A Multiuser Receiver for Trellis-Coded DS/CDMA Systems in Asynchronous Channels

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***Abstract*—In this paper, we propose and analyze a multiuser receiver using a decorrelating filter and Viterbi decoders for trellis-coded direct-sequence code-division multiple-access (DS/CDMA) systems with biorthogonal signal constellation in asynchronous channels. The biorthogonality is implemented by user signature waveforms and the decorrelating filter. The performance of the proposed system is investigated with emphasis on the asymptotic cases: it is shown that the proposed system provides us with some coding gain and near–far resistance. It is confirmed that we can enhance the performance of the proposed system by using base-station antenna arrays.**

***Index Terms*—Asynchronous channels, multiuser detector, trellis-coded modulation.**

I. Introduction

THERE has been a great deal of interest in direct-sequence code-division multiple-access (DS/CDMA) systems because of their useful applications in mobile communications. In addition, investigation of multiuser receivers for the DS/CDMA systems becomes an interesting research area since they can reduce interuser interference and resist the near–far effect, which are among the major defects of DS/CDMA systems [1]–[7].

Most of the work on multiuser receivers has been accomplished for uncoded systems: coding has been frequently used, however, for reliable communication systems. Recently, multiuser receivers for convolutionally coded DS/CDMA were investigated in [8] and [9], and it was shown that we could achieve some coding gain and near–far resistance simultaneously. Trellis-coded modulation (TCM) is also widely used in many communication systems because of its bandwidth efficiency. In addition, TCM can be used as a bandwidth-efficient inner code for concatenated-coded DS/CDMA systems which is expected to provide us with fast and reliable integrated information services for mobile communication systems in the near future. Thus, it is worthwhile investigating multiuser detection schemes for trellis-coded DS/CDMA systems. In [10], multiuser receivers for trellis-coded DS/CDMA systems using C-ary phase-shift keying (PSK) constellation were investigated, where it was again shown that we could get some coding gain with near–far resistance, as in the convolutionally coded DS/CDMA systems.

In [11], TCM with biorthogonal signal constellation was proposed for DS/CDMA systems: it was shown that the system has better performance than the trellis-coded DS/CDMA system using C-ary PSK signal constellation. In addition, the biorthogonality could be implemented by using BPSK and $C/2$ signature waveforms for each user. Thus, it is more suitable to employ biorthogonal signal constellation (than to employ PSK) in trellis-coded DS/CDMA systems, and employing multiuser receivers for the systems are more adequate because of the nature of the biorthogonal signal constellation. In this paper, we will investigate a multiuser receiver for trellis-coded DS/CDMA systems using biorthogonal signal constellation.

On the other hand, it was shown that we could improve the performance of multiuser receivers by using antenna arrays [12]–[15] not only for capturing more signal energy, but also for using spatial diversity, since we can discriminate different users by both signature waveforms and channel vectors.

In this paper, we propose a suboptimum multiuser receiver using a decorrelating filter and Viterbi decoders for trellis-coded DS/CDMA systems. In Section II, the system model considered in this paper is shown. In Section III, a suboptimum multiuser receiver using a decorrelating filter and Viterbi decoders for trellis-coded DS/CDMA systems is proposed, and the asymptotic performance of the proposed system is investigated in Section IV. In Section V, we investigate the performance enhancement of the proposed system by using base-station antenna arrays.

II. System Model

First, we consider the $m/(m+1)$ TCM with biorthogonal signal sets. A biorthogonal signal set can be defined as follows.

- For each signal in a biorthogonal signal set, there exists only one antipodal signal in the set.
- A signal in a biorthogonal signal set is orthogonal to all the other signals in the set except for its antipodal one.

Let K be the number of users and $2L = 2^{m+1}$ be the number of signal points in the biorthogonal set. One of the ways to achieve the biorthogonality mentioned above is to employ K sets of L mutually orthogonal signature sequences provided that the processing gain is larger than KL. In this case, low cross correlation between any two signature sequences of different sets should be guaranteed: one example of such sequences was shown in [11]. Another way is to use nearly

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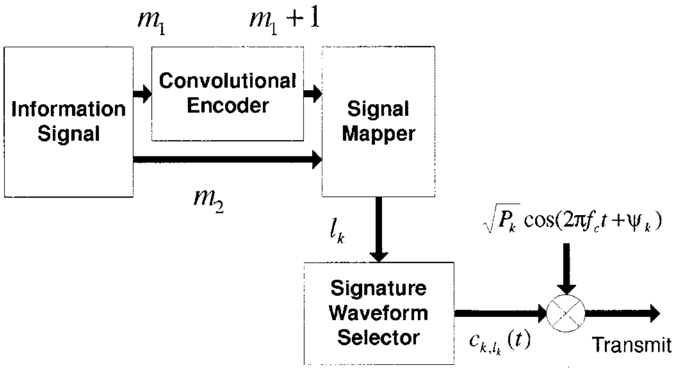

Fig. 1. Transmitter structure of the trellis-coded DS/CDMA system.

orthogonal pseudonoise (PN) sequences such as Gold sequence or m sequence. In this case, we have both low cross correlation and accommodation of more users at the expense of slightly imperfect biorthogonality. Furthermore, the perfect biorthogonality can be achieved by employing appropriate multiuser receivers. Thus, the second choice would be more feasible for trellis-coded DS/CDMA systems employing a multiuser receiver.

The transmitter scheme for the trellis-coded DS/CDMA system considered in this paper is shown in Fig. 1. First, m bits of an information signal is divided into m_1 and m_2 bits. The m_1 bits enter the $m_1/(m_1+1)$ convolutional encoder. Then, m_1+1 output bits from the convolutional encoder and the m_2 bits decide the signal point in the signal mapper. A signature waveform is then selected according to the signal mapper output in the signature waveform selector. After the selected signature waveform is modulated by a carrier, it is transmitted. Let $\{c_{k,l}(t)|k=1, 2, \cdots, K, l=1, 2, \cdots, L\}$ be the set of signature waveforms with period T_s. Then, we can construct the biorthogonal signal set of the kth user as $\{c_{k,l}(t)|l=1, 2, \cdots, 2L\}=\{c_{k,1}(t), \cdots, c_{k,L}(t), -c_{k,1}(t), \cdots, -c_{k,L}(t)\}$. The transmitted signal from the kth mobile is then

$$u_k(t)=\sqrt{2P_k}\,\mathrm{Re}\left\{c_{k,l_k}(t)e^{j(2\pi f_c t+\psi_k)}\right\} \tag{1}$$

where

- P_k transmitted power of the kth user signal;
- f_c carrier frequency;
- ψ_k random phase of the kth carrier;
- l_k output of the signal mapper of the kth user.

We assume that the channel is slowly varying nonselective Rayleigh and we use coherent reception. Then, the equivalent baseband signal received at the base-station is

$$r(t)=\sum_{k=1}^{K}\sum_{l=1}^{2L}\sqrt{P_k}\alpha_k e^{j\phi_k}I_{k,l}(t-\tau_k)c_{k,l}(t-\tau_k)+n(t) \tag{2}$$

where

- $\alpha_k e^{j\phi_k}$ complex fading process of the kth user;
- τ_k time delay of the kth user;
- $n(t)$ additive white Gaussian noise with variance σ_n^2;
- $I_{k,l}(t)$ indicator function of the kth user defined as

$$I_{k,l}(t)=\begin{cases}1, & \text{if } l_k=l\\ 0, & \text{otherwise.}\end{cases} \tag{3}$$

The receiver structure considered in this paper is shown in Fig. 2. It is easily seen that the $(L+l)$th matched filter output can be obtained by multiplying -1 with the output of the lth matched filter output at each filter bank. Therefore, only KL matched filters (or K filter banks each with L matched filters) and a $KL\times KL$ decorrelator are necessary: the received signal passes through the bank of KL matched filters, and the matched filter outputs are decorrelated by the decorrelating filter and then decoded by Viterbi decoders. The matched filter output vector sampled at $t=nT_s$ can be written as

$$Y(n)=[y_{1,1}(n)\cdots y_{1,L}(n)\cdots y_{K,1}(n)\cdots y_{K,L}(n)]^T \tag{4}$$

where

$$y_{p,q}(n)=\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=-1}^{1}\sqrt{P_k}\,\alpha_k e^{j\phi_k}x_{k,l}(n)\gamma_{p,q,k,l}^{(m)} + n_{p,q}(n) \tag{5}$$

