$$= [\tilde{\mathbf{U}}''_s : \mathbf{u}''_{r+1} : \tilde{\mathbf{U}}''^{\perp}] \cdot \begin{bmatrix} \tilde{\mathbf{R}}''' & \tilde{\mathbf{w}} & & & \\ & c_2 & & & \\ & & \bar{\sigma}_n & & \\ & & & \ddots & \\ & & & & \bar{\sigma}_n \end{bmatrix} \cdot [\tilde{\mathbf{V}}'_s : \mathbf{v}'_{r+1} : \tilde{\mathbf{V}}'^{\perp}]^H. \tag{4}$$

Comment on Iterative Refinement: The off-diagonal elements of the $r+1$th column of $\mathbf{R_2}$ and $\tilde{\mathbf{R}}_2$ represent blurring of the two subspaces. Steps 3 and 4 work to drive these elements toward zero, thus resolving the subspaces. Together, Steps 3 and 4 constitute a single TQR step (in the case of TQR-SVD) and a single refinement step (in the case of RO-FST).

Now, recall that we started with $\text{span}([\mathbf{V}_s : \mathbf{v}_{r+1}]) = \text{span}([\tilde{\mathbf{V}}_s : \tilde{\mathbf{v}}_{r+1}])$. Each algorithm then rotated these columns with the result that $\mathbf{v}'_{r+1} = \tilde{\mathbf{v}}'_{r+1}$. Thus, $\text{span}(\mathbf{V}'_s) = \text{span}(\tilde{\mathbf{V}}'_s)$ (i.e. the two algorithms compute the same subspace). Finally, note that the structure of (4) is the same as in (2). Thus, we may repeat Steps 3 and 4 to further resolve the subspaces while preserving the structure. In [1], it is shown that Steps 3 and 4 of the TQR algorithm are equivalent to an iteration of the symmetric QR algorithm. Thus, the FST refinement step is also equivalent to the symmetric QR algorithm (in terms of separating the two subspaces).

Step 5: In this final step, the off-diagonal elements of column $r+1$ are set to zero and the noise subspace is made spherical by reaveraging $\bar{\sigma}_n$

$$\bar{\sigma}_n^2 = \frac{(N-r-1)\bar{\sigma}_n^2 + c_2^2}{N-r}.$$

All that remains now is to show that zeroing the off-diagonal elements of column $r+1$ does not alter the relationship between the two factorizations. To do this, carry out the multiplication on each side of (4) and consider the terms corresponding to $\mathbf{v}'_{r+1}$

$$\begin{aligned} &\tilde{\mathbf{U}}''_s \cdot \mathbf{w} \cdot \mathbf{v}'^H_{r+1} + c_2 \cdot \mathbf{u}''_{r+1} \cdot \mathbf{v}'^H_{r+1} \\ &\quad = \tilde{\mathbf{U}}''_s \cdot \tilde{\mathbf{w}} \cdot \mathbf{v}'^H_{r+1} + c_2 \cdot \mathbf{u}''_{r+1} \cdot \mathbf{v}'^H_{r+1} \equiv \tilde{\mathbf{R}}_2. \end{aligned}$$

Now, observe that setting $\mathbf{w} = \tilde{\mathbf{w}} = 0$ does not alter the equality. □

III. Conclusion

We have shown that refinement only FST (RO-FST) produces the same subspaces as the adaptive TQR-SVD algorithm, while reducing the complexity. The tradeoffs for this reduction in complexity are as follows:

1) RO-FST does not track each individual singular vector. Instead, it tracks an orthonormal basis for the subspace spanned by the dominant singular vectors. Fortunately, this is all that is necessary for projection-based methods. This includes MUSIC, projection nulling, and the great majority of high resolution methods appearing in the literature.
2) RO-FST does not explicitly estimate the full set of dominant singular values. Fortunately, this is not a serious limitation because most high resolution methods do not require knowledge of any dominant singular values unless the rank of the dominant subspace must be tracked along with its subspace. Furthermore, even in situations where the rank must be tracked, it is usually sufficient to estimate only the smallest of the dominant singular values. RO-FST can do this in only $O(r^2)$ additional operations by using a condition estimator as described in [5].

