes the symbol index to be detected at the kth iteration. Let $\mathbf{H}^{[1]} = \mathbf{H}$ and $\mathbf{H}^{[k]}$, $k = 2, 3, \ldots, N$, be the modified channel matrices after nulling column vectors $\mathbf{h}_{z_1}, \mathbf{h}_{z_2}, \ldots, \mathbf{h}_{z_{k-1}}$ from $\mathbf{H}^{[1]}$. Then, the MMSE filter at the kth iteration is given as $\mathbf{G}^{[k]} = \mathbf{H}^{[k],H}\mathbf{R}^{[k],-1}$.[1] At the kth iteration, the first step is to choose the current best symbol x_{z_k} with the smallest error variance among undetected data symbols. This ordering can reduce performance degradation due to the error propagation. By computing $\mathbf{R}^{[k],-1}$, z_k can be determined as

$$z_k = \arg\min_m \mathbf{R}^{[k],-1}_{m,m}. \tag{3}$$

In the second step, the symbol is detected as

$$\hat{x}_{z_k} = \mathbf{g}^{[k],H}\mathbf{y}^{[k]} \tag{4}$$

where $\mathbf{g}^{[k],H}$ denotes the kth row vector of $\mathbf{G}^{[k]}$ and $\mathbf{y}^{[k]}$ is the ICI cancelled received signal vector at the kth iteration. Then, the hard-decision value $\bar{x}_{z_k}$ is obtained. Finally, the third step is to remove the ICI of the detected symbol $\bar{x}_{z_k}$ from the received signal vector as

$$\mathbf{y}^{[k+1]} = \mathbf{y}^{[k]} - \mathbf{h}_{z_k}\bar{x}_{z_k} \tag{5}$$

and replace the column vector $\mathbf{h}_{z_k}$ with a zero vector (nulling $\mathbf{h}_{z_k}$). This procedure is repeated for $k = 1, 2, \ldots, N$.

It is well known that a suboptimal ordering technique can be used to reduce the computational complexity of the MMSE-SIC with negligible performance degradation [10]. However, the MMSE filter should be recalculated at each iteration, which is computationally intensive due to matrix inversion. In the next subsection, a new recursive algorithm is proposed to compute the MMSE filter at each iteration.

B. Proposed Recursive Algorithm

In (2), the kth frequency-domain channel matrix $\mathbf{H}^{[k]}$ can be described as $\mathbf{H}^{[k]} = \mathbf{H}^{[1]}\mathbf{P}^{[k]}$, where $\mathbf{P}^{[k]}$ is an $N \times N$ diagonal matrix with the ith diagonal entry defined as

$$\mathbf{P}^{[k]}_{i,i} = \begin{cases} 0, & i \in \{z_1, z_2, \ldots z_{k-1}\} \\ 1, & \text{otherwise.} \end{cases} \tag{6}$$

Note that $\mathbf{P}^{[1]} = \mathbf{I}_N$. From (1) and (2), the kth MMSE filter $\mathbf{G}^{[k]}$ can be rewritten as

$$\mathbf{G}^{[k]} = \mathbf{P}^{[k]}\mathbf{M}^{[k],-1}\mathbf{H}^{-1} \tag{7}$$

where

$$\mathbf{M}^{[k]} = \mathbf{P}^{[k]} + \sigma^2\left(\mathbf{F}\mathbf{H}_\mathbf{t}^H\mathbf{H}_\mathbf{t}\mathbf{F}^H\right)^{-1}. \tag{8}$$

[1]Although such a nulling has been widely used in many MMSE-SIC schemes, $\mathbf{G}^{[k]}$ is not the exact MMSE filter unless all preceding symbol detections are correct, which may cause performance degradation. However, in this letter, we use $\mathbf{G}^{[k]}$ as did in most literature because an appropriate ordering can greatly reduce such degradation.

The derivations of (7) and (8) are given in Appendix A. From (7) and (8), a recursive expression can be invoked by using the Sherman–Morrison formula [11] as follows.

