

Use of Antenna Array Diversity in Multiuser Detection of QS-CDMA Systems

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Abstract

Recently, multiuser detection has been an interesting topic because of its capability of eliminating multiuser interference and resistance to the near-far problem. The decorrelator approach has been investigated as a suboptimum multiuser detector for low complexity in receiver at the expense of the enhancement and correlation of noise. In this paper, we consider a quasi maximum likelihood (quasi-ML) detector in the reverse link system which uses antenna arrays in quasi-synchronous channels. It is shown that the performance of the system is better than that of the conventional decorrelator and that the performance gain over the conventional decorrelator system increases as the number of active users and number of antenna arrays increase.

1 Introduction

Although the CDMA technology offers higher spectral efficiency than the others [1], its capacity would not satisfy the explosive demands for mobile communication in near future. Thus, some additional methods to increase the spectral efficiency should be considered. It has been shown that the capacity of the system can be increased [2][3] with antenna arrays.

As a technique to eliminate interuser interference and to resist the near-far problem, multiuser detection has been considered recently [4][5][6]. While the performance of an optimum multiuser detector gives the lower bound of error probability, the complexity of the decision algorithm increases exponentially with the number of users. Because of the complexity of an optimum multiuser detector, a decorrelating approach is considered

as a suboptimum multiuser detector in [6], and a system that uses decorrelator and antenna array is introduced in [7]. The performance of these systems is better than that of conventional systems because the multiuser interference can be eliminated at the expense of noise correlation and enhancement, when the cross correlation matrix of signature waveforms is perfectly known. It is natural that these systems have a resistance to the near-far problem because the multiuser interference is eliminated. These systems, however, have a drawback that decorrelating enhances and correlates the noise, and the enhancement and correlation of noise gets larger as the cross correlation gets larger or the number of user gets larger.

In considering multiuser detection for mobile communication systems, the channel should be considered to be asynchronous because of the difference among the distances from the mobile stations to the base station and because of multipath fading. However, we can treat it as a quasi-synchronous channel if the difference of time delays is small. Currently, DS/CDMA systems in quasi-synchronous channels in which all users try to transmit synchronously and use global positioning system (GPS)-derived clock become an interesting research area because of recent development in GPS receiver. In such a system, we can reduce the delay uncertainty into a few chip durations [8][9].

In this paper, we will propose a quasi maximum likelihood (quasi-ML) detector in the reverse link from the mobile to the base station which uses antenna arrays in quasi-synchronous channels and investigate the performance of the proposed system.

2 System model

We consider the reverse link from a mobile station to a base station with antenna arrays in quasi-synchronous channel. In this channel model, the delay uncertainty is much smaller than the symbol duration. Thus, the interference from previous user symbols in the received signal which is unavoidable in asynchronous systems can be ignored. In the mobile station, the information bits are multiplied by a spreading sequence. After multiplied by carrier, the signal is assumed to be transmitted with binary phase shift keying (BPSK). Then, the transmitter signal modulated by a carrier frequency f_c is

$$s_k(t) = \sqrt{P_k} \text{Re}\{x_k(t)c_k(t) \exp[j(\omega_c t + \psi_k)]\}, \quad (1)$$

where $x_k(t)$ is the k th user's baseband information symbol, $c_k(t)$ is the signature waveform of the k th user, ψ_k is the random phase of the k th carrier, and P_k is the k th user's transmitted power.

Then the equivalent complex baseband received signal vector at the receiver of the base station with antenna arrays can be written as

$$r(t) = \sum_{k=1}^K \sqrt{P_k} \alpha_k x_k(t - \tau_k) c_k(t - \tau_k) e^{j\phi_k} a_k + n(t), \quad (2)$$

where K is the number of users, α_k is the attenuation factor of the k th user, $\phi_k = \psi - \omega_c \tau_k$, in which we can assume that $\psi_k = 0$ without loss of generality, a_k is the $M \times 1$ channel vector of the k th user, and $n(t)$ is the $M \times 1$ additive temporally and spatially white complex Gaussian noise vector with covariance matrix $\sigma_n^2 I$.

