Correspondence

Performance Analysis of Forward Link Beamforming Techniques for DS/CDMA Systems Using Base Station Antenna Arrays

Kwang Soon Kim, Iickho Song, Seung Chan Bang, and Tae-Joong Kim

***Abstract*—The performance of DS/CDMA systems using forward link beamforming and rake diversity combining is investigated analytically in frequency selective fading channels, whereas previous studies have resorted to Monte Carlo simulations in flat fading channels. The capacity of DS/CDMA systems is considerably improved by employing forward link beamforming.**

***Index Terms*—Base station antenna array, DS/CDMA systems, forward link beamforming.**

I. INTRODUCTION

To improve the overall capacity of DS/CDMA mobile communication systems, smart antenna array techniques not only for the reverse link [1]–[4] but for the forward link [4]–[6] as well have been investigated. Since employing antenna arrays at the mobile station is not feasible in many practical applications, techniques using base station antenna arrays are desirable. Consequently, some estimation techniques for forward link channel vectors are required due to the different frequency bands used in the two links.

Among the techniques are to use feedback from mobiles to the base station [4], [5] and to retrieve forward link spatial information from the reverse link spatial information [6], [7]. Although some estimation methods for forward link channel vectors have been proposed in some studies, the performance enhancement by employing forward link beamforming techniques has not been fully investigated; in the investigation, frequency selective fading channels should be considered and not only simulation results, but analytical results should be obtained as well. In this correspondence, we investigate the performance enhancement by employing forward link beamforming analytically in frequency fading channels.

II. THE CHANNEL MODEL

The wireless channel is assumed to be frequency selective and time nonselective fading described by a wide sense stationary uncorrelated scattering (WSSUS) model. We also assume that the characteristic of the channel is Rayleigh and that the fading remains constant during a symbol interval. In wireless multipath channels, a lot of replicas of the transmitted signal arrive at the receiver. They are typically divided into several subpath clusters in which each ray is not resolvable.

If we assume an M-element uniform linear antenna array, the channel with resolvable paths can be described as $\mathbf{b} = \Sigma_n \alpha_n e^{j\phi_n}\mathbf{v}(\theta_n)$, where $\alpha_n e^{j\phi_n}$ and θ_n are the complex attenuation and direction of arrival (DOA) of the nth ray, respectively, $\mathbf{v}(\theta) = [1 \; e^{-j\Delta\sin\theta} \; \cdots \; e^{-j(M-1)\Delta\sin\theta}]^T$, $\Delta = (2\pi d/\lambda)$, d is the spacing between antennas, and λ is the wavelength of the arriving signal. Here, we make some physically reasonable assumptions, which have also been made in [1], [6], [8], and [9], that the number of rays and DOA distribution in a subpath cluster is sufficiently large and dense, respectively, so that the spatial distribution of the DOA does not change in a time interval in which we are interested (such as a frame period), and the spatial distribution of $\Delta\sin\theta$ is a normal distribution with mean $\Delta\sin\theta_c$ and variance η^2, where θ_c is called the center DOA, and η is called the dispersion parameter.

III. PERFORMANCE ANALYSIS

From now on, we will assume that the oth user is the desired user and that we can estimate the forward link channel by one of the estimation methods mentioned above. In addition, since the instantaneous channel gain is not available in most practical cases, we assume that the transmit power is divided equally into all available paths as a suboptimum solution.

Let $x_o(t)$, $c_o(t)$, and $\mathbf{w}_o$ be the information signal, signature waveform, and beamforming weight of the oth user in the forward link, respectively. Then, the transmitted signal of the oth user at the base station antenna array is

$$\mathbf{u}_o(t) = \sqrt{P_o}x_o(t)c_o(t)\mathbf{w}_o. \tag{1}$$

Here, we assume that a rake receiver is employed at mobile stations. Then, the equivalent baseband received signal at the oth mobile is

$$r_o(t) = \sum_{k=1}^{K}\sum_{l=1}^{L}\sqrt{P_k}\alpha_{o,l}e^{j\phi_{o,l}}x_k \cdot (t-\tau_{o,l})c_k(t-\tau_{o,l})\zeta_{o,l,k} + n(t). \tag{2}$$

In (2)

P_k	transmitted signal power of the kth user;
$\alpha_{o,l}e^{j\phi_{o,l}}$	complex fading process of the lth path of the oth user;
$\tau_{o,l}$	time delay of the lth path of the oth user;
$\zeta_{o,l,k}$	cross correlation between the lth path's channel of the oth user and the kth user's beamforming vector;
$n(t)$	additive white complex Gaussian noise with variance σ_n^2.