Suppose that $\mathbf{M}^{[k-1],-1}$ is known. Then, $\mathbf{M}^{[k],-1}$ can be expressed as

$$\begin{aligned}\mathbf{M}^{[k],-1} &= \left(\mathbf{M}^{[k-1]} + \mathbf{c}_k\mathbf{d}_k^T\right)^{-1} \\ &= \mathbf{M}^{[k-1],-1} - \frac{\mathbf{M}^{[k-1],-1}\mathbf{c}_k\mathbf{d}_k^T\mathbf{M}^{[k-1],-1}}{1 + \mathbf{d}_k^T\mathbf{M}^{[k-1],-1}\mathbf{c}_k}\end{aligned} \tag{9}$$

where $\mathbf{c}_k$ and $\mathbf{d}_k$ are $N \times 1$ vectors with only one nonzero element at the z_{k-1}th entry, satisfying $\mathbf{P}^{[k]} = \mathbf{P}^{[k-1]} + \mathbf{c}_k\mathbf{d}_k^T$. For convenience, set the z_{k-1}th entries of $\mathbf{c}_k$ and $\mathbf{d}_k$ to -1 and 1, respectively, and let $\alpha_k = (1 + \mathbf{d}_k^T\mathbf{M}^{[k-1],-1}\mathbf{c}_k)$. Then, (9) can be rewritten as

$$\mathbf{M}^{[k],-1} = \left[\mathbf{I}_N - \mathbf{A}^{[k]}\right]\mathbf{M}^{[k-1],-1} \tag{10}$$

where $\mathbf{A}^{[k]} = (1/\alpha_k)\mathbf{M}^{[k-1],-1}\mathbf{c}_k\mathbf{d}_k^T$ is a sparse matrix with nonzero elements only at the z_{k-1}th column. By substituting (10) into (7), $\mathbf{G}^{[k]}$ can be expressed as

$$\mathbf{G}^{[k]} = \left[\mathbf{I}_N - \mathbf{A}^{[k]}\right]\mathbf{G}^{[k-1]}. \tag{11}$$

Therefore, the MMSE filter $\mathbf{G}^{[k]}$, for $k = 2, 3, \ldots, N$, can be recursively calculated as

$$\mathbf{G}^{[k]} = \mathbf{T}^{[k]}\mathbf{G}^{[1]} \tag{12}$$

where

$$\begin{aligned}\mathbf{T}^{[k]} &= \left[\mathbf{I}_N - \mathbf{A}^{[k]}\right]\left[\mathbf{I}_N - \mathbf{A}^{[k-1]}\right]\cdots\left[\mathbf{I}_N - \mathbf{A}^{[1]}\right] \\ &= \left[\mathbf{I}_N - \mathbf{A}^{[k]}\right]\mathbf{T}^{[k-1]},\end{aligned} \tag{13}$$

$$\mathbf{A}^{[k]}_{:,z_{k-1}} = -(1/\alpha_k)\mathbf{P}^{[k]}\mathbf{T}^{[k-1]}\mathbf{M}^{[1],-1}_{:,z_{k-1}}. \tag{14}$$

Note that $\mathbf{T}^{[1]} = \mathbf{I}_N$ and $\mathbf{A}^{[1]} = \mathbf{0}_N$. According to (2) and (7), the z_{k-1}th column vector of $\mathbf{M}^{[1],-1}$ can be calculated as $\mathbf{M}^{[1],-1}_{:,z_{k-1}} = \mathbf{G}^{[1]}\mathbf{H}_{:,z_{k-1}}$. Also, note that $\alpha_k \neq 0$ because $\mathbf{G}^{[1]}_{z_{k-1},:}\mathbf{H}_{:,z_{k-1}} \neq 1$. Using (12)–(14), the estimate of the kth symbol can be described as

$$\hat{x}_{z_k} = \mathbf{T}^{[k]}_{z_k,:}\mathbf{G}^{[1]}\mathbf{y}^{[k]} = \mathbf{T}^{[k]}_{z_k,:}\bar{\mathbf{y}}^{[k]} \tag{15}$$