$x_{k,l}(n)=x_{k,l}(t)|_{t=nT_s}$, and $x_{k,l}(t)=I_{k,l}(t)-I_{k,L+l}(t)$, $l=1, 2, \cdots, L$. Here, $\gamma_{p,q,k,l}^{(-1)}$, $\gamma_{p,q,k,l}^{(0)}$, and $\gamma_{p,q,k,l}^{(1)}$ are the cross correlations between the qth signature waveform of the pth user during the nth symbol and the lth signature waveform of the kth user during the $(n-1)$th, nth, and $(n+1)$th symbols, respectively, and $n_{p,q}(n)$ is a zero mean Gaussian random variable with

$$E\{n_{p,q}(n)n_{k,l}^*(n+m)\}=\begin{cases}\dfrac{\sigma_n^2\gamma_{p,q,k,l}^{(m)}}{T_s}, & m=-1, 0, 1\\ 0, & \text{otherwise.}\end{cases} \tag{6}$$

III. The Decorrelating Filter and Viterbi Decoders

In this section, we will consider the decorrelating filter and Viterbi decoders for trellis-coded DS/CDMA systems. We can rewrite the output of the bank of matched filters in vector notation as

$$Y(n)=\Gamma^{(-1)}WX(n-1)+\Gamma^{(0)}WX(n) + \Gamma^{(1)}WX(n+1)+N(n) \tag{7}$$

where

$$W=\mathrm{diag}([w_{1,1}\cdots w_{1,L}\cdots w_{K,1}\cdots w_{K,L}]) \tag{8}$$

$$w_{k,l}=\sqrt{P_k}\alpha_k e^{j\phi_k} \tag{9}$$

$$X(n)=[x_{1,1}(n)\cdots x_{1,L}(n)\cdots x_{K,1}(n)\cdots x_{K,L}(n)]^T \tag{10}$$

$$N(n)=[n_{1,1}(n)\cdots n_{1,L}(n)\cdots n_{K,1}(n)\cdots n_{K,L}(n)]^T \tag{11}$$

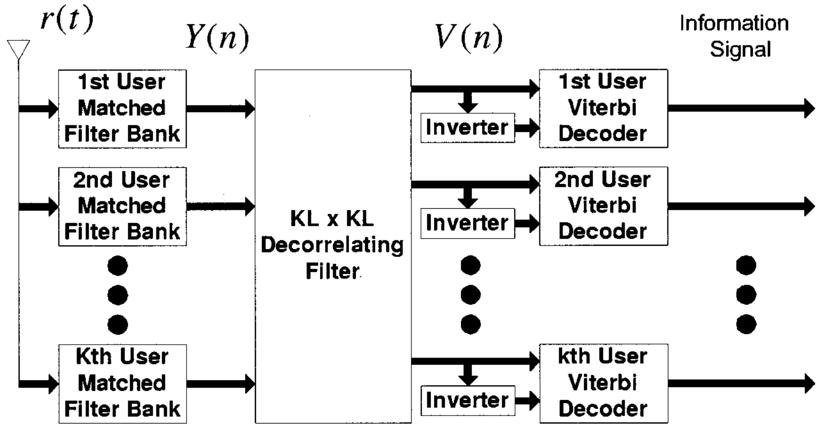

Fig. 2. The receiver structure of the proposed system.

and $\Gamma^{(m)}$ is the $KL \times KL$ matrix whose $(p \circ q,\ k \circ l)$th element is $\gamma^{(m)}_{p,q,k,l}$, with $k \circ l = (k-1)L + l$. Let $y_{k,l}(Z)$ be the Z transform of $y_{k,l}(n)$. Then, the Z transform

$$Y(Z) = [y_{1,1}(Z) \cdots y_{1,L}(Z) \cdots y_{K,1}(Z) \cdots y_{K,L}(Z)]^T$$

of $Y(n)$ is

$$Y(Z) = \left(\Gamma^{(-1)}Z^{-1} + \Gamma^{(0)} + \Gamma^{(1)}Z\right) WX(Z) + N(Z) \quad (12)$$

where $X(Z)$ and $N(Z)$ are the Z transforms of $X(n)$ and $N(n)$, respectively. Let

$$G(Z) = \sum_{k=-\infty}^{\infty} T(k)Z^{-k} = \left(\Gamma^{(-1)}Z^{-1} + \Gamma^{(0)} + \Gamma^{(1)}Z\right)^{-1}$$

where $\{T(k)\}$ is a sequence of the impulse response matrices of the decorrelating filter. Then the decorrelating filter output $V(Z)$ is

$$\begin{aligned} V(Z) &= \begin{bmatrix} G(Z)Y(Z) \\ -G(Z)Y(Z) \end{bmatrix} \\ &= \begin{bmatrix} X(Z) + G(Z)N(Z) \\ -WX(Z) - G(Z)N(Z) \end{bmatrix} \\ &= W'X'(Z) + N'(Z) \end{aligned} \quad (13)$$

where

$$W' = \mathrm{diag}\left([w_{1,1} \cdots w_{1,2L} \cdots w_{K,1} \cdots w_{K,2L}]\right)$$
$$X'(Z) = \left[X^T(Z) - X^T(Z)\right]^T$$

and

$$N'(Z) = \left[N^T(Z)G^T(Z) - N^T(Z)G^T(Z)\right]^T.$$

Then, the lth output $v_{k,l}(n)$ of the kth user decorrelating filter multiplied by $e^{-j\phi_k}$ (assuming coherent reception) is

$$v_{k,l}(n) = \sqrt{P_k}\alpha_k x'_{k,l}(n) + \mathrm{Re}\left\{e^{-j\phi_k} n'_{k,l}(n)\right\} \quad (14)$$

where $x'_{k,l}(n)$ and $n'_{k,l}(n)$ are the $(k \circ l)$th elements of the inverse Z transforms of $X'(Z)$ and $N'(Z)$, respectively. Since the decorrelating filter $G(Z)$ is noncausal, it is necessary to approximate it by appropriate long delay and truncation of the noncausal part in practice [5].

First, we assume that no channel state information (CSI) is available. Let Ξ be the set of all allowable paths in the trellis code. Then, the kth user's metric of $\xi_i \in \Xi$ at epoch n is

$$\Delta_{k,\xi_i} = \sum_{j=-\infty}^{n} v_{k,l^i_j}(j) \quad (15)$$

where $\xi_i = \{l^i_j\}^{\infty}_{j=-\infty}$ and l^i_j is the index of the signature waveform during the jth symbol of the ith path of the kth user. Note that, without noise, the normalized decorrelator output with respect to the symbol energy due to the desired, orthogonal, and antipodal signals are 1, 0, and -1, respectively, while the normalized Euclidean distances with respect to the symbol energy between the same, orthogonal, and antipodal signal points are 0, 2, and 4, respectively. Thus, the decorrelator output is itself the (scaled and shifted) Euclidean distance. Therefore, for given outputs of the decorrelator, the maximum likelihood metric should be the weighted sum of the decorrelator outputs since the noise in the decorrelator outputs is correlated. Fortunately, noise correlation of the decorrelator output is not significant in most cases. Thus, we choose the near maximum likelihood metric (15) to avoid additional burden of weight calculations. Then, we can choose the largest Δ_{k,ξ_i} among all $\xi_i \in \Xi$ by Viterbi soft-decision algorithm.

Next, let us assume that the CSI is known perfectly at the receiver, which can be achieved by several methods including use of pilot symbols [16]. Since interuser interference is eliminated by the decorrelator, the optimal weight is the value of the fading variable [17]. Then the kth user's metric of $\xi_i \in \Xi$ at epoch n is

$$\Delta_{k,\xi_i} = \sum_{j=-\infty}^{n} \alpha_k(j) v_{k,l^i_j}(j) \quad (16)$$

where $\alpha_k(j)$ is the fading process of the kth user in the jth symbol interval. Then, we can choose the largest Δ_{k,ξ_i} among all $\xi_i \in \Xi$ by Viterbi soft-decision algorithm.