The receiver schemes considered in this paper is shown in Fig. 1. We use a uniform linear array which has M element. Let $Y(n)$ be the $M \times K$ output matrix from the code correlator, then we can write $Y(n)$ as

$$Y(n) = [y_1(n) \ y_2(n) \ \cdots \ y_K(n)]^T, \quad (3)$$

$$y_p(n) = \sum_{k=1}^K \sqrt{P_k} \alpha_k x_k(n) e^{j\phi_k} \gamma_{p,k} a_k + n_p(n), \quad (4)$$

where

$$\gamma_{p,k} = \frac{1}{T_s} \int_{\tau_p}^{\tau_p + T_s} c_k(t - \tau_k) c_p(t - \tau_p) dt, \quad (5)$$

and

$$n_p(n) = \frac{1}{T_s} \int_{\tau_p + (n-1)T_s}^{\tau_p + nT_s} n(t) c_p(t - \tau_p) dt. \quad (6)$$

If we assume an ideal low pass filter with bandwidth $B = \frac{1}{T_c}$ and the same amount of energy regardless of the chip waveform, the cross covariance matrix between $n_p(n)$ and $n_k(n)$ is

$$E\{n_p(n)n_k^H(n)\} = \gamma_{p,k} \sigma_n'^2 I, \quad (7)$$

where $\sigma_n'^2 = \frac{\sigma_n^2}{T_c}$.

3 Quasi-ML Detection

In this section, we will investigate quasi-ML detection of user signals. The $M \times 1$ code correlated output vector $y_p(n)$ of the p th user is

$$\begin{aligned} y_p(n) &= \int_{\tau_p + (n-1)T_s}^{\tau_p + nT_s} r(t) c_p(t - \tau_p) dt \\ &= \sum_{k=1}^K \sqrt{P_k} \alpha_k x_k(n) e^{j\phi_k} \gamma_{p,k} a_k + n_p(n), \end{aligned} \quad (8)$$

Then we can set the likelihood function $L_p(n)$ as

$$\begin{aligned} L_p(n) &= \frac{1}{\|\pi R_{nn,p}\|} \exp \left[- (y_p(n) - \sum_{k=1}^K \sqrt{P_k} \alpha_k x_k(n) e^{j\phi_k} \gamma_{p,k} a_k)^H R_{nn,p}^{-1} (y_p(n) - \sum_{k=1}^K \sqrt{P_k} \alpha_k x_k(n) e^{j\phi_k} \gamma_{p,k} a_k) \right], \end{aligned} \quad (9)$$

where $R_{nn,p} = E\{n_p(n)n_p^H(n)\} = \sigma_n'^2 I$, $p = 1, \dots, K$. Taking the partial derivative of $\log L_p(n)$ and setting the result to zero yields

$$\begin{aligned} 0 &= 2 \left(\sqrt{P_p} \alpha_p e^{j\phi_p} \gamma_{p,p} a_p \right)^H R_{nn,p}^{-1} \cdot \\ &\quad \left(y_p(n) - \sum_{k=1}^K \sqrt{P_k} \alpha_k x_k(n) e^{j\phi_k} \gamma_{p,k} a_k \right) \\ &= 2 \sqrt{P_p} \alpha_p e^{-j\phi_p} \gamma_{p,p} \left[a_p^H R_{nn,p}^{-1} y_p(n) - \sum_{k=1}^K \sqrt{P_k} \alpha_k x_k(n) e^{j\phi_k} \gamma_{p,k} a_p^H R_{nn,p}^{-1} a_k \right], \end{aligned} \quad (10)$$

for $p = 1, \dots, K$. Since the noise is spatially and temporally white, i.e. $R_{nn,p} = \sigma_n' I$, the ML estimates can be obtained as

$$\begin{aligned} \hat{x}_p(n) &= \frac{1}{(\sqrt{P_p} \alpha_p e^{j\phi_p} \gamma_{p,p} \|a_p\|)} \left[a_p^H y_p(n) / \|a_p\| - \sum_{k=1, k \neq p}^K \sqrt{P_k} \alpha_k x_k(n) e^{j\phi_k} \gamma_{p,k} a_p^H a_k / \|a_p\| \right]. \end{aligned} \quad (11)$$

In (11), we need to know $x_k(n)$, $k = 1, \dots, p-1, p+1, \dots, K$, in order to estimate $x_p(n)$.

If we replace the estimates $\hat{x}_k(n)$ for $x_k(n)$, $k = 1, \dots, p-1, p+1, \dots, K$, in the righthand side of (11), and consider all the likelihood functions of code correlated outputs of the users, we can obtain

$$\hat{X}(n) = Q^{-1} Y_a(n), \quad (12)$$

where Q is a $K \times K$ matrix whose ij th element is $\sqrt{P_j} \alpha_j e^{j\phi_j} \gamma_{i,j} a_i^H a_j / \|a_i\|$, $\hat{X}(n) =$

$[\hat{x}_1(n) \ \hat{x}_2(n) \ \cdots \ \hat{x}_K(n)]^T$, and $Y_a(n) = [a_1^H y_1(n)/\|a_1\| \ a_2^H y_2(n)/\|a_2\| \ \cdots \ a_K^H y_K(n)/\|a_K\|]^T$. If we know a_1, a_2, \dots, a_K , $\phi_1, \phi_2, \dots, \phi_K$, and $\sqrt{P_1}\alpha_1, \sqrt{P_2}\alpha_2, \dots, \sqrt{P_K}\alpha_K$, we can obtain the quasi-ML estimates of the user signals.