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Identification of Nonminimum Phase FIR Systems Using the Third- and Fourth-Order Cumulants

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***Abstract*—An algorithm is proposed for MA system identification using the third- and fourth-order cumulants. Asymmetric non-Gaussian input and Gaussian measurement noise are assumed. Simulation results show that the proposed algorithm is useful when the characteristics of noise are not known or when the SNR is low.**

I. Introduction

Various methods for the identification of MA systems using cumulants have recently been developed by many authors [1]–[8]. For example, the GM-method (Giannakis–Mendel method) was proposed in [3] and modified in [4] to overcome certain deficiencies. In [6], another modified GM-method was proposed to avoid numerical ill-conditioning, and a detailed analysis of the performance of the GM-method has been presented in [7].

The GM-method and its modifications use the correlation and third-order (or fourth-order, depending on the input distribution) cumulants. Because correlation is not blind to Gaussian noise, some equations containing estimates corrupted by the Gaussian measurement noise should be discarded in these methods. As a result, the number of equations available is reduced, and the performance of these algorithms degrades when the data are corrupted by Gaussian noise. In addition, when the noise is colored Gaussian, the number of equations to be discarded increases, resulting in much more degraded performance.

We present an algorithm to identify MA systems driven by independent and identically distributed (i.i.d.) non-Gaussian input. The proposed algorithm has the following properties.

- 1) The algorithm uses the third- and fourth-order cumulants but not the correlation.

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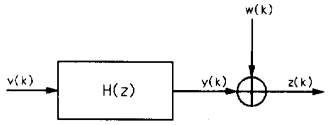

Fig. 1. Single-channel system.

- 2) The algorithm is blind to Gaussian measurement noise even if it is not known whether the noise is white or colored.
- 3) The algorithm is useful when the characteristics of noise are not known or when the signal-to-noise ratio (SNR) is low.

II. Problem Definition and Algorithm Derivation

In the MA channel model of Fig. 1, it is assumed that the channel is stable, linear, and time invariant. The output $z(k)$ is expressed as the following MA(q) model,

$$z(k) = y(k) + w(k) = \sum_{i=0}^{q} h(i)v(k-i) + w(k) \tag{1}$$

where $y(k)$ is the system output, $\{h(k)\}$ is the impulse response of the system with transfer function $H(z)$, and the input signal $\{v(k)\}$ is i.i.d. non-Gaussian and has an asymmetric probability density function (pdf) with $E\{v(k)\} = 0$, $\gamma_{3,v} \triangleq E\{v^3(k)\} \neq 0$, and $\gamma_{4,v} \triangleq E\{v^4(k)\} - 3[E\{v^2(k)\}]^2 \neq 0$. The measurement noise $\{w(k)\}$ is Gaussian and independent of $\{v(k)\}$. We also assume that the system order q is known or has already been estimated by some methods [9]. (The problem of system order overfit [7] is not addressed in this paper.) Our objective is to estimate $h(k)$, $k = 1, 2, \cdots, q$, from the third- and fourth-order cumulants of $\{z(k)\}$.

The third- and fourth-order cumulants of $\{y(k)\}$ are

$$c_{3,y}(m_1, m_2) = \gamma_{3,v} \sum_{i=0}^{q} h(i)h(i+m_1)h(i+m_2) \tag{2}$$

and

$$c_{4,y}(m_1, m_2, m_3) = \gamma_{4,v} \sum_{i=0}^{q} h(i)h(i+m_1)h(i+m_2)h(i+m_3) \tag{3}$$

respectively. As in [7], these can be straightforwardly extended to complex valued systems. The third-order cumulant at lags m and $m+n$ can be expressed as

$$c_{3,y}(m, m+n) = \gamma_{3,v} \sum_{i=0}^{q} h(i)g(i+m; n) \tag{4}$$

where $g(k; n) = h(k)h(k+n)$. Let $G(z; n)$ and $C_{3,y}(z; n)$ denote the z transforms of $\{g(k; n)\}$ and $\{c_{3,y}(m, m+n)\}$, respectively, for $n = 0, 1, 2, \cdots .q$. Then, from (4) we get

$$\begin{aligned} C_{3,y}(z; n) &= \gamma_{3,v} \sum_{m=-q}^{q} \sum_{i=0}^{q} h(i)g(i+m; n)z^{-m} \\ &= \gamma_{3,v} H(z^{-1})G(z; n). \end{aligned} \tag{5}$$