IV. Performance Analysis

In this section, we will analyze the performance of the proposed system. In Section IV-A, we will introduce the asymptotic diversity order (ADO) and asymptotic gain (AG) as the performance measures for the proposed system. In Section IV-B, we will consider the case where interleaving is not used and the CSI is not available. In Section IV-C, we will consider the case where interleaving is used and the CSI is available. Although the two cases may or may not represent a real situation, we can get the lower and upper bounds of the performance from the analyses.

A. The Asymptotic Diversity Order (ADO) and Asymptotic Gain (AG)

As a measure of the performance of multiuser receivers, the asymptotic efficiency (AE) has been widely used [1]–[5]. In coded systems, however, the asymptotic efficiency cannot be used directly since it does not reflect the effect of coding gain. Furthermore, coded systems with interleaving can provide a form of diversity in fading channels. Therefore, it is reasonable to measure the asymptotic performance of the multiuser receivers for coded systems with its equivalent diversity order and equivalent coding gain. In the following definitions, Rayleigh fading channel is assumed.

Definition 1: Let P_b be the bit error probability of the kth user of a coded system when the powers of interferers are all zero, P_b^s be the bit error probability of the conventional single user uncoded BPSK system, and σ_n^2 be the noise power. Then, the ADO of the kth user is defined as

$$\mathrm{ADO}_k = \sup_{O} \left\{ O \,\middle|\, \lim_{\sigma_n^2 \to 0} \frac{P_b}{(P_b^s)^O} < \infty \right\}. \tag{17}$$

Now, we define the AG of the trellis-coded DS/CDMA system using multiuser receiver as the required signal energy of the uncoded single-user BPSK system with diversity order O, which achieves the same performance as the trellis-coded DS/CDMA system using a multiuser receiver with unit symbol energy when the noise power approaches zero.

Definition 2: Let P_b be the bit error probability of the kth user, E_k the symbol energy of the kth user of a coded system, $P_b^s(E, O)$ the bit error probability of the conventional single-user uncoded BPSK system with symbol energy E and diversity order O, and σ_n^2 the noise power. Then, the AG of the kth user is defined as

$$\mathrm{AG}_k = \max \left\{ \arg_G \lim_{\sigma_n^2 \to 0} \frac{P_b}{P_b^s(GE_k, O)} = 1, 0 \right\}. \tag{18}$$

It should be mentioned that the AE was proposed for an uncoded system employing multiuser receivers in [2], and asymptotic multiuser coding gain (AMCG) was proposed for a coded system employing multiuser receivers in [8]. These two measures are very useful and intuitive in no-fading channels. Since coding in fading channels can provide not only some coding gain, but also some degrees of diversity, however, the AE and AMCG are no longer useful for coded systems in fading channels: the ADO and AG proposed in this paper can play a similar role for a coded system in fading channels to that of the AE and AMCG, respectively, in no-fading channels.

Let the ADO_k and AG_k of the kth user of a coded system be O and G, respectively. Then, the log bit error probability of the kth user goes to zero with the same slope as that of a single-user BPSK system with diversity order O and energy GE_k. In other words, a coded DS/CDMA system with ADO O and AG G can be considered as a system which has coding gain G over the uncoded single-user BPSK system with diversity order O.

B. No Interleaving Without Channel State Information

Consider the case where interleaving is not used. Since we assumed a slowly varying channel, the fading process α_k is constant during an error event. Let ξ_d be the correct path and ξ_c be an error event with η_c different symbols. Without loss of generality, we can assume that an error event begins at the first symbol for an ergodic system. Then, the distance between ξ_d and ξ_c is

$$D_{\xi_d, \xi_c} = \sum_{j=1}^{\eta_c} D_{\xi_d, \xi_c}(j) = \sum_{j=1}^{\eta_c} \left(x'_{k, l_j^d}(j) - x'_{k, l_j^c}(j) \right). \tag{19}$$

The pairwise error probability is then

$$\begin{aligned} P_{\xi_d \to \xi_c} &= \Pr\left\{ \Delta_{k, \xi_d} < \Delta_{k, \xi_c} \right\} \\ &= \Pr\left\{ \sum_{j=1}^{\eta_c} v_{k, l_j^d}(j) - \sum_{j=1}^{\eta_c} v_{k, l_j^c}(j) < 0 \right\} \\ &= \Pr\left\{ \sqrt{P_k}\alpha_k D_{\xi_d, \xi_c} + \overline{n}_{k, \xi_d, \xi_c} < 0 \right\} \end{aligned} \tag{20}$$

where

$$\overline{n}_{k, \xi_d, \xi_c} = \mathrm{Re}\left\{ e^{-j\phi_k} \sum_{j=1}^{\eta_c} \left(n'_{k, l_j^d}(j) - n'_{k, l_j^c}(j) \right) \right\}.$$

Let us define Q_a^b and R_a^b as

$$Q_a^b = \begin{cases} 1, & l_a^b > L \\ 0, & l_a^b \le L \end{cases} \tag{21}$$

and

$$R_a^b = \begin{cases} l_a^b - L, & l_a^b > L \\ l_a^b, & l_a^b \le L. \end{cases} \tag{22}$$

Then, after slight manipulations as shown in Appendix A, it is easy to see that $\overline{n}_{k, \xi_d, \xi_c}$ is a Gaussian random variable with mean

$$E\left\{ \overline{n}_{k, \xi_d, \xi_c} \right\} = 0 \tag{23}$$

and variance

$$\mathrm{Var}\left\{ \overline{n}_{k, \xi_d, \xi_c} \right\} = \frac{\sigma_n^2 \beta_{\xi_d, \xi_c}}{2T_s} \tag{24}$$

where

$$\begin{aligned} \beta_{\xi_d, \xi_c} = \sum_{j_1=1}^{\eta_c} \sum_{j_2=1}^{\eta_c} \Big\{ &(-1)^{Q_{j_1}^d + Q_{j_2}^d} \left[T^k(j_2 - j_1) \right]_{R_{j_1}^d, R_{j_2}^d} \\ &+ (-1)^{Q_{j_1}^c + Q_{j_2}^c} \left[T^k(j_2 - j_1) \right]_{R_{j_1}^c, R_{j_2}^c} \\ &- (-1)^{Q_{j_1}^d + Q_{j_2}^c} \left[T^k(j_2 - j_1) \right]_{R_{j_1}^d, R_{j_2}^c} \\ &- (-1)^{Q_{j_1}^c + Q_{j_2}^d} \left[T^k(j_2 - j_1) \right]_{R_{j_1}^c, R_{j_2}^d} \Big\} \end{aligned} \tag{25}$$

$[\cdot]_{i,j}$ denotes the ijth element of a matrix, and $T^k(\cdot)$ is the kth $L \times L$ submatrix of the $KL \times KL$ matrix $T(\cdot)$ such that $[T^k(\cdot)]_{i,j} = [T(\cdot)]_{k\circ i, k\circ j}$.

Then, the instantaneous signal-to-noise ratio (SNR) ν_k of the kth user is

$$\nu_k = \frac{E_k \alpha_k^2 D^2_{\xi_d, \xi_c}}{\sigma_n^2 \beta_{\xi_d, \xi_c}} \tag{26}$$

where $E_k = P_k T_s$ is the transmitted symbol energy and the conditional pairwise error probability is

$$P_{\xi_d \to \xi_c}(\nu_k) = \tfrac{1}{2}\mathrm{erfc}(\nu_k). \tag{27}$$

Therefore, the pairwise error probability can be obtained as

$$P_{\xi_d \to \xi_c} = \frac{1}{2}\left(1 - \sqrt{\frac{\kappa}{\kappa+1}}\right) \tag{28}$$

where $\kappa = ((E_k E\{\alpha_k^2\} D^2_{\xi_d, \xi_c})/(\sigma_n^2 \beta_{\xi_d, \xi_c}))$. We can approximate (28) as

$$P_{\xi_d \to \xi_c} \approx \frac{1}{4\kappa} \tag{29}$$

when the SNR is high. We can see from (28) or (29) that the performance of the kth user is independent of the energy level of the other users. In other words, the proposed system is near–far resistant.

