LETTER

Robust Space Time Code for Channel Coded MIMO Systems**

Ilmu BYUN†, Hae Gwang HWANG†, Young Jin SANG†, *Nonmembers*, *and* Kwang Soon KIM†*a), *Member*

SUMMARY Various space time code (STC) designs have been proposed to obtain full diversity at full rate in multiple-input multiple-output (MIMO) channels for uncoded systems. However, commercial wireless systems typically employ powerful channel codes such as turbo codes and low density parity check (LDPC) codes together with an STC. For these applications, an STC optimized for uncoded systems may not provide the best performance. In this paper, an STC with relatively good performance over a wide range of code rates is proposed. Simulation results show that the performance of the proposed robust STC is very close to the best performance of the SM and the Golden code in various code rates.
key words: *MIMO, STC, channel code, robust STC*

1. Introduction

Conventional MIMO wireless systems employ a transmission scheme designed to maximize either diversity gain or multiplexing gain. Diversity transmission schemes, like orthogonal space time block codes (OSTBCs), are designed to overcome fading channel by combining the channel gain of each antenna link [1], [2]. However, OSTBC cannot achieve the full channel capacity except for the case with two transmit antennas and one receive antenna. The spatial multiplexing (SM) transmission scheme (or Bell labs layered space-time [3]) can achieve the full channel capacity by transmitting independent data stream through each transmit antenna. However, the diversity order of the SM is worse than that of the OSTBC.

Recently, linear dispersion codes (LDCs) have been proposed to obtain full diversity and full rate (FDFR) of MIMO channel [4]. These codes use a linear transformation matrix, and a transmitted codeword is constructed by a linear combination of symbols. In [5], Belfiore et al. developed the Golden code, which is known optimal for 2×2 uncoded MIMO systems. In [6], a generalized code structure was proposed to provide both full diversity and full rate for $N \times N$ uncoded MIMO systems. However, commercial wireless systems typically employ powerful channel codes such as turbo codes and LDPC codes together with an STC. In these cases, an STC optimized for uncoded systems may not provide the best performance.

On the other hand, there have been efforts to develop an STC in conjunction with channel codes [7]–[12]. In [7], space time trellis coded modulation (STTCM) schemes were proposed. However, the performance of an STTCM is much worse than a channel coded MIMO system using a powerful channel code. To overcome this, jointly designed STCs with a powerful channel code (such as space time turbo trellis code modulation [8], [9]) have been investigated [10]–[12]. Although the jointly designed STCs can provide good coding gain as well as full diversity at full rate, a special encoder and decoder structure with high complexity is required [8] or different set of encoder and decoder is required as the number of transmit antennas varies [9]. Thus, these schemes are not attractive for a practical wireless communication system since various sets of modulation, coding, and antenna scheme should be supported in a system with reasonable complexity. One reasonable alternative is to use powerful channel codes designed for SISO channels and design a good STC for the given channel codes. This approach is attractive since it can be directly applied to current commercial systems to improve the system performance.

In this paper, we focus on the facts that i) the log likelihood ratio (LLR) distribution of coded bits determines the performance with a given channel code and ii) the LLR distribution depends on the STC structure and the signal to noise (SNR). Also, it is found that a single previously known STC cannot provide the best performance over a wide range of code rates (or target SNRs). The proposed STC, which will be referred to as robust STC, is designed to average two different types of LLR distributions. By carefully selecting the two different types of LLR distribution, the proposed robust STC is expected to provide robust performance over a wide range of code rates in a coded MIMO system.

2. MIMO System Model

In a MIMO system model with N_t transmit antennas and N_r receive antennas, the receiver is assumed to have perfect channel state information while the transmitter has no information about the channel. The block-fading Gaussian matrix channel model is assumed with channel matrix $\mathbf{H} \in C^{N_r \times N_t}$ (C denoting the field of complex number) whose (i,j)th elements, $H_{i,j}$, is independent and identically distributed (i.i.d) circularly symmetric complex Gaussian random variable (RV) with distribution $CN(0, 1)$. Let $\mathbf{R} \in C^{N_r \times T}$ be the received signal, then it can be represented