Similarly, let us define the diagonal slice of the fourth-order cumulant

$$d_{4,y}(m) \triangleq c_{4,y}(m, m, m) = \gamma_{4,v} \sum_{i=0}^{q} h(i)h^3(i+m). \tag{6}$$

Then, the z transform of $\{d_{4,y}(m)\}$ is

$$\begin{aligned} C_{4,y}(z) &= \gamma_{4,v} \sum_{m=-q}^{q} \sum_{i=0}^{q} h(i)h^3(i+m)z^{-m} \\ &= \gamma_{4,v} H(z^{-1})H_3(z) \end{aligned} \tag{7}$$

where $H_3(z)$ is the z transform of $\{h^3(k)\}$. By eliminating $H(z^{-1})$ from (5) and (7), we have

$$C_{3,y}(z; n)H_3(z) + \epsilon C_{4,y}(z)G(z; n) = 0 \tag{8}$$

where $\epsilon \triangleq -\gamma_{3,v}/\gamma_{4,v}$. When $n = 0$, (8) reduces to

$$C_{3,y}(z; 0)H_3(z) + \epsilon C_{4,y}(z)H_2(z) = 0 \tag{9}$$

where $H_2(z) = G(z; 0)$ denotes the z transform of $\{h^2(k)\}$. The inverse z transform of $C_{3,y}(z; 0)$, $d_{3,y}(m) \triangleq c_{3,y}(m, m)$, is called the diagonal slice of the third-order cumulant. Using the definition of z transform, we can write (9) as

$$\begin{aligned} &\Big(\sum_{i=0}^{q} h^3(i)z^{-i}\Big) \cdot \Big(\sum_{m=-q}^{q} d_{3,y}(m)z^{-m}\Big) \\ &\quad + \epsilon \cdot \Big(\sum_{i=0}^{q} h^2(i)z^{-i}\Big) \cdot \Big(\sum_{m=-q}^{q} d_{4,y}(m)z^{-m}\Big) = 0. \end{aligned} \tag{10}$$

$$\mathbf{x} = [\epsilon \;\, \epsilon h^2(1) \;\, \epsilon h^2(2) \;\cdots\; \epsilon h^2(q) \;\, h^3(1) \;\, h^3(2) \;\cdots\; h^3(q) \;\, \epsilon h(q)]',$$

$$\mathbf{b} = [d_{3,y}(-q) \;\, d_{3,y}(-q+1) \;\cdots\; d_{3,y}(q) \;\, 0 \;\cdots\; 0 \;\, c_{3,y}(q,q) \;\, c_{3,y}(q-1,q) \;\cdots\; c_{3,y}(0,q) \;\, 0 \;\cdots\; 0],$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_1} \\ \mathbf{A_2} \end{bmatrix},$$

$$\mathbf{A_1} = \begin{bmatrix}
d_{4,y}(-q) & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
d_{4,y}(-q+1) & d_{4,y}(-q) & \cdots & 0 & d_{3,y}(-q) & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
d_{4,y}(0) & d_{4,y}(-1) & \ddots & d_{4,y}(-q) & d_{3,y}(-1) & \ddots & d_{3,y}(-q) & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
d_{4,y}(q) & d_{4,y}(q-1) & \cdots & d_{4,y}(0) & d_{3,y}(q-1) & \cdots & d_{3,y}(0) & 0 \\
0 & d_{4,y}(q) & \cdots & d_{4,y}(1) & d_{3,y}(q) & \cdots & d_{3,y}(1) & 0 \\
0 & 0 & \cdots & d_{4,y}(2) & 0 & \cdots & d_{3,y}(2) & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & d_{4,y}(q) & 0 & \cdots & d_{3,y}(q) & 0
\end{bmatrix}$$

TABLE I
SIMULATION RESULTS FROM 30 MONTE CARLO RUNS (q =2)