Next, we obtain the union bound of the bit error probability. We will assume that the trellis code is geometrically uniform [18]. Then, we can evaluate the performance of the proposed system by assuming that one particular codeword has been sent: From now on, we thus assume that the correct path ξ_d is the all-zero code word. (Three examples of geometrically uniform trellis codes using biorthogonal signal constellation were shown in [11].)

Let Ξ_e be the set of all indecomposable error events and $g(\xi_i, \xi_j)$ be the number of different information bits between ξ_i and ξ_j per the number of information bits in a symbol. Then, the bit error probability P_b of the proposed system satisfies

$$P_b \le \sum_{\xi_i \in \Xi_e} g(\xi_i, \xi_d) P_{\xi_d \to \xi_i}. \tag{30}$$

At moderate to high values of SNR, the bit error probability is dominated by the free-distance error events [19]. Thus, the bound of the bit error probability can be approximated as

$$P_b \overset{<}{\approx} g_f P_{\xi_d \to \xi_f} \tag{31}$$

where $g_f = \sum_{\xi_i \in \Xi_f} g(\xi_i, \xi_d)$ and Ξ_f is the set of the free-distance error events.

Proposition 1: The ADO of the proposed system is one.

Proof: It is well known that the bit error probability of the uncoded single-user BPSK system is $P_b^s = (1/2)(1 - \sqrt{\kappa_s/(\kappa_s+1)})$ and $\kappa_s = E_k E\{\alpha_k^2\}/\sigma_n^2$, which can be approximated as $P_b^s \approx 1/4\kappa_s$ when the SNR is high. Thus, the ADO can be obtained as

$$\begin{aligned}\mathrm{ADO}_k &= \sup_O \left\{ O \middle| \lim_{\sigma_n^2 \to 0} \frac{\frac{g_f}{4\kappa}}{\left(\frac{1}{4\kappa_s}\right)^O} < \infty \right\} \\ &= \sup_O \left\{ O \middle| \lim_{\kappa \to \infty} \frac{\frac{g_f}{4\kappa}}{\left(\frac{1}{4D^2\kappa/\beta}\right)^O} < \infty \right\} \\ &= 1. \end{aligned} \tag{32}$$

■

Proposition 2: The AG of the proposed system is $D^2/g_f\beta$, where $D^2 = D^2_{\xi_d, \xi_f}$, $\beta = \beta_{\xi_d, \xi_f}$, and ξ_f is a free-distance error event.

Proof: Since the ADO of the proposed system is 1, the AG of the proposed system is

$$\begin{aligned}\mathrm{AG}_k &= \max\left\{ \arg_G \lim_{\sigma_n^2 \to 0} \frac{\frac{g_f}{4\kappa}}{\frac{1}{4G\kappa_s}} = 1, 0 \right\} \\ &= \max\left\{ \arg_G \lim_{\kappa \to 0} \frac{g_f G}{D^2/\beta} = 1, 0 \right\} \\ &= \frac{D^2}{g_f \beta}. \end{aligned} \tag{33}$$

■

To show the outperformance of the proposed system over other systems, we will compare it with the system using the same TCM scheme and Viterbi decoders, but without a decorrelating filter (from now on, such a system will be denoted as the conventional system): for the conventional system, a perfect power control is assumed (i.e., the received powers of users are all equal) in order to get an upper bound on the performance of the conventional system.

Proposition 3: The ADO of the conventional system is one.

Proof: It is easily seen that the bit error probability of the kth user of the conventional system without interuser interference can be approximated to $g_f/4\kappa_c$, where $\kappa_c = (E_k E\{\alpha_k\} D^2)/\sigma_n^2$. Thus, the ADO of the kth user of the conventional system is

$$\begin{aligned}\mathrm{ADO}_k &= \sup_O \left\{ O \middle| \lim_{\sigma_n^2 \to 0} \frac{\frac{g_f}{4\kappa_c}}{\left(\frac{1}{4\kappa_s}\right)^O} < \infty \right\} \\ &= \sup_O \left\{ O \middle| \lim_{\kappa_c \to \infty} \frac{\frac{g_f}{4\kappa_c}}{\left(\frac{1}{4D^2\kappa_c}\right)^O} < \infty \right\} \\ &= 1. \end{aligned} \tag{34}$$

■

Proposition 4: The AG of the conventional system is zero.

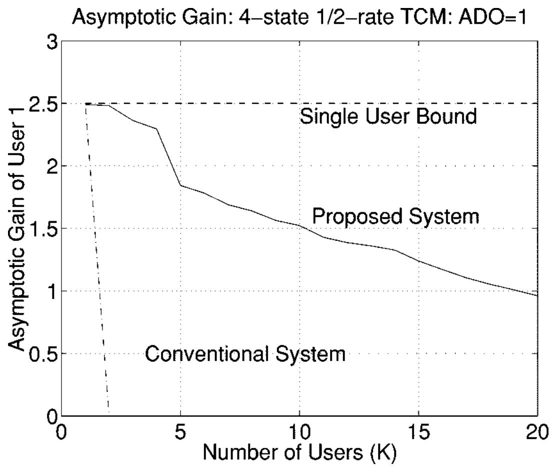

Fig. 3. Asymptotic gain of the proposed and conventional systems with no interleaving.

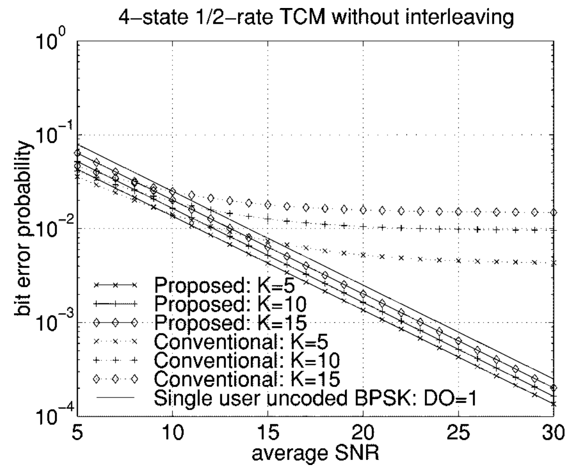

Fig. 4. Bit error probability bounds of the proposed and conventional systems with no interleaving.

Proof: In many previous studies on CDMA, Gaussian approximation to the interuser interference has been widely used. Since the received powers of users are assumed to be equal, the bit error probability of the kth user of the conventional system can be approximated to $g_f/4\overline{\kappa}_c$, where $\overline{\kappa}_c = ((E_k E\{\alpha_k\} D^2)/\overline{\sigma}_n^2)$, $\overline{\sigma}_n^2 = \sigma_n^2 + ((2(K-1)E_k)/3N)$, and N is the length of signature sequences. We can easily see that the bit error probability of the system does not approach zero as the noise power goes to zero. Thus, the AG of the system is zero. ■

In Fig. 3, the AG's of the proposed and conventional systems are plotted as a function of the number of users when we use four-state 1/2-rate TCM without interleaving. We assume that the decorrelator is ideal (i.e., the truncation error is ignored): we evaluate β from (25) and AG from (33) [the finite support of $T(k)$ is sufficient for the evaluation]. Note that the ADO's of the proposed and conventional systems are both one. We use Gold sequences of period 63 as the user signature sequences and the time delays (τ_k) of users are assumed to be uniform over $[0, T_s]$. It is clearly seen that the performance of the proposed system is much better than that of the conventional system.

In Fig. 4, the bound (31) on the bit error probabilities of the proposed and conventional systems is plotted when we use four-state 1/2-rate TCM without interleaving. We again use Gold sequences of period 63 as the user signature sequences and the time delays (τ_k) of users are uniform over $[0, T_s]$. It is clearly seen that the performance of the conventional system suffers from the error floor and degrades severely as the number of users increases, while the performance of the proposed system is affected only a little by the increase of the number of interferers.