where $\mathbf{y}^{[k]} = \mathbf{y}^{[k-1]} - \mathbf{h}_{z_{k-1}}\bar{x}_{z_{k-1}}$ and $\mathbf{y}^{[1]} = \mathbf{y}$. It is shown that the column vectors, $\bar{\mathbf{y}}^{[k]} = \mathbf{G}^{[1]}\mathbf{y}^{[k]}$ and $\mathbf{M}^{[1],-1}_{:,z_{k-1}} = \mathbf{G}^{[1]}\mathbf{H}_{:,z_{k-1}}$, are the outputs of the first MMSE filter $\mathbf{G}^{[1]}$ when the inputs are $\mathbf{y}^{[k]}$ and $\mathbf{H}_{:,z_{k-1}}$, respectively, which can be easily calculated as follows. From (1) and (2), we can describe $\mathbf{G}^{[1]}$ with the time-domain channel matrix $\mathbf{H}_t$ as

$$\begin{aligned}\mathbf{G}^{[1]} &= \mathbf{F}\mathbf{H}_t^H\mathbf{F}^H\left(\mathbf{F}\mathbf{H}_t\mathbf{H}_t^H\mathbf{F}^H + \sigma^2\mathbf{I}_N\right)^{-1} \\ &= \mathbf{F}\mathbf{H}_t^H\left(\mathbf{H}_t\mathbf{H}_t^H + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{F}^H.\end{aligned} \tag{16}$$

Clearly, $\mathbf{R}_t = (\mathbf{H}_t\mathbf{H}_t^H + \sigma^2\mathbf{I}_N)$ is a Hermitian and sparse matrix. By performing the LDL^H factorization [11], $\mathbf{R}_t$ is represented as $\mathbf{R}_t = \mathbf{L}\mathbf{D}\mathbf{L}^H$, where $\mathbf{D}$ is a diagonal matrix and $\mathbf{L}$ is a lower triangular matrix with unit diagonal elements. Then,

TABLE I
PROPOSED RECURSIVE ALGORITHM

1) Compute $\mathbf{R}_t$ and perform the LDL^H factorization of $\mathbf{R}_t$.
2) Compute $\bar{\mathbf{y}}^{[1]}$ by forward and backward substitutions.
3) Make a hard decision on $\hat{x}_{z_1}$.
4) For k=2,3,...,N
   a) Compute the ICI cancelled signal vector $\mathbf{y}^{[k]}$ from (5).
   b) Compute $\mathbf{M}^{[1],-1}_{:,z_{k-1}}$ by forward and backward substitutions.
   c) Compute $\mathbf{A}^{[k]}_{:,z_{k-1}}$ from (14).
   d) Compute $\mathbf{T}^{[k]}$ from (13).
   e) Compute $\bar{\mathbf{y}}^{[k]}$ by forward and backward substitutions.
   f) Compute decision statistics from (15) and make a hard decision.
   End

Initialization: $\mathbf{P}^{[1]}=\mathbf{I}_N$, $\mathbf{A}^{[1]}=\mathbf{0}_N$, $\mathbf{T}^{[1]}=\mathbf{I}_N$ and $\mathbf{z}=[z_1, z_2, ..., z_N]$

$\mathbf{R}_t^{-1}$ can be solved by forward and backward substitutions. The steps for computing $\bar{\mathbf{y}}^{[k]}$ is described as follows.

1) Compute $\mathbf{R}_t$ and perform the LDL^H factorization.
2) Solve the linear equation $\mathbf{R}_t\mathbf{a} = (\mathbf{F}^H\mathbf{y}^{[k]})$ by solving $\mathbf{L}\mathbf{b} = (\mathbf{F}^H\mathbf{y}^{[k]})$, $\mathbf{D}\mathbf{c} = \mathbf{b}$, and $\mathbf{L}^H\mathbf{a} = \mathbf{c}$, respectively.
3) Calculate $\bar{\mathbf{y}}^{[k]} = \mathbf{F}\mathbf{H}_t^H\mathbf{a}$.

Step 1) is done only once at the first iteration. By replacing $\mathbf{y}^{[k]}$ with $\mathbf{H}_{:,z_{k-1}}$, we can also compute $\mathbf{M}^{[1],-1}_{:,z_{k-1}}$. It is known that the complexity of the FFT (or IFFT) operation is $(1/2)N\log_2 N$. Since $\mathbf{L}$ is sparse due to the spareness of $\mathbf{R}_t$, the complexity of steps 2) and 3) is substantially reduced. The proposed algorithm is summarized in Table I. The major computational complexity at the kth iteration is in calculating $\mathbf{A}^{[k]}_{:,z_{k-1}}$ and $\mathbf{T}^{[k]}$, whose overall complexity will be shown as $(1/2)N^3 + \mathcal{O}(N^2\log_2 N)$ in complexity analysis. Thus, the proposed algorithm can reduce the complexity of the MMSE-SIC scheme by a factor of N.