Then, the mth matched filter output is

$$\begin{aligned} y_{o,m}(n) &= \mathrm{Re}\left\{\frac{1}{T_s}\int_{(n-1)T_s+\tau_{o,m}}^{nT_s+\tau_{o,m}} r_o(t)c_o(t-\tau_{o,m})\,dt\right\} \\ &= d_{o,m}(n) + i_{o,m}(n) + n_{o,m}(n) \end{aligned} \tag{3}$$

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where

$$d_{o,m}(n) = \sqrt{P_o}\alpha_{o,m}e^{j\phi_{o,l}}\zeta_{o,l,o}x_o(n)$$

$$i_{o,m}(n) = \text{Re}\left\{\sum_{\substack{k=1\\(k,l)\neq(o,m)}}^{K}\sum_{l=1}^{L} \cdot \sqrt{P_k}\alpha_{o,l}e^{j\phi_{o,l}}\zeta_{o,l,k}x_k(n)\gamma_{o,m,k,l}\right\}$$

and

$$n_{o,m}(n) = \text{Re}\left\{\frac{1}{T_s}\int_{(n-1)T_s+\tau_{o,m}}^{nT_s+\tau_{o,m}} n(t)c_o(t-\tau_{o,m})\,dt\right\}$$

$$\gamma_{o,m,k,l} = \frac{1}{T_s}\int_{(n-1)T_s+\tau_{o,m}}^{nT_s+\tau_{o,m}} c_o(t-\tau_{o,m})c_k(t-\tau_{o,l})\,dt$$

$$\text{Var}\,\{n_{o,m}\} = \frac{\sigma_n^2}{2T_s}$$

and T_s is the symbol duration. Let $\boldsymbol{\alpha}_o = (\alpha_{o,1},\cdots,\alpha_{o,L})$, $\boldsymbol{\theta}_o = (\theta_{o,1},\cdots,\theta_{o,L})$, $\theta_{k,l}$ be the center angle of arrival of the kth user's signal through the lth path. Then, it is straightforward to see that $E\{i_{o,m}(n)\} = 0$ and

$$\text{Var}\,\{i_{o,m}(n)|\boldsymbol{\alpha}_o,\boldsymbol{\theta}_o\} = \frac{1}{2T_s}\sum_{\substack{k=1\\(k,l)\neq(o,m)}}^{K}\sum_{l=1}^{L} E_k\alpha_{o,l}^2|\zeta_{o,l,k}|^2E\{\gamma_{o,m,k,l}^2\} \tag{4}$$

where $E_k = P_kT_s$ is the symbol energy of the kth user. If we assume that the signature waveforms are i.i.d. binary random sequences multiplied by a rectangular pulse train, we have $E\{\gamma_{o,m,k,l}^2\} = (2/3N)$, where N is the processing gain [2]. Then, we can approximate (4), by the weak law of large numbers, as

$$\begin{aligned}\text{Var}\,\{i_{o,m}|\boldsymbol{\alpha}_o,\boldsymbol{\theta}_o\} &= \frac{1}{3NT_s}\sum_{\substack{k=1\\(k,l)\neq(o,m)}}^{K}\sum_{l=1}^{L} E_k\alpha_{o,l}^2|\zeta_{o,l,k}|^2\\ &\approx \frac{E_t}{3NT_s}\sum_{l=1}^{L}\alpha_{o,l}^2\zeta_{o,l}\end{aligned} \tag{5}$$

where

$$\zeta_{o,l} = E\{|\zeta_{o,l,k}|^2|\theta_{o,l}\} = \frac{1}{\Upsilon}\sum_{p=0}^{M-1}\chi(p)e^{-p^2\eta^2}J_0 \cdot (p\Delta)\cos(p\Delta\sin(\theta_{o,l}))$$

$$\chi(0) = M,\ \chi(p) = 2(M-p)\qquad p = 1,\cdots,M-1$$

$$\Upsilon = \sum_{p=0}^{M-1}\chi(p)e^{-p^2\eta^2},\quad E_t = \sum_{k=1}^{K}E_k$$

and $J_0(\cdot)$ is the zeroth-order modified Bessel function. The derivation of $\zeta_{o,l}$ is in Appendix A.