C. Interleaving with Channel State Information

Let us assume that the interleaving is perfect so that the fading process affecting one symbol is independent of that affecting other symbols and the CSI is known perfectly at the receiver. Let

$$D^c_{\xi_d,\,\xi_c} = \sum_{j=1}^{\eta_c} \alpha_k^2(j)\left(x'_{k,\,l_j^d}(j) - x'_{k,\,l_j^c}(j)\right).$$

Then, the pairwise error probability is

$$\begin{aligned} P_{\xi_d\to\xi_c} &= \Pr\left\{\Delta^c_{k,\,\xi_d} < \Delta^c_{k,\,\xi_c}\right\} \\ &= \Pr\left\{\sum_{j=1}^{\eta_c}\alpha_k(j)\left(v_{k,\,l_j^d}(j) - v_{k,\,l_j^c}(j)\right) < 0\right\} \\ &= \Pr\left\{\sqrt{P_k}D^c_{\xi_d,\,\xi_c} + \overline{n}^c_{k,\,\xi_d,\,\xi_c} < 0\right\} \end{aligned} \tag{35}$$

where

$$\overline{n}^c_{k,\,\xi_d,\,\xi_c} = \sum_{j=1}^{\eta_c}\alpha_k(j)\mathrm{Re}\left\{e^{-j\phi_k(j)}\left(n'_{k,\,l_j^d}(j) - n'_{k,\,l_j^c}(j)\right)\right\}. \tag{36}$$

Then, after slight manipulations as shown in Appendix B, the instantaneous SNR ν_k of the kth user is

$$\nu_k = \frac{E_k D^c_{\xi_d,\,\xi_c}}{\beta\sigma_n^2}. \tag{37}$$

Let us assume that $E\{\alpha_k^2(j)\} = E\{\alpha_k^2\}$, $j = 1, 2, \cdots, \eta_c$. Since ν_k is the sum of independent chi-square random variables, the moment generating function (mgf) of ν_k is obtained by producting the mgf's of the chi-square random variables. Due to the biorthogonal constellation, $D_{\xi_d,\,\xi_c}(j)$ can take on one of the three values, 0, 1, and 2, when the jth symbols of ξ_c and ξ_d are the same, orthogonal, and antipodal ones, respectively. Then, the mgf of ν_k is

$$\begin{aligned} \Phi_k(s) &= \frac{1}{(1-s\kappa)^{L_1(\xi_c)}(1-2s\kappa)^{L_2(\xi_c)}} \\ &= \sum_{l=1}^{L_1(\xi_c)}\frac{\pi_{1,\,l}}{(1-s\kappa)^l} + \sum_{l=1}^{L_2(\xi_c)}\frac{\pi_{2,\,l}}{(1-2s\kappa)^l} \end{aligned} \tag{38}$$

where

$L_1(\xi_c)$ number of the index j such that $D_{\xi_d,\,\xi_c}(j) = 1$;
$L_2(\xi_c)$ number of the index j such that $D_{\xi_d,\,\xi_c}(j) = 2$;
$\pi_{1,\,l}$ $= (-1)^{L_2(\xi_c)}2^{L_1(\xi_c)-l}\binom{L_1(\xi_c)+L_2(\xi_c)-l-1}{L_2(\xi_c)-1}$;
$\pi_{2,\,l}$ $= (-1)^{L_2(\xi_c)-l}2^{L_1(\xi_c)}\binom{L_1(\xi_c)+L_2(\xi_c)-l-1}{L_1(\xi_c)-1}$.

Thus, the pairwise error probability can be obtained as

$$\begin{aligned} P_{\xi_d \to \xi_c} &= \sum_{l=1}^{L_1(\xi_c)} \frac{\pi_{1,l}}{2^l} \left(1 - \sqrt{\frac{\kappa}{\kappa+1}}\right)^l \sum_{i=0}^{l-1} \binom{l-1+i}{i} \\ &\quad \times \left(\frac{1+\sqrt{\frac{\kappa}{\kappa+1}}}{2}\right)^i + \sum_{l=1}^{L_2(\xi_c)} \frac{\pi_{1,l}}{2^l} \left(1 - \sqrt{\frac{2\kappa}{2\kappa+1}}\right)^l \\ &\quad \times \sum_{i=0}^{l-1} \binom{l-1+i}{i} \left(\frac{1+\sqrt{\frac{2\kappa}{2\kappa+1}}}{2}\right)^i \end{aligned} \tag{39}$$

where $\kappa = ((E_k E\{\alpha_k^2\})/(\sigma_n^2 \beta))$. When the SNR is high, we can approximate (39) as [20]

$$P_{\xi_d \to \xi_c} \approx \frac{\binom{2L_1(\xi_c)+2L_2(\xi_c)-1}{L_1(\xi_c)+L_2(\xi_c)}}{(4\kappa)^{L_1(\xi_c)}(8\kappa)^{L_2(\xi_c)}}. \tag{40}$$

We can again see from (39) or (40) that the performance of the kth user is independent of the energy level of other users: in other words, the proposed system is near–far resistant. Let $L_1 = L_1(\xi_f)$, $L_2 = L_2(\xi_f)$, and $L = L_1 + L_2$. Then, the bound on the bit error probability can be approximately expressed as

$$P_b \lesssim \frac{g_f \binom{2L-1}{L}}{(4\kappa)^{L_1}(8\kappa)^{L_2}}. \tag{41}$$

Proposition 5: The ADO of the kth user of the proposed system is L.

Proof: From (40) and (41), the ADO of the kth user of the proposed system is

$$\begin{aligned} \mathrm{ADO}_k &= \sup_O \left\{ O \,\middle|\, \lim_{\sigma_n^2 \to 0} \frac{\frac{g_f \binom{2L-1}{L}}{(4\kappa)^{L_1}(8\kappa)^{L_2}}}{\left(\frac{1}{4\kappa_s}\right)^O} < \infty \right\} \\ &= \sup_O \left\{ O \,\middle|\, \lim_{\kappa \to \infty} \frac{\frac{g_f \binom{2L-1}{L}}{(4\kappa)^{L_1}(8\kappa)^{L_2}}}{\left(\frac{1}{4D^2\beta\kappa}\right)^O} < \infty \right\} \\ &= L. \end{aligned} \tag{42}$$

■

Proposition 6: The AG of the kth user of the proposed system is $((2^{L_2/L})/(g_f^{1/L}\beta))$.

Proof: The bit error probability of an uncoded single-user BPSK system with diversity order O can be approximated as $P_b^s \approx (\binom{2O-1}{O})/(4\kappa_s)^O)$ when the SNR is high. Since the ADO of the proposed system is L, the AG of the kth user can be obtained as

$$\begin{aligned} \mathrm{AG}_k &= \max \left\{ \arg_G \lim_{\sigma_n^2 \to 0} \frac{\frac{g_f \binom{2L-1}{L}}{(4\kappa)^{L_1}(8\kappa)^{L_2}}}{\frac{\binom{2L-1}{L}}{(4G\kappa_s)^L}} = 1, 0 \right\} \\ &= \max \left\{ \arg_G \lim_{\kappa \to 0} \frac{g_f (G\beta)^L}{2^{L_2}} = 1, 0 \right\} \\ &= \frac{2^{L_2/L}}{g_f^{1/L}\beta}. \end{aligned} \tag{43}$$

■

Proposition 7: The ADO of the conventional system is L.

Proof: It is easily seen that the bit error probability of the kth user of the conventional system without interuser interference can be approximated to $((g_f \binom{2L-1}{L}))/((4\kappa)^{L_1}(8\kappa)^{L_2}))$, where $\kappa_c = ((E_k E\{\alpha_k\})/\sigma_n^2)$. Thus, the ADO of the kth user of the conventional system is

$$\begin{aligned} \mathrm{ADO}_k &= \sup_O \left\{ O \,\middle|\, \lim_{\sigma_n^2 \to 0} \frac{\frac{g_f \binom{2L-1}{L}}{(4\kappa_c)^{L_1}(8\kappa_c)^{L_2}}}{\left(\frac{1}{4\kappa_s}\right)^O} < \infty \right\} \\ &= \sup_O \left\{ O \,\middle|\, \lim_{\kappa_c \to \infty} \frac{0\frac{g_f \binom{2L-1}{L}}{(4\kappa_c)^{L_1}(8\kappa_c)^{L_2}}}{\left(\frac{1}{4D^2\kappa}\right)^O} < \infty \right\} \\ &= L. \end{aligned} \tag{44}$$

■

Proposition 8: The AG of the conventional system is zero.