Parameter		$C(q, k)$	GMT1		GMT2		Proposed method	SNR
			(NF)	(WG)	(NF)	(WG)		
$h(1)$	mean	−1.424	−1.402	−1.410	−1.376	−1.407	−1.465	∞
−1.40	σ	0.099	0.073	0.079	0.118	0.177	0.163	
$h(2)$	mean	1.018	0.994	0.986	0.991	0.998	0.980	
0.98	σ	0.109	0.050	0.038	0.086	0.110	0.069	
$h(1)$	mean	−1.433	−1.305	−1.407	−1.026	−1.389	−1.430	10dB white
−1.40	σ	0.137	0.069	0.083	0.071	0.218	0.138	
$h(2)$	mean	1.017	1.004	0.985	0.892	0.977	0.956	
0.98	σ	0.150	0.053	0.045	0.097	0.149	0.086	
$h(1)$	mean	−1.429	−1.306	−1.458	−0.941	−1.494	−1.422	10dB colored
−1.40	σ	0.132	0.075	0.094	0.069	0.247	0.109	
$h(2)$	mean	1.014	1.014	0.984	0.835	0.974	0.951	
0.98	σ	0.139	0.053	0.045	0.090	0.149	0.081	
$h(1)$	mean	−1.455	−1.255	−1.406	−0.596	−1.355	−1.331	5dB white
−1.40	σ	0.209	0.190	0.097	0.060	0.307	0.351	
$h(2)$	mean	1.031	1.188	0.987	0.685	0.961	0.926	
0.98	σ	0.221	0.202	0.059	0.097	0.183	0.111	
$h(1)$	mean	−1.450	−1.292	−1.627	−0.456	−1.730	−1.327	5dB colored
−1.40	σ	0.205	1.005	0.154	0.064	0.411	0.176	
$h(2)$	mean	1.024	1.525	0.989	0.536	0.951	0.913	
0.98	σ	0.202	1.130	0.056	0.092	0.208	0.105	
$h(1)$	mean	−1.726	−0.461	−1.354	−0.207	−1.230	−1.063	0dB white
−1.40	σ	1.317	2.692	0.530	0.076	0.569	0.520	
$h(2)$	mean	1.273	−1.119	0.965	0.369	0.915	0.851	
0.98	σ	1.234	16.697	0.238	0.124	0.237	0.126	
$h(1)$	mean	−1.642	−0.021	−3.566	−0.003	−2.608	−1.067	0dB colored
−1.40	σ	0.926	0.700	4.610	0.111	1.959	0.333	
$h(2)$	mean	1.172	−1.010	0.834	0.115	0.856	0.848	
0.98	σ	0.783	2.959	1.286	0.110	0.442	0.101	

Without loss of generality, we take $h(0) = 1$. Then, we obtain from (10)

$$\sum_{i=1}^{q} h^3(i) d_{3,y}(l-i) + \sum_{i=0}^{q} \epsilon h^2(i) d_{4,y}(l-i) = -d_{3,y}(l), \quad \text{for} - q \le l \le 2q. \tag{11}$$

Since this system of $3q+1$ equations alone does not produce a unique least square solution to the $2q+1$ unknown variables $\epsilon, \epsilon h^2(k), h^3(k), k = 1, 2, \cdots, q$ when the nonzero coefficients of the MA system are all unity, let us obtain more equations.

If we let $n = q$ in (8), and then take the same steps as when $n = 0$, we get

$$\sum_{i=1}^{q} h^3(i) c_{3,y}(i-l,q) + \epsilon h(q) d_{4,y}(l) = -c_{3,y}(-l,q), \quad \text{for} -q \le l \le q \tag{12}$$

where we used the fact that $c_{3,y}(l-i, l-i+q) = c_{3,y}(i-l, q)$. Note that to use the third- and fourth-order cumulants simultaneously, the input sequence is assumed to be asymmetrically distributed: otherwise, the proposed algorithm may be used after some transformation [11] of the input sequence. Combining (11) and (12), we get the desired system of $5q+2$ equations for $2q+2$ unknown variables

$$\mathbf{A}\mathbf{x} = -\mathbf{b} \tag{13}$$

where the first group of equalities at the bottom of the previous page are a $(3q+1) \times (2q+2)$ matrix, and $\mathbf{A_2}$, as given at the bottom of the page, is a $(2q+1) \times (2q+2)$ matrix.