In Fig. 1, the values of $\zeta_{o,m}$ normalized by their maximum values are plotted versus $\theta_{o,m}$ for various values of M and η. Here, we will use the two-value approximation. Let $\sigma_0^2 = \min_{\theta_{o,l}}\ \zeta_{o,l}$, $\sigma_1^2 = \max_{\theta_{o,l}}\ \zeta_{o,l}$, and $\varepsilon = (E\{\zeta_{o,l}\} - \sigma_0^2/\sigma_1^2 - \sigma_0^2)$. Then, $\zeta_{o,l}$ is approximated as a random variable that takes the value σ_0^2 with probability $1-\varepsilon$ and σ_1^2 with probability ε. In addition, we assume that the loss due to the channel estimation is a constant β so that $\zeta_{o,l,o} \approx \sqrt{\beta/L}$. From now on, we drop the subscript o for simple notations whenever it does not cause confusion. Then, after maximal ratio combining, we have

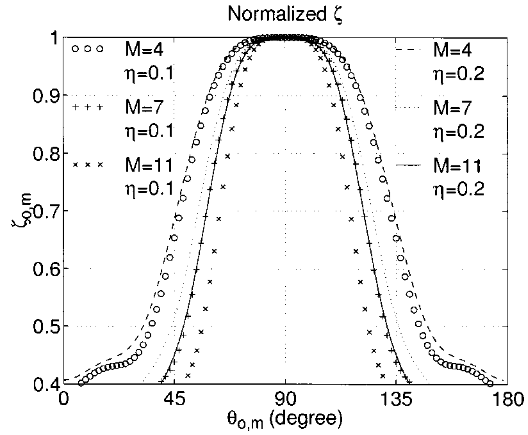

Fig. 1. Normalized $\zeta_{o,m}$ for various values of M and η.

$$\begin{aligned}\rho(n) &= \sum_{m=1}^{L}\alpha_m e^{-j\phi_m}y_m\\ &= d(n) + i(n) + n(n)\end{aligned} \tag{6}$$

where

$$d(n) = \sqrt{\frac{P\beta}{L}}\left(\sum_{l=1}^{L}\alpha_m^2\right)x(n)$$

$$\text{Var}\,\{i(n)|\boldsymbol{\alpha},\boldsymbol{\theta}\} = \sum_{m=1}^{L}\alpha_m^2\ \text{Var}\,\{i_m|\boldsymbol{\alpha},\boldsymbol{\theta}\}$$

and

$$\text{Var}\,\{n(n)|\boldsymbol{\alpha},\boldsymbol{\theta}\} = \frac{\sigma_n^2}{2T_s}\sum_{m=1}^{L}\alpha_m^2.$$

Then, the instantaneous signal to noise and interference ratio (SINR) is

$$\begin{aligned}\nu(\boldsymbol{\alpha},\boldsymbol{\theta}) &= \frac{d^2(n)}{2\ \text{Var}\,\{i(n)|\boldsymbol{\alpha},\boldsymbol{\theta}\} + 2\ \text{Var}\,\{n(n)\}}\\ &\approx \frac{E\beta\sum_{m=1}^{L}\alpha_m^2}{\frac{2LE_t}{3N}\sum_{m=1}^{L}\alpha_m^2\zeta_m + L\sigma_n^2}.\end{aligned} \tag{7}$$