In the optimal MMSE-SIC scheme, the ordering sequence $\mathbf{z}$ should be recalculated at each iteration. However, it is already known in [10] that the performance degradation is negligible if the initially optimal ordering sequence is used throughout the whole iterations. Thus, such a suboptimal ordering is adopted in the proposed scheme. The MMSE filter coefficients are designed to maximize the signal-to-interference-plus-noise ratio (SINR) between the output of the filter and the transmitted signal. At the first iteration, the SINR of the kth symbol is given in [2] as

$$SINR_k = \frac{\left|\mathbf{M}^{[1],-1}_{k,k}\right|^2}{\sum\limits_{m,m\neq k}\left|\mathbf{M}^{[1],-1}_{m,k}\right|^2 + \sigma^2\left\|\mathbf{G}^{[1]}_{k,:}\right\|^2}. \quad (17)$$

We approximate the SINRs as the signal terms $|[\mathbf{M}^{[1],-1}_{k,k}|^2$ and determine a ordering vector $\mathbf{z}$ as the descending order of the signal terms. To obtain $\mathbf{z}$, it is sufficient to compute $\mathbf{M}^{[1],-1}$ before entering the iteration loop in Table I.

IV. Complexity and Performance Evaluation

A. Complexity Analysis

We evaluate the complexity of the proposed recursive algorithm by counting the number of complex multiplications (CMs) and the number of complex additions (CAs). The LDL^H factorization of an $N \times N$ Hermitian matrix $\mathbf{A}$ is described in [11]. The complexity to compute $\mathbf{R}_t$ and perform LDL^H factorization depends on the number of channel taps L and the maximum delay of the channel L_{max}. To evaluate the upper bound of the complexity, we set $L = L_{max} + 1$. The sparse matrix $\mathbf{R}_t$ contains $(2L_{max}+1)N$ nonzero elements. The complexity to compute $\mathbf{R}_t$ is $(1/2)(L_{max}^2+3L_{max}+2)N$ CMs and $(1/2)(L_{max}^2 + L_{max})N$ CAs. The LDL^H factorization of $\mathbf{R}_t$ requires roughly $(2L_{max}^2 + 7L_{max} + 2)N$ CMs and $(2L_{max}^2 - 2L_{max} + 3)N$ CAs, where $\mathbf{L}$ has $(2L_{max})N$ nonzero elements except the unit diagonal elements. The complexity of computing $\mathbf{y}^{[k]}$, for $k = 1, 2, \ldots, N$, is $N^2 - N$ CMs and $N^2 - N$ CAs. The main computational complexity of the proposed algorithms is summarized in Table II, where $C[\cdot]$ denotes the number of CMs or the number of CAs to calculate a matrix or a vector. The total numbers of CMs and CAs required for the proposed algorithm are, respectively, given as

$$C_{CM} = \frac{1}{3}N^3 + (2log_2 N + 10L_{max} + 4)N^2 + \left(\frac{5}{2}L_{max}^2 + \frac{17}{2}L_{max} - \frac{1}{3}\right)N + 1 \quad (18)$$

$$C_{CA} = \frac{1}{6}N^3 + (10L_{max} - 3)N^2 + \left(\frac{5}{2}L_{max}^2 - \frac{3}{2}L_{max} + \frac{17}{6}\right)N - 2. \quad (19)$$