Proof: Similarly to the proof of Proposition 4, the bit error probability of the kth user of the conventional system can be approximated to $((g_f \binom{2L-1}{L}))/((4\overline{\kappa}_c)^{L_1}(8\overline{\kappa}_c)^{L_2})$, where $\overline{\kappa}_c = ((E_k E\{\alpha_k\})/\overline{\sigma}_n^2)$. It is easily seen again that the bit error probability of the system does not approach zero as the noise power goes to zero. Thus, the AG of the conventional system is zero. ■

In Fig. 5, the AG's of the proposed and conventional systems are plotted as a function of the number of users when we use four-state 1/2-rate TCM ($L = 3$) with interleaving and perfect CSI. We again assume that the decorrelator is ideal. Then, from Propositions 5 and 7, the ADO's of the proposed and conventional systems are both three. We use Gold sequences of period 63 as the user signature sequences and the time delays (τ_k) of the users are uniform over $[0, T_s]$. Again, it is clear that the performance of the proposed system is much better than that of the conventional system.

In Fig. 6, the upper bound (41) on the bit error probabilities of the proposed and conventional systems is plotted when we use four-state 1/2-rate TCM ($L = 3$) with interleaving and perfect CSI. We use Gold sequences of period 63 as the user signature sequences and the time delays (τ_k) of users are uniform over $[0, T_s]$. Clearly, the performance of the proposed system is free

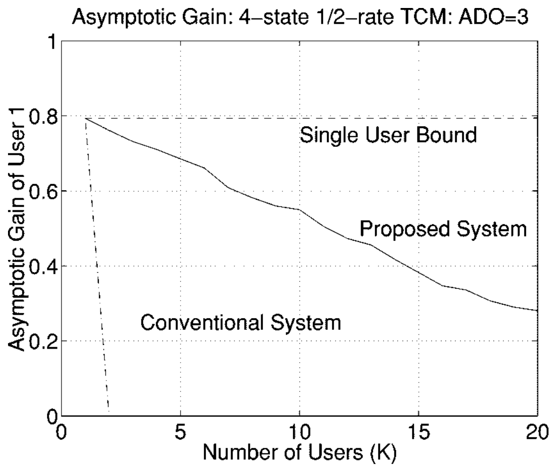

Fig. 5. Asymptotic gain of the proposed and conventional systems with interleaving and perfect CSI.

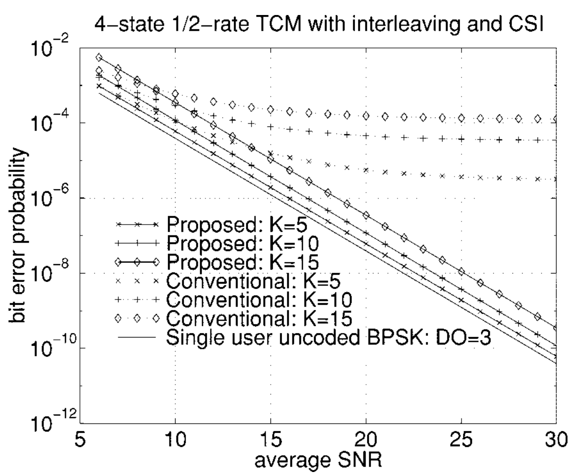

Fig. 6. Bit error probability bounds of the proposed and conventional systems with interleaving and perfect CSI.

from the error floor and does not degrade much as the number of users increases.

V. Performance Enhancement with Base-Station Antenna Array

To combat multipath fading and to enhance the performance of mobile communication systems, diversity reception has been widely used. Among such techniques, multiple reception using antenna arrays has been frequently used. The main idea is that we can reduce the effect of multipath fading by combining received signals at multiple antennas spaced far apart enough so that their fading envelopes are uncorrelated. In mobile communication channels, however, received signals at a base-station antenna array are angularly spread. Thus, the fading envelopes of the received signals may be correlated and the amount of the correlation depends on the channel. It has also been reported that use of base-station antenna arrays can reduce the effect of noise enhancement and correlation caused by decorrelating [12]–[15].

Let a_k be the channel vector, which represents the relative amplitude and phase of the signal received at the antenna array with respect to the signal received at the first antenna element (as defined in [13]–[15]), of the kth user and define $d_{p,k} = ((a_p^H a_k)/(\|a_p\|\,\|a_k\|))$. Then, the transfer function of the decorrelating filter is given as [15]

$$\begin{aligned} G(Z) &= \sum_{k=-\infty}^{\infty} \overline{T}(k) Z^{-k} \\ &= \left(\Lambda^{(-1)} Z^{-1} + \Lambda^{(0)} + \Lambda^{(1)} Z\right)^{-1} \end{aligned} \tag{45}$$

where $[\Lambda^{(m)}]_{p\circ q,\,k\circ l} = \gamma_{p,q,k,l}^{(m)} d_{p,k}$. Thus, it is easily seen that the asymptotic performance of the proposed system gets better as the cross correlation of the signature waveforms gets smaller. From now on, we will assume that we use an M-element uniform linear array and then consider the ADO of the proposed system using antenna array.

Proposition 9: The ADO of the proposed system using an antenna array without interleaving satisfies $1 \le \text{ADO} \le M$.

Proof: The ADO of the proposed system depends on the correlation of the fading envelopes of the received signals at the base-station antenna array. In the worst (coherent) case, the fading processes of the signals at all antennas of a user are the same. Then, the instantaneous SNR ν_k of the kth user is a chi-square random variable with two degrees of freedom and the ADO is one. In the best (uncorrelated) case, the fading processes are independent. Then, the instantaneous SNR ν_k of the kth user is a chi-square random variable with $2M$ degrees of freedom and the ADO is M. Therefore, the ADO of the proposed system satisfies $1 \le \text{ADO} \le M$. ■

Proposition 10: The ADO of the proposed system using an antenna array with interleaving and perfect CSI satisfies $L \le \text{ADO} \le ML$.

Proof: Similarly to the proof of Proposition 9, the instantaneous SNR ν_k of the kth user is a chi-square random variable with $2L$ degrees of freedom and the ADO is L in the worst case. The instantaneous SNR ν_k of the kth user is a chi-square random variable with $2ML$ degrees of freedom and the ADO is ML in the best case. Therefore, the ADO of the proposed system satisfies $L \le \text{ADO} \le ML$. ■

Now, let us consider the AG of the proposed system. As is the ADO, AG is also dependent on such characteristics of the channel as the amount of correlation among the fading envelopes of the received signals at the antenna array and the angle dispersion of the channel. Thus, it is convenient to consider the uncoded single-user BPSK system (in Definition 2) employing the same antenna array. Then, the definition of the AG is now modified as the signal energy required for the uncoded single-user BPSK system using an antenna array to possess the same performance as the trellis-coded DS/CDMA system using multiuser receiver and the same antenna array with unit symbol energy when the noise power approaches zero in the no interleaving case. In the interleaving with CSI case, the definition of the AG is modified as the signal energy required for the uncoded single-user BPSK system using an antenna array and such other form of diversity as Rake diversity of order L to achieve the same performance as the trellis-coded DS/CDMA system using multiuser receiver and the same antenna array with unit symbol energy when the noise power approaches zero.

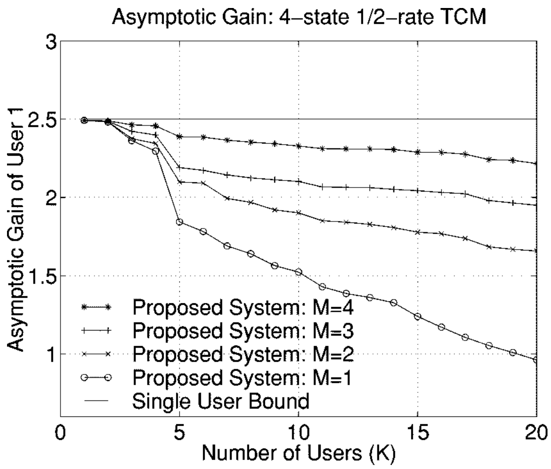

Fig. 7. Asymptotic gain of the proposed system with base-station antenna array with no interleaving.