Since $\mathbf{A}$ has full rank $2q+2$ as shown in the Appendix, the unique least squares solution of (13) is

$$\hat{\mathbf{x}} = -(A'A)^{-1} A' b. \tag{14}$$

Here, $h^2(k)$ and $h^3(k)$ are treated to be independent as in [3], [4], [6], although there might exist some problems as described in [7]. Since the estimates of $h^2(k)$ do not tell us the sign of $h(k)$ and the division by the estimate of ϵ as a process of obtaining the estimates of $h^2(k)$ may give rise to a very large magnitude of estimates when ϵ is estimated to have small absolute value, we will take

$$\widehat{h}(k) = \sqrt[3]{\widehat{h^3(k)}} \tag{15}$$

as the estimate of $h(k)$.

When Gaussian measurement noise $w(k)$ is added to $y(k)$, we can still use (14) by replacing the third- and fourth-order cumulants of $y(k)$ with those of $z(k)$, because $c_{3,z}(m_1, m_2) = c_{3,y}(m_1, m_2)$ and $c_{4,z}(m_1, m_2, m_3) = c_{4,y}(m_1, m_2, m_3)$. This is the advantage of the proposed method: note that the algorithms using autocorrelation may be affected significantly by Gaussian measurement noise, because $c_{2,z}(m) = c_{2,y}(m) + c_{2,w}(m)$.

$$\mathbf{A_2} = \begin{bmatrix} 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & d_{4,y}(-q) \\ 0 & \cdots & 0 & c_{3,y}(q,q) & 0 & \cdots & 0 & d_{4,y}(-q+1) \\ 0 & \cdots & 0 & c_{3,y}(q-1,q) & c_{3,y}(q,q) & \cdots & 0 & d_{4,y}(-q+2) \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & c_{3,y}(1,q) & c_{3,y}(2,q) & \cdots & c_{3,y}(q,q) & d_{4,y}(0) \\ 0 & \cdots & 0 & c_{3,y}(0,q) & c_{3,y}(1,q) & \cdots & c_{3,y}(q-1,q) & d_{4,y}(1) \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & c_{3,y}(-q+1,q) & c_{3,y}(-q+2,q) & \cdots & c_{3,y}(0,q) & d_{4,y}(q) \end{bmatrix}$$

TABLE II
SIMULATION RESULTS FROM 30 MONTE CARLO RUNS (q =3)