As one of the performance measures, outage probability has been widely used. Let P_b be the required bit error probability and Q be the corresponding SINR. Then, the outage probability is defined as $P^{out} = \text{Pr}\ \{BER > P_b\} = \text{Pr}\ \{\nu(\boldsymbol{\alpha},\boldsymbol{\theta}) < Q\}$. Then, we obtain (see Appendix B)

$$\begin{aligned}P^{out}(\boldsymbol{\theta}) &= \text{Pr}\ \{\nu(\boldsymbol{\alpha},\boldsymbol{\theta}) < Q|\boldsymbol{\theta}\}\\ &\approx \begin{cases}1, & \bar{\iota}_0 = 0\\ 1 - \sum_{l\in\iota_0}\pi_l^{(0)}e^{-(1/\kappa_l)}, & \bar{\iota}_1 = 0\\ 1 - \sum_{l\in\iota_1}\sum_{m\in\iota_0}\pi_l^{(1)}\pi_m^{(0)}\frac{\kappa_m}{\kappa_l+\kappa_m}e^{-(1/\kappa_m)} & \text{otherwise}\end{cases}\end{aligned} \tag{8}$$

where

$$\kappa_l = E\{\alpha_l^2\}|\xi_l|,\quad \xi_l = \frac{E\beta}{LQ\sigma_n^2} - \frac{2E_t\zeta_l}{3N\sigma_n^2}$$

$$\pi_l^{(i)} = \prod_{j\in\iota_i, j\neq l}\frac{\kappa_l}{\kappa_l - \kappa_j}$$

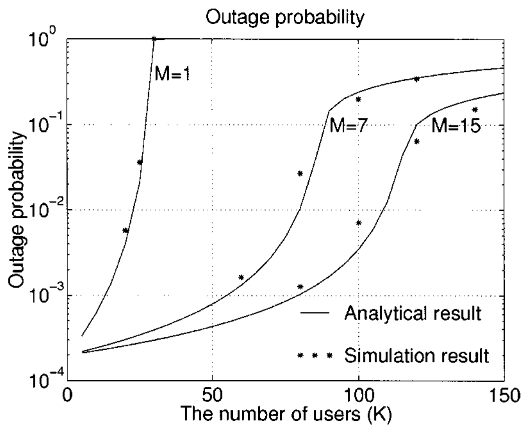

Fig. 2. Outage probability for various values of M.

and ι_i is the set of the index l such that $(-1)^i \xi_l > 0$, and $\bar{\iota}_i$ is the cardinality of ι_i. Let us define

$$\mathbf{q} = (q_1, \cdots, q_L), \quad \kappa_l(\mathbf{q}) = \kappa_l|_{\zeta_l = \sigma_{q_l}^2}$$

$$\pi_l^{(i)}(\mathbf{q}) = \pi_l^{(i)}|_{\zeta_1 = \sigma_{q_1}^2, \cdots, \zeta_L = \sigma_{q_L}^2}, \quad \text{and}$$

$$q_t = \sum_{l=1}^{L} q_l.$$

Then, we can use $P^{out}(\mathbf{q})$ instead of $P^{out}(\boldsymbol{\theta})$ in (8) by the two-value approximation. Then, we finally obtain

$$P^{out} \approx \sum_{\mathbf{q} \in \{0,1\}^L} \varepsilon^{q_t}(1-\varepsilon)^{L-q_t}(\mathbf{q}) P^{out}(\mathbf{q}). \tag{9}$$

In Fig. 2, the outage probability curves obtained from (9) and 10^5-run Monte Carlo simulations are plotted as a function of the number of users for various values of M. We generated $\boldsymbol{\alpha}$, $\boldsymbol{\theta}$, and $|\zeta_{o,l,k}|^2$ in each iteration to evaluate (4). Then, the instantaneous SINR is evaluated from (6) and compared with the threshold Q. We assume that

- $P_k = P$;
- $L = 3$;
- $E\{\alpha_1^2\} = 0$ dB;
- $E\{\alpha_2^2\} = -1$ dB;
- $E\{\alpha_3^2\} = -9$ dB;
- average received SNR is 25 dB;
- $\beta = 0.95$;
- $\eta = 0.1$;
- $d = 0.5\lambda$.