When $L_{max} \ll N$, the complexity of the proposed algorithm is roughly $(1/2)N^3 + \mathcal{O}(N^2\log_2 N)$. To compute the upper bound on the complexity for outdoor OFDM systems such as IEEE 802.16e [12], L_{max} is set to $L_{max} = L_{CP} = (1/8)N$, where L_{CP} is the length of cyclic prefix (CP). Then, the total numbers of CMs and CAs are, respectively, upper-bounded as

$$C_{CM} \leq \left(\frac{19}{12} + \frac{5}{128}\right)N^3 + \left(2log_2 N + \frac{81}{16}\right)N^2 - \frac{1}{3}N + 1, \quad (20)$$

$$C_{CA} \leq \left(\frac{17}{12} + \frac{5}{128}\right)N^3 - \frac{51}{16}N^2 + \frac{17}{6}N - 2. \quad (21)$$

Then, the total number of complex operations (COs) is upper bounded as $C_{Total} = (3 + (5/64))N^3 + \mathcal{O}(N^2\log_2 N)$. The complexity of previously reported low-complexity schemes in [4]–[9] is summarized in Table III.[2] It is seen that even the upper-bound of the complexity of the proposed scheme is smaller than the complexity of existing schemes. Furthermore, L_{max} is smaller than the CP length and L is typically much smaller than L_{max}. Thus, the complexity of the proposed scheme is typically much smaller than the upper bounds.

B. Simulation Results

To evaluate the BER performance of the proposed algorithm, an uncoded OFDM system with $N = 64$ and QPSK constellation is used. The length of CP is set to eight samples. The wide-sense stationary uncorrelated scattering (WSSUS) tapped-delay-line channel model is used with six paths $(L = 6)$ and an exponential power delay profile whose decay factor is 1 dB/sample. The path gain for each channel tap is independently generated from the Jakes' model, and perfect channel estimation is assumed at the receiver.

[2]The additional complexity (in CM) for the optimal ordering is $(2/3)N^3$ [4], [5] or $(1/2)N^3$ [6], which is much smaller than the complexity for computing MMSE filter coefficients.

TABLE II
NUMBER OF CMs AND CAs

Calculations	Computational Complex
$C[\sum_{k=1,\dots,N} \bar{\mathbf{y}}^{[k]}]$	$(log_2 N + 5L_{max} + 2)N^2$ CMs $(5L_{max} - 2)N^2$ CAs
$C[\sum_{k=1,\dots,N} \mathbf{M}^{[1],-1}_{:,k}]$	$(log_2 N + 5L_{max} + 2)N^2$ CMs $(5L_{max} - 2)N^2$ CAs
$C[\sum_{k=1,\dots,N} \mathbf{A}^{[k]}]$	$(1/6)N^3 - N^2 - (13/6)N + 1$ CMs $2N - 3$ CAs
$C[\sum_{k=1,\dots,N} \mathbf{T}^{[k]}]$	$(1/6)N^3 - (1/2)N^2 + (1/3)N$ CMs $(1/6)N^3 - (1/2)N^2 + (1/3)N$ CAs
$C[\sum_{k=1,\dots,N} \hat{x}_{z_k}]$	$(1/2)N^2 - (1/2)N$ CMs $(1/2)N^2 - (3/2)N + 1$ CAs

TABLE III
COMPLEXITY OF PREVIOUSLY REPORTED ALGORITHMS

Algorithm	Computational Complexity
$C_{[4]}$	$(5+\frac{2}{3})N^3+\mathcal{O}(N^2)$ CMs, $(5+\frac{2}{3})N^3+\mathcal{O}(N^2)$ CAs
$C_{[5]}$	$(4+\frac{1}{6})N^3+\mathcal{O}(N^2)$ CMs, $(4+\frac{1}{6})N^3+\mathcal{O}(N^2)$ CAs
$C_{[6]}$	$(3+\frac{1}{2})N^3+\mathcal{O}(N^2)$ CMs, unknown CAs
$C_{[7]}$	$(3+\frac{2}{3})N^3+\mathcal{O}(N^2)$ CMs, $3N^3+\mathcal{O}(N^2)$ CAs
$C_{[8]}$	$(2+\frac{11}{12})N^3+\mathcal{O}(N^2)$ CMs, $(3+\frac{1}{6})N^3+\mathcal{O}(N^2)$ CAs
$C_{[9]}$	$(3+\frac{1}{3})N^3 + \mathcal{O}(N^2)$ COs