Parameter		$C(q, k)$	GMT1 (NF)	GMT1 (WG)	GMT2 (NF)	GMT2 (WG)	Proposed method	SNR
$h(1)$	mean	0.875	0.870	0.879	0.867	0.883	0.867	∞
0.90	σ	0.090	0.084	0.076	0.103	0.079	0.087	
$h(2)$	mean	0.798	0.766	0.747	0.764	0.743	0.745	
0.79	σ	0.099	0.104	0.139	0.108	0.148	0.053	
$h(3)$	mean	−0.762	−0.719	−0.702	−0.713	−0.724	−0.726	
−0.745	σ	0.135	0.068	0.098	0.091	0.101	0.108	
$h(1)$	mean	0.891	0.862	0.876	0.849	0.876	0.858	10dB
0.90	σ	0.113	0.088	0.081	0.129	0.086	0.102	white
$h(2)$	mean	0.808	0.796	0.745	0.832	0.729	0.736	
0.79	σ	0.114	0.116	0.158	0.137	0.155	0.067	
$h(3)$	mean	−0.778	−0.733	−0.703	−0.751	−0.715	−0.711	
−0.745	σ	0.149	0.073	0.109	0.114	0.111	0.128	
$h(1)$	mean	0.877	0.839	0.871	0.805	0.950	0.852	10dB
0.90	σ	0.109	0.084	0.087	0.132	0.102	0.096	colored
$h(2)$	mean	0.793	0.766	0.634	0.754	0.659	0.744	
0.79	σ	0.114	0.104	0.261	0.141	0.161	0.070	
$h(3)$	mean	−0.778	−0.687	−0.623	−0.650	−0.670	−0.715	
−0.745	σ	0.139	0.067	0.107	0.093	0.120	0.127	
$h(1)$	mean	0.908	0.861	0.870	0.744	0.866	0.853	5dB
0.90	σ	0.151	0.108	0.100	0.140	0.102	0.110	white
$h(2)$	mean	0.818	0.861	0.752	0.869	0.722	0.732	
0.79	σ	0.154	0.151	0.191	0.185	0.188	0.076	
$h(3)$	mean	−0.794	−0.758	−0.699	−0.748	−0.708	−0.702	
−0.745	σ	0.188	0.095	0.136	0.115	0.133	0.145	
$h(1)$	mean	0.880	0.790	0.832	0.617	1.046	0.839	5dB
0.90	σ	0.134	0.093	0.109	0.155	0.176	0.112	colored
$h(2)$	mean	0.793	0.763	−0.207	0.687	0.488	0.737	
0.79	σ	0.151	0.117	0.620	0.153	0.188	0.088	
$h(3)$	mean	−0.791	−0.628	−0.159	−0.497	−0.501	−0.706	
−0.745	σ	0.169	0.076	0.469	0.095	0.151	0.148	
$h(1)$	mean	0.964	0.907	0.825	0.468	0.793	0.783	0dB
0.90	σ	0.309	0.231	0.289	0.206	0.158	0.294	white
$h(2)$	mean	0.866	1.022	0.739	0.903	0.717	0.693	
0.79	σ	0.317	0.369	0.525	0.308	0.340	0.144	
$h(3)$	mean	−0.853	−0.821	−0.706	−0.681	−0.670	−0.646	
−0.745	σ	0.360	0.211	0.255	0.184	0.208	0.258	
$h(1)$	mean	0.908	0.517	0.295	0.195	0.573	0.690	0dB
0.90	σ	0.233	1.041	1.373	0.181	0.544	0.354	colored
$h(2)$	mean	0.830	0.886	−5.794	0.505	0.348	0.664	
0.79	σ	0.296	1.260	20.203	0.159	0.177	0.246	
$h(3)$	mean	−0.854	−0.429	4.713	−0.191	0.091	−0.613	
−0.745	σ	0.343	0.313	14.263	0.084	0.238	0.292	

III. SIMULATION RESULTS

Under various noise environments, the proposed algorithm is compared with the $C(q,k)$, GMT1 [4], and GMT2 [5] methods. The $C(q,k)$ method estimates the parameters of MA systems from cumulants with a simple calculation [1]. The GMT1 and GMT2 methods use the second- and third-order (or fourth-order depending on the input distribution) cumulants.

As the input sequence $\{v(k)\}$, an independent exponentially distributed zero-mean random sequence ($\sigma_v^2 = 1$, $\gamma_{3,v} = 2$, and $\gamma_{4,v} = 6$) was generated, and Gaussian measurement noise was added to the system output. We performed 30 Monte Carlo (MC) runs for each of the examples, where 5120 data were used to estimate the third- and fourth-order cumulants for each run.

Tables I and II show the true and estimated MA parameters and the standard deviation σ of the estimated parameters. The SNR was defined as

$$\mathrm{SNR} = 10\log_{10}\left(\frac{\sum_{i=0}^{q} h^2(i)}{E\{w^2(k)\}}\right). \tag{16}$$

When it is known that the noise is a colored Gaussian MA (p) sequence, where $p \leq \overline{q} = \lfloor (q-1)/2 \rfloor$, the GMT1 and GMT2 methods can produce consistent estimates after discarding some equations, resulting in much degraded performance. If $p > \overline{q}$, however, there is no way to obtain consistent estimates in the GMT1 and GMT2 methods. On the contrary, the proposed method gives consistent estimates in this case also. The colored noise sequences used in the following examples were generated to satisfy $p > \overline{q}$. Therefore, we considered the GMT1 and GMT2 methods operating in noise-free (NF) and white-Gaussian (WG) noise cases.