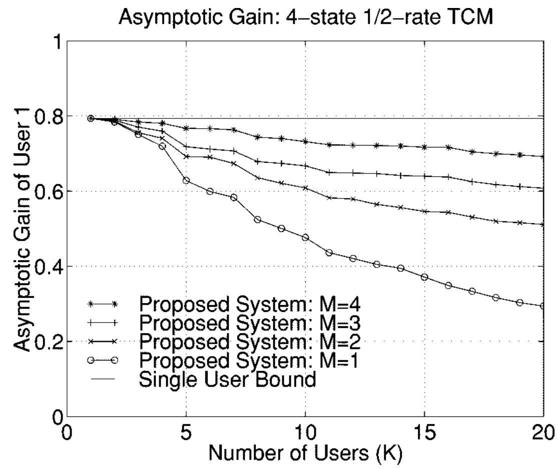

Fig. 8. Asymptotic gain of the proposed system with base-station antenna array with interleaving and perfect CSI.

Proposition 11: The AG of the proposed system using an antenna array without interleaving is $(D^2/(g_f^{1/\overline{M}}\overline{\beta}))$, where $\overline{\beta}$ is obtained from (25) by replacing $T^k(\cdot)$ with the kth $L \times L$ submatrix $\overline{T}^k(\cdot)$ of $\overline{T}(\cdot)$ and $\overline{M}$ is the diversity order obtained from using the antenna array.

Proof: The instantaneous SNR of the proposed system is $\nu_k = ((E_k \sum_{m=1}^{M} \|a_{k,m}\|^2 \alpha_{k,m}^2 D^2)/(\sigma_n^2 \overline{\beta}))$, where $a_{k,m}$ is the mth element of the channel vector a_k and $\alpha_{k,m}$ is the fading envelope of the received signal of the kth user at the mth antenna. In addition, the instantaneous SNR of the conventional system (the single-user BPSK system using the same antenna array) is $\nu_k^c = ((E_k \sum_{m=1}^{M} \|a_{k,m}\|^2 \alpha_{k,m}^2)/(\sigma_n^2))$. Thus, it is easily seen that $\nu_k = ((D^2 \nu_k^c)/\overline{\beta})$. Therefore, the mgf of the instantaneous SNR of the proposed system is the same as that of the conventional system except that the parameter representing the SNR in the mgf is scaled by the constant $(D^2/\overline{\beta})$. Since the diversity order obtained from the use of the antenna array is $\overline{M}$, the AG of the proposed system is $(D^2/(g_f^{1/\overline{M}}\overline{\beta}))$. ■

Proposition 12: The AG of the proposed system using an antenna array with interleaving and CSI is $((2^{L_2/L})/(g_f^{1/\overline{M}L}\overline{\beta}))$, where $(\overline{\beta}/2)$ is the diagonal element of $\overline{T}^k(0)$.

Proof: Similarly to the proof of Proposition 11, the mgf of the proposed system with interleaving and CSI is the same as that of the single-user BPSK system using the same antenna array and another form of diversity of order L except for constant multiples. Thus, it is straightforward to see that the AG of the proposed system is $((2^{L_2/L})/(g_f^{1/\overline{M}L}\overline{\beta}))$. ■

In Figs. 7 and 8, the asymptotic gains of the proposed system with base-station antenna arrays are plotted as functions of the number of users when we use four-state 1/2-rate TCM without and with interleaving and CSI, respectively: uniform linear arrays with $M = 1, 2, 3,$ and 4 are considered. We use Gold sequences of period 63 as the user signature sequences and the time delays (τ_k) of users are uniform over $[0, T_s]$. The channel vectors are generated by the weighted combination of ten array response vectors whose angles are uniformly distributed over $[0, 2\pi]$ and weights are randomly generated with complex Gaussian pdf as in [15]. The decorrelator is assumed to be ideal. It is clearly seen that the performance of the proposed system is considerably enhanced by using base-station antenna arrays and that the performance gain increases as the number of active users increases.

VI. Concluding Remarks

In this paper, we proposed a multiuser receiver using a decorrelating filter and Viterbi decoders for trellis-coded DS/CDMA systems with biorthogonal signal constellation in asynchronous channels. It is shown that the biorthogonality can be implemented by user signature waveforms and a decorrelating filter in asynchronous channels.

The performance of the proposed system is investigated in the measures of the asymptotic diversity order and asymptotic gain. It is shown that the proposed system is near–far resistant and has some coding gain over uncoded systems. The performance of the proposed system degrades as the number of active users increases: the performance is, however, still much better than that of the conventional trellis-coded DS/CDMA systems. It is also observed that we can enhance the performance of the proposed system by using base-station antenna arrays and the performance gain increases as the number of antenna increases and the number of active users increases.

In [21], it has been shown that, with fixed throughput and complexity, the performance of lower rate convolutional codes is better than that of trellis codes in DS/CDMA systems. The reason is as follows. We should use shorter signature sequences to employ a lower rate convolutional code. If we use the $1/m$-rate convolutional code, the symbol energy and the length of signature sequences are reduced by the factor m. In [22], it has been shown that the interuser interference level is proportional to the symbol energy of interferers and inversely proportional to the length of signature sequences. Thus, the interuser interference level of the DS/CDMA system using the $1/m$-rate convolutional code is still almost the same as that of the DS/CDMA system using the trellis code. The performance enhancement comes from the fact that the lower rate convolutional code can provide additional coding gain larger than m so that the reduced symbol energy of the desired user can be overcome. When a decorrelating receiver is employed, the situation is different. In order to produce better performance, the additional coding gain of a lower rate convolutional code should overcome not only the symbol energy loss, but also the increased noise enhancement due to the increased cross correlations between signature sequences, which gets larger as the number of users increases. Thus, the relative performance of the proposed system to that of the DS/CDMA system using lower rate convolutional codes depends on the signature sequence, relative rate (m), and number of users.

For example, assume that the cross correlation between signature sequences is $-(1/N)$, where N is the length of the sequence. We assume that the number of users is 30. Let us compare the four-state 1/2-rate biorthogonal TCM with $N = 256$ with the four-state 1/8-rate convolutional code with $N = 32$. In this case, the Euclidean free distances of the trellis and convolutional codes are $10E_s$ and $84E_s$, respectively, where E_s is the symbol energy. Since the Euclidean distance between antipodal signals (uncoded BPSK) is $4E_s$, the coding gains of the trellis and convolutional codes are $10\log_{10}(10E_s/4E_s) = 3.98$ and $10\log_{10}(84E_s/4E_s) = 13.22$ dB, respectively. The additional coding gain of the convolutional code is thus 9.24 dB. On the other hand, the loss of symbol energy is $10\log_{10}(8) = 9.03$ dB and the loss of noise enhancement due to the increased value of β is 1.11 dB. Thus, the performance of the proposed system is better by 0.9 dB than that of the convolutional code in this case. In addition, using signature sequences of shorter length would keep us from fully exploiting the benefit of the significant capacity improvement of the multiuser receiver. Therefore, when a multiuser receiver is employed, the biorthogonal TCM has some advantage over the lower rate convolutional code.

When interleaving is used, it has been shown that TCM may be worse than convolutional codes since convolutional codes can provide more coding diversity in fading channels. To overcome this, bit-wise interleaved coded modulation was proposed in [23] and [24]. It is possible to employ bit interleaving in the proposed system by performing bit interleaving before signal mapping: the output of the decorrelating detector and the metric should be correspondingly changed into bit-wise ones in such a case. It would be very fruitful to investigate the detailed analysis of the proposed system when the bit interleaving is employed.