Manuscript received April 5, 2007.
Manuscript revised August 2, 2007.
†The authors are with the Department of Electrical and Electronic Engineering, Yonsei University, 134 Shinchon-dong, Seodaemun-gu, Seoul 120-749, Korea.
*Corresponding author.
**This work was supported by the MIC, Korea, under the ITRC support program supervised by the IITA. (IITA-2006-(C1090-0602-0011))
a) E-mail: ks.kim@yonsei.ac.kr
DOI: 10.1093/ietcom/e91–b.1.381

as

$$\mathbf{R} = \sqrt{\frac{E_s}{N_t}}\mathbf{H}\mathbf{S} + \mathbf{N}, \tag{1}$$

where $\mathbf{S} \in C^{N_t \times T}$ denotes a space time codeword matrix over a symbol duration T and $\mathbf{N} \in C^{N_r \times T}$ is a noise matrix in which each element is an i.i.d. circularly symmetric complex Gaussian RV with distribution $CN(0, N_0)$. Also, E_s is the symbol energy and the symbol energy is equally divided to each transmit antenna. The vector form of the above received signal can be represented as

$$\begin{aligned} \mathbf{r} &= \sqrt{\frac{E_s}{N_t}}\mathbf{H}_{stack}\mathbf{s} + \mathbf{n} \\ &= \sqrt{\frac{E_s}{N_t}}\mathbf{H}_{stack}\mathbf{C}\mathbf{x} + \mathbf{n}, \end{aligned} \tag{2}$$

where $\mathbf{r} \doteq vec(\mathbf{R})$, $\mathbf{s} \doteq vec(\mathbf{S})$, $\mathbf{n} \doteq vec(\mathbf{N})$, and $\mathbf{H}_{stack} \doteq \mathbf{I}_T \otimes \mathbf{H}$ is an $N_rT \times N_tT$ stacked channel matrix. The space time codeword vector $\mathbf{s}$ is generated by multiplying the data symbol vector $\mathbf{x} \in C^{M \times 1}$ with the linear dispersion matrix $\mathbf{C} \in C^{N_tT \times M}$, where M is the number of transmitted symbols over one space time codeword period. For a full-rate code, $M = N_tT$.

In channel coded MIMO systems, the LLR is calculated for the channel decoding procedure from the received signal as

$$\begin{aligned} LLR(k) &= \log\frac{p(b_k = 0|\mathbf{r})}{p(b_k = 1|\mathbf{r})} = \log\frac{f(\mathbf{r}|b_k = 0)}{f(\mathbf{r}|b_k = 1)} \\ &= \log\frac{\Sigma_{\hat{\mathbf{x}} \in \mathbf{X}(k,0)}\ \exp\left(-\frac{|\mathbf{r}-\hat{\mathbf{r}}|^2}{2\sigma_n^2}\right)}{\Sigma_{\hat{\mathbf{x}} \in \mathbf{X}(k,1)}\ \exp\left(-\frac{|\mathbf{r}-\hat{\mathbf{r}}|^2}{2\sigma_n^2}\right)} \\ &\simeq \log\frac{\min_{\hat{\mathbf{x}} \in \mathbf{X}(k,0)}\ \exp\left(-\frac{|\mathbf{r}-\hat{\mathbf{r}}|^2}{2\sigma_n^2}\right)}{\min_{\hat{\mathbf{x}} \in \mathbf{X}(k,1)}\ \exp\left(-\frac{|\mathbf{r}-\hat{\mathbf{r}}|^2}{2\sigma_n^2}\right)} \\ &= -\frac{|\mathbf{r} - \hat{\mathbf{r}}(b_k = 0)|^2}{2\sigma_n^2} + \frac{|\mathbf{r} - \hat{\mathbf{r}}(b_k = 1)|^2}{2\sigma_n^2}, \end{aligned} \tag{3}$$

where $\sigma_n^2 = \frac{N_0}{2}$ is the noise variance. Here, b_k is the kth bit, $LLR(k)$ is the LLR of b_k, and $\mathbf{X}(b_k = i)$ is the set of all vector symbols satisfying $b_k = i$. Also, $\hat{\mathbf{r}}(b_k = i)$ is defined as

$$\hat{\mathbf{r}}(b_k = i) = \sqrt{\frac{E_s}{N_t}}\mathbf{H}_{stack}\mathbf{C}\hat{\mathbf{x}}(b_k = i)\ , i = 0, 1, \tag{4}$$

where $\hat{\mathbf{x}}(b_k = i)$ denotes the vector symbol which makes $|\mathbf{r} - \hat{\mathbf{r}}|^2$ minimum in the set of all possible vector symbols $\mathbf{X}(k, i)$.