Example 1: The true MA system is

$$y(k) = v(k) - 1.4v(k-1) + 0.98v(k-2)$$

with zeroes at $0.7 \pm 0.7i$, and the colored measurement noise is generated by

$$w(k) = e(k) + 0.5e(k-1) - 0.25e(k-2),$$

where $e(k)$ is i.i.d. Gaussian. Results from an MC experiment are given in Table I. In Table I, (NF) and (WG) in the columns of GMT1 and GMT2 mean that these methods are used assuming the noise-free and white-Gaussian noise environment, respectively. The GMT1 method sometimes becomes unstable and produces estimates with very large magnitude; the main motivation of the GMT2 method was to overcome such instability [5]. Table I shows that the performance of the GMT1 and GMT2 methods depends significantly on the knowledge of the noise environment. On the contrary, the proposed method shows consistent performance without having to assume *a priori* knowledge of noise environment. In addition, when the noise is colored, the GMT1 and GMT2 methods are quite affected; the proposed algorithm is less affected than the GMT1 or GMT2 methods for colored noise.

Example 2: The true MA system is

$$y(k) = v(k) + 0.9v(k-1) + 0.79v(k-2) - 0.745v(k-3)$$

with zeroes at 0.5 and $-0.7 \pm i$, and the colored measurement noise is generated by

$$w(k) = e(k) + 0.5e(k-1) - 0.25e(k-2) + 0.5e(k-3).$$

Results from an MC experiment are given in Table II. In Example 2, we can make observations similar to those made in Example 1.

Normally, more number of data and MC runs are necessary to analyze the performance of the proposed method than for other methods. The proposed method is more useful for colored Gaussian, low SNR, asymmetric input, large data sample, and no channel diversity. (An application of the second order statistic for channel diversity environment has recently been investigated in [11] and [12]).

IV. Concluding Remarks

A cumulant-based method with which we can identify MA systems was considered. The proposed algorithm uses the third- and fourth-order cumulants simultaneously at the expense of a large number of data and more processing time. The input sequence was assumed to have asymmetric pdf: when the input sequence is symmetrically distributed, the proposed algorithm may be used after some transformation of the input sequence.

Simulation results showed that the proposed algorithm was more useful than the other methods: the proposed algorithm showed satisfactory performance that did not depend on the knowledge of whether the measurement noise was white or colored Gaussian.

Appendix

Let us first rewrite $\mathbf{A}$ as $\mathbf{A} = \begin{bmatrix}\mathbf{A_1}\\ \mathbf{A_2}\end{bmatrix} = \begin{bmatrix}\mathbf{D_4} & \mathbf{D_3}\\ \mathbf{0_L} & \mathbf{C_3}\end{bmatrix}$, where $\mathbf{A_1} = [\mathbf{D_4}\ \mathbf{D_3}]$, $\mathbf{A_2} = [\mathbf{0_L}\ \mathbf{C_3}]$,

$$\mathbf{D_4} = \begin{bmatrix} d_{4,y}(-q) & 0 & \cdots & 0 \\ d_{4,y}(-q+1) & d_{4,y}(-q) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ d_{4,y}(0) & d_{4,y}(-1) & \ddots & d_{4,y}(-q) \\ \vdots & \vdots & \ddots & \vdots \\ d_{4,y}(q) & d_{4,y}(q-1) & \cdots & d_{4,y}(0) \\ 0 & d_{4,y}(q) & \cdots & d_{4,y}(1) \\ 0 & 0 & \cdots & d_{4,y}(2) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_{4,y}(q) \end{bmatrix}$$

is a $(3q+1)\times(q+1)$ matrix, and $\mathbf{0_L}$ is the all-zero $(2q+1)\times(q+1)$ matrix. It is easy to see that the ranks of $\mathbf{D_4}$ and $\mathbf{C_3}$ are both $q+1$. Thus, the rank of $\mathbf{A}$ is $2q+2$.

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Fourth-Order Criteria for Blind Sources Separation

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***Abstract*—Various criteria based on contrast functions as well as higher order statistics are used for solving the problem of blind separation of sources. In this correspondence, for the case of instantaneous mixtures of two sources, it is proved that the minimization or the cancellation of a really simple criterion, a fourth-order cross-cumulant, leads to a set of solutions whose spurious ones can be simply cancelled by using a decorrelation.**

I. Introduction

A. Problem Description

The problem of blind separation of independent sources consists in retrieving the sources from the observation of unknown mixtures of the unknown sources. It is only assumed that the sources are non-Gaussian, and not necessarily i.i.d., otherwise solutions based on second-order statistics are possible [1], [2]. Separation of sources

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