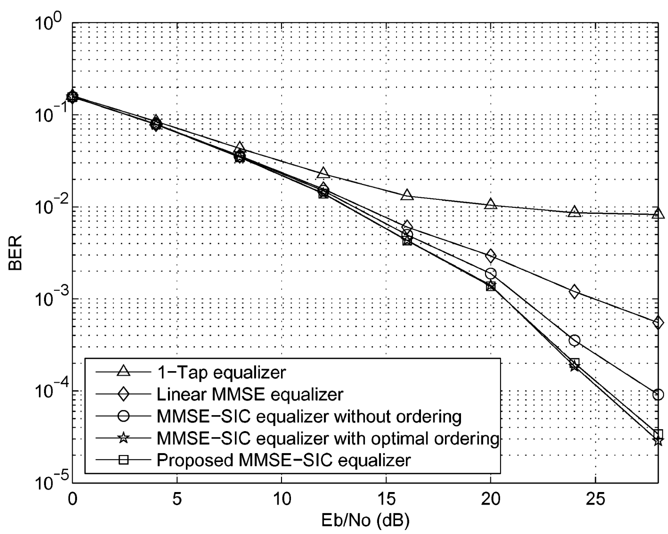

Fig. 1. Performance comparisons of the proposed MMSE-SIC equalizer with the classical MMSE-SIC equalizer when $f_n = 0.1$.

Fig. 1 shows the BER performance comparison when $f_n = 0.1$. There is no great difference in performance at low SNR regime ($E_b/N_o \leq 8$). However, as the SNR increases, the classical 1-tap equalizer shows an error floor quickly. The MMSE-SIC techniques outperform the linear MMSE equalizer due to the ICI cancellation. The difference between the MMSE-SIC equalizer and the proposed equalizer comes from the suboptimal ordering, which is negligible as expected.

V. CONCLUSION

In this letter, we proposed a novel time-domain recursive algorithm for reducing the complexity of the MMSE-SIC scheme in OFDM systems. Taking the advantage of the spareness of the time-domain channel matrix, the proposed scheme can achieve a significant complexity reduction, and it can provide an MMSE-SIC equalizer for OFDM systems with complexity much smaller than existing low-complexity MMSE-SIC schemes. Also, the performance loss due to the suboptimal ordering was confirmed to be negligible.

APPENDIX A
DERIVATIONS OF (7) AND (8)

Using properties of the IFFT/FFT matrix, ($\mathbf{F}^H = \mathbf{F}^{-1}$ and $\mathbf{F}^H\mathbf{F} = \mathbf{F}\mathbf{F}^H = \mathbf{I}_N$), $\mathbf{G}^{[k]}$ can be described as

$$\mathbf{G}^{[k]} = \mathbf{P}^{[k]}\mathbf{F}\mathbf{H}_t^H\mathbf{F}^H\left[\mathbf{F}\mathbf{H}_t\mathbf{F}^H\mathbf{P}^{[k]}\mathbf{F}\mathbf{H}_t^H\mathbf{F}^H + \sigma^2\mathbf{I}_N\right]^{-1}. \tag{22}$$

For convenience, let $\mathbf{J} = \left[\mathbf{F}\mathbf{H}_t\mathbf{F}^H\mathbf{P}^{[k]}\mathbf{F}\mathbf{H}_t^H\mathbf{F}^H + \sigma^2\mathbf{I}_N\right]^{-1}$. Then, $\mathbf{J}$ can be rewritten as

$$\mathbf{J} = \left[\mathbf{F}\mathbf{H}_t\mathbf{F}^H\left\{\mathbf{P}^{[k]} + \sigma^2\left(\mathbf{F}\mathbf{H}_t^H\mathbf{H}_t\mathbf{F}^H\right)^{-1}\right\}\mathbf{F}\mathbf{H}_t^H\mathbf{F}^H\right]^{-1}. \tag{23}$$

Substituting (23) into (22) gives

$$\mathbf{G}^{[k]} = \mathbf{P}^{[k]}\left[\mathbf{P}^{[k]} + \sigma^2\left(\mathbf{F}\mathbf{H}_t^H\mathbf{H}_t\mathbf{F}^H\right)^{-1}\right]^{-1}\mathbf{F}\mathbf{H}_t^{-1}\mathbf{F}^H. \tag{24}$$

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