Previously, STCs are designed to minimize the pairwise error probability of the maximum likelihood (ML) detection using the determinant and the rank criteria [2], [6]. However, the determinant and the rank criteria were developed for uncoded MIMO systems. In channel coded MIMO systems, it is difficult to analyze the performance because they are decoded through an iterative procedure. In [13], the asymptotic performance of an LDPC code using a message passing decoder was analyzed by tracking the evolution of the LLR distribution. From the density evolution, it is straightforward that an LLR distribution (assuming that '0' is sent) gets better as the mean increases and/or the variance decreases. Once a channel code and its decoding algorithm is given, the initial LLR distribution determines the performance of the code. From this motivation, we compare the initial LLR distribution of the SM and the Golden code in the 2×2 MIMO systems through simulation. As shown in Table 1, the LLR mean of the SM is greater than that of the Golden code and the LLR standard deviation (STD) of the SM is also greater than that of the Golden code. However, the LLR mean of the Golden code is quite close to that of the SM while the STD of the Golden code is still somewhat smaller than that of the SM as SNR increases. Since the required SNR for a given target performance increases as the code rate increases, it is easily seen that the relative performance of different STCs varies with the code rate. For example as shown in Fig. 1, the performance of the SM is better than that of the Golden code when the code rate is 1/2. However, the performance of the Golden code is better than that of the SM when the code rate is 3/4.

Table 1 LLR mean and STD of STCs in 2×2 vehicular A channel with 120 km/h mobile speed.

	SM		Golden		Robust	
SNR	mean	STD	mean	STD	mean	STD
3	3.01	3.52	2.76	3.05	2.98	3.36
5	4.91	5.04	4.49	4.35	4.77	4.80
7	7.87	7.39	7.38	6.36	7.75	7.08
9	12.63	11.08	12.20	9.45	12.59	10.59
11	20.20	16.88	19.98	14.21	20.38	16.14

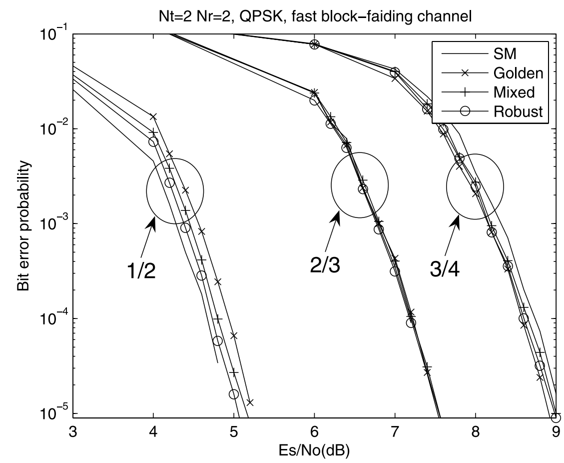

Fig. 1 Performance of STCs under 2×2 fast block-fading channel.

3. The Proposed Robust Space Time Code

The main idea of the proposed robust STC is to use different STC for each transmit antenna for obtaining different LLR distributions at the receiver. In the decoding procedure, information bits are recovered from the LLRs of the received coded bits. Thus, the different LLR distributions can be combined through the decoding procedure, which results in a robust performance.

In 2×2 MIMO systems, two different space time coded symbols can be transmitted from 2 transmit antennas by using the following STC structure.

$$\mathbf{C}_{mixed} = \begin{pmatrix} \alpha & jA\alpha & 0 & 0 \\ 0 & 0 & B\beta & \beta \\ 0 & 0 & \beta & -B\beta \\ jA\alpha & \alpha & 0 & 0 \end{pmatrix}, \tag{5}$$

where $\alpha = (1 + A^2)^{-1/2}$ and $\beta = (1 + B^2)^{-1/2}$. Note that the LLR distribution of the coded bits transmitted from antennas 1 and 2 in (5), which will be referred to as mixed STC, are determined by the parameters A and B, respectively. Then, the two distributions are combined during the decoding procedure. However, the combining effect is reduced as the code rate increases. To remedy this, the mixed STC is multiplied by a unitary matrix to average the LLR distributions. One of unitary matrices that divide the symbol energy equally may be given as

$$\mathbf{u} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix}. \tag{6}$$

Finally, multiplying the mixed STC with the unitary matrix **u**, the proposed robust STC is obtained as

$$\begin{aligned} \mathbf{C}_{robust} &= \mathbf{C}_{mixed}\mathbf{u} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha & jA\alpha & jA\alpha & \alpha \\ -B\beta & -\beta & \beta & B\beta \\ -\beta & B\beta & -B\beta & \beta \\ jA\alpha & \alpha & \alpha & jA\alpha \end{pmatrix}. \end{aligned} \tag{7}$$