We can see that the analytical results obtained from (9) are quite close to those obtained from simulations. If the outage probability requirement is 0.01, we can see from Fig. 2 that the system capacity increases up to 110 users with 15 antennas, whereas that with a single antenna is 25. Thus, it is again confirmed that we can considerably increase the capacity of the DS/CDMA systems by employing forward link beamforming techniques.

IV. Concluding Remark

In this correspondence, analytical results on the outage probability of the DS/CDMA systems employing forward link beamforming techniques and rake diversity combining were obtained in frequency selective channels. It was shown that the analytical results were quite close to those obtained from Monte Carlo simulations. It was also shown that we could improve the capacity of the mobile communication systems considerably by employing forward link beamforming techniques.

Appendix A

Let $b_{k,l} = \Sigma_n \alpha_{k,l,n} e^{j\phi_{k,l,n}} \mathbf{v}(\theta_{k,l,n})$ and $\theta_{k,l}$ be the center angle. Then, we get

$$\begin{aligned} E\{|b_{o,l}^H b_{k,m}|^2 | \theta_{o,l}, \theta_{k,m}\} &= \sum_n \sum_s E\{\alpha_{o,l,n}^2 \alpha_{k,m,s}^2 |\mathbf{v}(\theta_{o,l,n})^H \mathbf{v}(\theta_{k,m,s}) \\ &\quad \cdot |^2 | \theta_{o,l}, \theta_{k,m}\}. \end{aligned} \tag{10}$$

Since we assumed the normal spatial distribution and sufficiently large number of rays, as in [9], we can rewrite (10) as

$$\begin{aligned} E\{|b_{o,l}^H b_{k,m}|^2 | \theta_{o,l}, \theta_{k,m}\} = \Upsilon E\{\alpha_{o,l}^2\} E\{\alpha_{k,m}^2\} E\{|\mu_{o,l,k,m}|^2 | \theta_{o,l}, \theta_{k,m}\} \end{aligned}$$

where $\mu_{o,l,k,m}$ is the cross correlation between the two channels

$$\begin{aligned} E\{|\mu_{o,l,k,m}|^2 | \theta_{o,l}, \theta_{k,m}\} &= \frac{1}{4\pi^2 \Upsilon} \sum_{p=0}^{M-1} \sum_{q=0}^{M-1} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{j(p-q)(\nu_1 - \nu_2)} \\ &\quad \cdot g(\nu_1; \Delta \sin\theta_{o,l}, \eta^2) \\ &\quad \cdot g(\nu_2; \Delta \sin\theta_{k,m}, \eta^2)\, d\nu_1\, d\nu_2 \\ &= \frac{1}{\Upsilon} \sum_{p=0}^{M-1} \sum_{q=0}^{M-1} e^{-(p-q)^2 \eta^2} e^{j(p-q)(\Delta \sin\theta_{o,l} - \Delta \sin\theta_{k,m})} \\ &= \frac{1}{\Upsilon} \sum_{p=0}^{M-1} \chi(p) e^{-p^2 \eta^2} \cos[p(\Delta \sin\theta_{o,l} - \Delta \sin\theta_{k,m})] \end{aligned} \tag{11}$$

and $g(x; m, \sigma^2)$ denotes the normal probability density function with mean m and variance σ^2. If we assume that the center angles are i.i.d. uniform random variables over $[-\pi, \pi]$, we have

$$\begin{aligned} E\{|\mu_{o,l,k,m}|^2 | \theta_{o,l}\} = \frac{1}{\Upsilon} \sum_{p=0}^{M-1} \chi(p) e^{-p^2 \eta^2} J_0(p\Delta) \cos(p\Delta \sin\theta_{o,l}). \end{aligned}$$

In [4], it was shown that $\mu_{o,l,k,m}$ can be successfully modeled as a Bernoulli random variable with success probability $E\{|\mu_{o,l,k,m}|^2 | \theta_{o,l}, \theta_{k,m}\}$. Since the transmitted signal power is divided equally into L paths, we can approximate that $|\zeta_{o,l,k}|^2 \approx (1/L)\ \Sigma_{m=1}^{L}\ \mu_{o,l,k,m}$. Thus, we get $\zeta_{o,l} \approx (1/L)\ \Sigma_{m=1}^{L}\ E\{\mu_{o,l,k,m} | \theta_{o,l}\} = E\{|\mu_{o,l,k,m}|^2 | \theta_{o,l}\}$.