APPENDIX A

Since the Z transform $R_n(Z)$ of the covariance matrix of $N(n)$ is $G^{-1}(Z)$ and $G(Z) = G^H(1/Z^*)$, the Z transform $R_{n'}(Z)$ of the covariance matrix of $N'(n)$ is

$$\begin{aligned} R_{n'}(Z) &= \begin{bmatrix} G(Z)R_n(Z)G^H(1/Z^*) & -G(Z)R_n(Z)G^H(1/Z^*) \\ -G(Z)R_n(Z)G^H(1/Z^*) & G(Z)R_n(Z)G^H(1/Z^*) \end{bmatrix} \\ &= \begin{bmatrix} G(Z) & -G(Z) \\ -G(Z) & G(Z) \end{bmatrix}. \end{aligned}$$

Thus, the filtered noise covariance matrix is

$$E\left\{N'(n){N'}^H(n+r)\right\} = \frac{\sigma_n^2}{T_s}T'(r)$$

where

$$T'(r) = \begin{bmatrix} T(r) & -T(r) \\ -T(r) & T(r) \end{bmatrix}.$$

Let $T^k(n)$ be the kth $L \times L$ submatrix of $T(n)$ such that $[T^k(n)]_{i,j} = [T(n)]_{k\circ i,\, k\circ j}$ and

$$\overline{n}_{k,\,\xi_d,\,\xi_c} = \mathrm{Re}\left\{ e^{-j\phi_k} \sum_{j=1}^{\eta_c} \left(n'_{k,\,l_j^d}(j) - n'_{k,\,l_j^c}(j) \right) \right\}.$$

Since $n'_{k,\,L+l} = -n'_{k,\,l}$, it is easily seen that

$$\begin{aligned} &E\left\{ n'_{k,\,l_{a_1}^{b_1}}(n) {n'_{k,\,l_{a_2}^{b_2}}}^*(n+r) \right\} \\ &\quad = (-1)^{Q_{a_1}^{b_1}+Q_{a_2}^{b_2}} \frac{\sigma_n^2}{T_s} \left[T^k(r)\right]_{R_{a_1}^{b_1},\, R_{a_2}^{b_2}}. \end{aligned}$$

Therefore, we get

$$E\left\{\overline{n}_{k,\,\xi_d,\,\xi_c}\right\} = 0$$

and

$$\begin{aligned} &\mathrm{Var}\left\{\overline{n}_{k,\,\xi_d,\,\xi_c}\right\} \\ &= \tfrac{1}{2} E\left\{ \left| \sum_{j=1}^{\eta_c} n'_{k,\,l_j^d}(j) - n'_{k,\,l_j^c}(j) \right|^2 \right\} \\ &= \tfrac{1}{2} \sum_{j_1=1}^{\eta_c} \sum_{j_2=1}^{\eta_c} E\left\{ n'_{k,\,l_{j_1}^d}(j_1) {n'_{k,\,l_{j_2}^d}}^*(j_2) \right. \\ &\qquad\qquad \left. + n'_{k,\,l_{j_1}^c}(j_1) {n'_{k,\,l_{j_2}^c}}^*(j_2) \right\} \\ &\quad - E\left\{ n'_{k,\,l_{j_1}^d}(j_1) {n'_{k,\,l_{j_2}^c}}^*(j_2) + n'_{k,\,l_{j_1}^c}(j_1) {n'_{k,\,l_{j_2}^d}}^*(j_2) \right\} \\ &= \frac{\sigma_n^2}{2T_s} \beta_{\xi_d,\,\xi_c}. \end{aligned}$$

APPENDIX B

Since the interleaving is perfect, the noise processes from adjacent signaling intervals are transmitted sufficiently far apart in time. Thus, we can assume that the noise processes are independent. Then, we get

$$\begin{aligned} &E\left\{ n'_{k,\,l_{a_1}^{b_1}}(n) {n'_{k,\,l_{a_2}^{b_2}}}^*(n+r) \right\} \\ &\quad = (-1)^{Q_{a_1}^{b_1}+Q_{a_2}^{b_2}} \frac{\sigma_n^2}{T_s} \left[T^k(0)\right]_{R_{a_1}^{b_1},\, R_{a_2}^{b_2}} \delta(r) \end{aligned}$$

where $\delta(\cdot)$ is the Kronecker delta. Thus, we have

$$E\left\{\overline{n}^c_{k,\,\xi_d,\,\xi_c}\right\} = 0$$

and

$$\begin{aligned} &\mathrm{Var}\left\{\overline{n}^c_{k,\,\xi_d,\,\xi_c}\right\} \\ &= \tfrac{1}{2} E\left\{ \left| \sum_{j=1}^{\eta_c} \alpha_k(j) e^{-j\phi_k(j)} \left(n'_{k,\,l_j^d}(j) - n'_{k,\,l_j^c}(j) \right) \right|^2 \right\} \\ &= \tfrac{1}{2} \sum_{j_1=1}^{\eta_c} \sum_{j_2=1}^{\eta_c} \alpha_k(j_1)\alpha_k(j_2) \\ &\quad \times \left(E\left\{ n'_{k,\,l_{j_1}^d}(j_1) {n'_{k,\,l_{j_2}^d}}^*(j_2) + n'_{k,\,l_{j_1}^c}(j_1) {n'_{k,\,l_{j_2}^c}}^*(j_2) \right\} \right. \\ &\quad \left. - E\left\{ n'_{k,\,l_{j_1}^d}(j_1) {n'_{k,\,l_{j_2}^c}}^*(j_2) + n'_{k,\,l_{j_1}^c}(j_1) {n'_{k,\,l_{j_2}^d}}^*(j_2) \right\} \right) \\ &= \frac{\sigma_n^2}{2T_s} \beta^c_{\xi_d,\,\xi_c} \end{aligned}$$

where

$$\begin{aligned}\beta^c_{\xi_d,\,\xi_c} &= \gamma\gamma \sum_{j=1}^{\eta_c} \beta^c_{\xi_d,\,\xi_c}(j) \\ &= \sum_{j=1}^{\eta_c} \alpha_k^2(j) \Big(\left[T^k(0)\right]_{R_j^d,\,R_j^d} + \left[T^k(0)\right]_{R_j^c,\,R_j^c} \\ &\quad - (-1)^{Q_j^d+Q_j^c} \left[T^k(0)\right]_{R_j^d,\,R_j^c} \\ &\quad - (-1)^{Q_j^c+Q_j^d} \left[T^k(0)\right]_{R_j^c,R_j^d} \Big). \end{aligned} \tag{46}$$

Here, we assume that $[T^k(0)]_{a,\,a}$ is a constant value $\beta/2$ for $a = 1,\, 2,\, \cdots,\, L$, and the off-diagonal elements of $T^k(0)$ are much smaller than the diagonal elements. These assumptions make sense if the signature waveforms are well designed. Since we use biorthogonal signal constellation, the value of $x'_{k,\,l_j^d}(j) - x'_{k,\,l_j^c}(j)$ is one of three values: it is zero when $l_j^d = l_j^c$, two when $\|l_j^d - l_j^c\| = L$, and one when $\|l_j^d - l_j^c\| \neq 0,\, L$. When $l_j^d = l_j^c$, it is easily seen that $\beta^c_{\xi_d,\,\xi_c}(j) = 0$. When $\|l_j^d - l_j^c\| = L$, $R_j^d = R_j^c$ and $Q_j^c + Q_j^d = 1$. Since we assume that $[T^k(0)]_{a,\,a} = \beta/2$ regardless of a, we get $\beta^c_{\xi_d,\,\xi_c}(j) = 2\alpha_k^2(j)\beta$. Last, when $\|l_j^d - l_j^c\| \neq 0,\, L$, $R_j^d \neq R_j^c$, and we can ignore the last two terms on the right-hand side of (46). Then, we get $\beta^c_{\xi_d,\,\xi_c}(j) = \alpha_k^2(j)\beta$. From these observations, we can see that $\beta^c_{\xi_d,\,\xi_c} = \beta D^c_{\xi_d,\,\xi_c}$. Therefore, the instantaneous SNR ν_k of the kth user is

$$\nu_k = \frac{E_k D^c_{\xi_d,\,\xi_c}}{\beta \sigma_n^2}. \tag{47}$$

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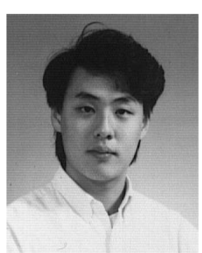

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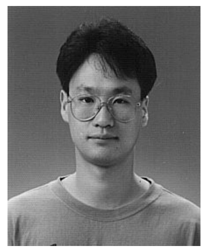

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