Note that $\mathbf{C}_{robust}$ is a unitary matrix because $\mathbf{C}_{mixed}$ and **u** are unitary matrices. Thus, the proposed robust STC satisfies the capacity criterion and the rank criterion in [4]. Instead of the determinant criterion, which is for an uncoded case or a coded case with very high code rate, the proposed robust STC is designed to have relatively good LLR distribution over a wide range of code rates (or target SNRs) by carefully selecting the two parameters, A and B. To obtain different LLR distributions in the robust STC matrix in (7), the parameter A is selected as 0.6183 and the parameter B is selected 0. Note that $A = 0.6183$ comes from the constant for the Golden code [5], [6] which is known as optimal for uncoded cases and as very good when code rate is high (see Fig. 1). Also, $B = 0$ come from the SM, which is very good when the code rate is low (see Fig. 1).

Remark 1: The optimal values of A and B vary with channel codes and code rates and we may change the values of A and B according to the given set of channel codes. However, although not shown explicitly, the performance gain is negligible compared to the increased complexity. Thus, in the proposed scheme, the values of A and B are fixed so that only a single STC is required.

Remark 2: The proposed robust STC can be extended to a general $n \times n$ MIMO systems. In $n \times n$ MIMO systems, a mixed STC can be constructed by transmitting n different space time coded symbols at n transmit antennas, similarly in [6] with different parameters. Then, the robust STC is obtained by multiplying a unitary matrix with the mixed STC. However, the selection of the n good space time codes and the n parameters is beyond the scope of this paper.

4. Simulation Results

First, the proposed STCs are evaluated in a fast block-fading (channel is constant over the space time codeword duration) for a 2 × 2 MIMO system. QPSK is used as a modulation method and the LLR is calculated using the max-log LLR calculation method. Also, the LDPC codes in the IEEE 802.16e standard [14] with block length of 1536 and code rates 1/2, 2/3, and 3/4 are used. The performance of the proposed robust STC under fast block-fading channel is shown in Fig. 1. The robust STC is better than the mixed STC in all cases. Also, it is shown that the robust STC shows the second best performance among the robust STC, the Golden code, and the SM over all range of code rates. When the code rate is 2/3, the performance of the SM is similar to that of the Golden code. Thus, the mixed STC and the robust STC also show similar performance since they are mixed versions of the SM and the Golden code. Figures 2 and 3 show the performance of the proposed robust STC in an OFDM system under 2 × 2 frequency selective fading channel (the ITU-R Vehicular A channel model) with Jake's Doppler spectrum, when the mobile speed is 3 km/h and 120 km/h, respectively. The OFDM parameters used in this simulation are listed in Table 2. From the figures, it is clearly seen that the proposed robust STC shows the performance very close to the best one in each case in a practical channel model.

To compare the LLR distribution of the proposed robust STC, the SM, and the Golden code, the mean and the STD of the LLR distritbuion of each STC are listed in Ta-

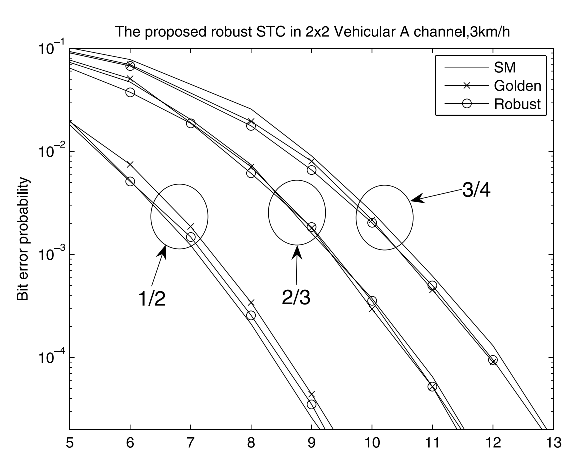

Fig. 2 Performance of the proposed robust STC when the mobile speed is 3 km/h.

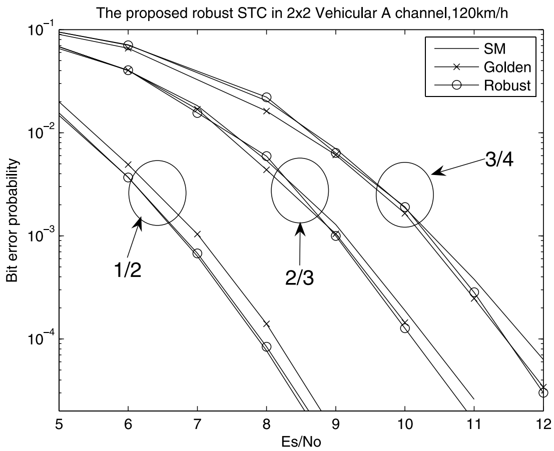

Fig. 3 Performance of the proposed robust STC when the mobile speed is 120 km/h.