Appendix B

The conditional outage probability is $P^{out}(\boldsymbol{\alpha}, \boldsymbol{\theta}) = \Pr\{\nu(\boldsymbol{\alpha}, \boldsymbol{\theta}) < Q\} \approx \Pr\ \{X < Y\}$, where $X = \Sigma_{l \in \iota_0}\ \alpha_{o,l}^2 \xi_{o,l}$, and $Y = 1 - \Sigma_{l \in \iota_1}\ \alpha_{o,l}^2 \xi_{o,l}$. Then, the probability density functions of X and Y are

$$f_X(x) = \begin{cases} \displaystyle\sum_{l \in \iota_0} \frac{\pi_l^{(0)}}{\kappa_l} e^{-(x/\kappa_l)} U(x), & \bar{\iota}_0 \geq 1 \\ \delta(x), & \bar{\iota}_0 = 0 \end{cases} \tag{12}$$

and

$$f_Y(y) = \begin{cases} \displaystyle\sum_{l \in \iota_1} \frac{\pi_l^{(1)}}{\kappa_l} e^{-(y-1/\kappa_l)} U(y-1), & \bar{\iota}_1 \geq 1 \\ \delta(y-1), & \bar{\iota}_1 = 0 \end{cases} \tag{13}$$

where $U(\cdot)$ is the unit step function, and $\delta(\cdot)$ is the Dirac delta function. Then, it is easily seen that $\Pr\ \{X < Y | Y = y > 0\} = 1 - \Sigma_{l \in \iota_0}\ \pi_l^{(0)} e^{-(y/\kappa_l)}$. Then, by evaluating $E\{\Pr\ \{X < Y | Y = y > 0\}\}$, we can easily get (8).

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The Estimation of Stable Distribution Parameters from Teletraffic Data

Stephen Bates and Steve McLaughlin

***Abstract*—This correspondence is concerned with the estimation of the parameters that describe a stable distribution from teletraffic data. Stable distributions are characterized by four parameters that can be estimated using a number of methods.**

This correspondence provides a short introduction to the stable distribution and a summary of some of the estimation techniques developed for them. It presents a comprehensive performance comparison between four techniques and a study of how these techniques can be applied to real data. In the case of this correspondence, the data under consideration is that expected on broadband digital networks.

***Index Terms*—Stable distributions, teletraffic modeling.**

I. INTRODUCTION

The Gaussian distribution has always been the preferred choice for statistical signal processing. This is because the use of the distribution can be heuristically justified by the central limit theorem (CLT) and the fact that it is characterized completely by the first two moments of the distribution. In addition, in many cases, the use of the Gaussian distribution greatly reduces the analytical complexity of a system. Recently, interest has grown in the more general stable distribution. This family of distributions includes the Gaussian as one of its limits but, in all other cases, is more impulsive than it. It is this ability to capture impulsive behavior that has driven the interest in stable distributions.

This correspondence begins by introducing stable distributions and some of the estimation techniques. Then, the well-known Norros model for teletraffic modeling is introduced and used to demonstrate why stable rather than Gaussian distributions is more appropriate for teletraffic modeling. The estimation techniques are then applied to the teletraffic datasets studied in this correspondence. Finally, the correspondence draws conclusions and presents a discussion of future work in section.

II. STABLE DISTRIBUTIONS

The characteristic function for a stable distribution is given by

$$\phi(t) = \exp\{jat - |\gamma t|^{\alpha}[1 + j\beta \ \text{sign}(t)\omega(t, \alpha)]\} \qquad (1)$$

where

$$\omega(t, \alpha) = \begin{cases} \tan(\alpha\pi/2), & \text{for } \alpha \neq 1 \\ (2/\pi)\log|t|, & \text{for } \alpha = 1 \end{cases} \qquad (2)$$

and

$$\text{sign}(t) = \begin{cases} 1, & \text{for } t > 0 \\ 0, & \text{for } t = 0 \\ -1, & \text{for } t < 0. \end{cases} \qquad (3)$$

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