Table 2 The OFDM parameters used in the simulation.

Carrier frequency	2 GHz
FFT size	1024
Sampling frequency	10 MHz
Data subcarrier	768
Guard subcarrier	256
Cyclic prefix time	12.8 μs
OFDM symbol time	115.2 μs
subchannel structure	distributed 24 subcarriers × 32 OFDM symbols

ble 1. The LLR STD of the robust STC is somewhere between that of the Golden code and that of the SM in all SNR values. However, the LLR mean of the robust STC is quite close to that of SM. Thus, the LLR distribution shown in Table 1 confirms the results shown in Fig. 3.

5. Conclusion

In this paper, the robust STC was proposed to obtain relatively good performance over a wide range of code rates in a channel coded MIMO system. In the robust STC, different STCs are mixed by a unitary matrix to average the LLR distribution of each STC. It was shown that the performance of the robust STC is very close to the best one among the SM and the Golden code over a wide range of code rates. Thus, the proposed STC can be considered as a good alternative for commercial wireless communication systems employing adaptive transmission scheme based on open-loop control or closed-loop control using a long-term feedback.

References

[1] S.M. Alamouti, “A simple transmit diversity technique for wireless communication,” IEEE J. Sel. Areas Commun., vol.16, no.8, pp.1451–1458, Oct. 1998.

[2] V. Tarokh, H. Jafarkhani, and A.R. Calderbank, “Space-time block codes from orthogonal designs,” IEEE Trans. Inf. Theory, vol.45, no.5, pp.1456–1476, July 1999.

[3] G.J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas,” Bell Labs Tech. J., vol.2, pp.41–59, Autumn 1996.

[4] R.W. Heath, Jr. and A.J. Paulraj, “Linear dispersion codes for MIMO systems based on frame theory,” IEEE Trans. Signal Process., vol.50, no.10, pp.2429–2441, Oct. 2002.

[5] J.C. Belfiore, G. Rekaya, and E. Viterbo, “The golden code: A 2 × 2 full-rate space-time code with nonvanishing determinants,” IEEE Trans. Inf. Theory, vol.51, no.4, pp.1432–1436, April 2005.

[6] M.I. Lee, S.K. Oh, and D.S. Kwon, “Design of space-time codes achieving generalized optimal diversity,” Proc. IEEE Global Telecommun. Confer. (GLOBECOM), vol.5, pp.3027–3031, St. Louis, MO, Nov. 2005.

[7] V. Tarokh, N. Seshadri, and A.R. Calderbank, “Space time codes for high data rate wireless communication: Performance criterion and code construction,” IEEE Trans. Inf. Theory, vol.44, no.2, pp.744–765, March 1998.

[8] Y. Li, B. Vucetic, Q. Zhang, and Y. Huang, “Assembled space-time turbo trellis codes” IEEE Trans. Veh. Technol., vol.54, no.5, pp.1768–1772, Sept. 2005.

[9] Y. Hong, J. Yuan, Z. Chen, and B. Vucetic, “Space-time turbo trellis codes for two, three, and four transmit antennas,” IEEE Trans. Veh. Technol., vol.53, no.2, pp.318–328, March 2004.

[10] K. Ishii and R. Kohno, “Space-time-frequency turbo code over time-varying and frequency-selective fading channel,” IEICE Trans. Fundamentals, vol.E88-A, no.10, pp.2885–2895, Oct. 2005.

[11] L. Goulet and H. Leib, “Serially concatenated space-time codes with iterative decoding and performance limits of block-fading channels,” IEEE J. Sel. Areas Commun., vol.21, no.5, pp.765–773, June 2003.

[12] Z. Wang, S. Zhou, and G. Giannakis, “Joint coding-precoding with low-complexity turbo-decoding,” IEEE Trans. Wireless Commun., vol.3, no.3, pp.832–842, May 2004.

[13] T.J. Richardson and R.L. Urbabke, “The capacity of low-density parity-check codes under message-passing decoding,” IEEE Trans. Inf. Theory, vol.47, no.2, pp.518–618, Feb. 2001.

[14] IEEE std. 802.16e, “Air interface for fixed and mobile broadband wireless access systems, amendment 2: Physical and medium access control layers for combined fixed and mobile operation in licensed bands,” Dec. 2005.