We can rewrite (3) as

$$Y(n) = S(n)\Gamma + N(n), \quad (13)$$

where

$$S(n) = [s_1(n) \ s_2(n) \ \cdots \ s_K(n)], \quad (14)$$

$$s_k(n) = \sqrt{P_k}\alpha_k x_k(n) e^{j\phi_k} a_k \quad (15)$$

$$N = [n_1(n) \ n_2(n) \ \cdots \ n_K(n)], \quad (16)$$

and Γ is a $K \times K$ matrix whose ij th element is $\gamma_{i,j}$. If we post-multiply $Y(n)$ by Γ^{-1} , we obtain

$$\begin{aligned} \hat{S}(n) &= Y(n)\Gamma^{-1} \\ &= S(n) + N(n)\Gamma^{-1}. \end{aligned} \quad (17)$$

In order to estimate a_k , consider the k th column $\hat{s}_k(n)$ of $\hat{S}(n)$, and evaluate the covariance matrix of $\hat{s}_k(n)$ as

$$\begin{aligned} R_{\hat{s}_k, k} &= E\{\hat{s}_k(n)\hat{s}_k^H(n)\} \\ &= P_k E\{\alpha_k^2\} \cdot a_k a_k^H + \sigma_n'^2 [\Gamma^{-1}]_{k,k} I, \end{aligned} \quad (18)$$

where $[\cdot]_{i,j}$ denotes the ij th element of a matrix. Thus, we can estimate a_k as the eigenvector corresponding to the largest eigenvalue. If we consider the covariance matrix $R_{\hat{s}_k, k}$ as a function of τ_k , we can estimate τ_k and P_k by computing the maximum largest eigenvalue of $R_{\hat{s}_k, k}(\tau)$ as τ changes over the range of possible time delays. The value of τ that corresponds to the maximum of the largest eigenvalue is a good estimate of τ_k , and the maximum largest eigenvalue is a good estimate of $P_k \alpha_k^2$. Since $\phi_k = -\omega_c \tau_k$, the estimate of ϕ_k can be obtained, too.

Now, let W be the $K \times K$ diagonal matrix whose k th diagonal is $\sqrt{P_k}\alpha_k e^{j\phi_k} \|a_k\|$ and Λ be the $K \times K$ matrix whose ij th element is $\gamma_{i,j} a_i^H a_j / (\|a_i\| \|a_j\|)$. Then we can write the $K \times 1$ beamformed signal vector $Y_a(n)$ as

$$Y_a(n) = \Lambda W X(n) + N_a(n), \quad (19)$$

where $N_a(n) = [a_1^H n_1(n)/\|a_1\| \ a_2^H n_2(n)/\|a_2\| \ \cdots \ a_K^H n_K(n)/\|a_K\|]^T$ and $R_{n_a n_a} = E\{N_a(n)N_a^H(n)\} = \sigma_n'^2 \Lambda$. Then we can see that the proposed quasi-ML detector is a decorrelator with decorrelating matrix Λ^{-1} .

4 Comparison to the conventional decorrelating approach

In conventional decorrelator system using antenna arrays, decorrelating precedes beamforming. The system proposed in this paper, on the other hand, can be considered as a system which performs beamforming first

and decorrelating later. Specifically, let $V(n)$ be the $K \times 1$ output vector of the decorrelator shown in the previous section, then

$$\begin{aligned} V(n) &= \Lambda^{-1} Y_a(n) \\ &= W X(n) + N'(n), \end{aligned} \quad (20)$$

where $N'(n) = \Lambda^{-1} N_a(n)$ is a $K \times 1$ column vector. Thus, we can get $Y_a(n)$ from $Y(n)$ by beamforming, and $V(n)$ from $Y_a(n)$ by decorrelating. Note that the off-diagonal terms of the decorrelating matrix Λ are smaller than those of the conventional decorrelating matrix because $\|a_p^H a_k\|/\|a_p\| \|a_k\|$, $k \neq p$, are less than 1 provided that a_p is not a constant multiple of a_k . This implies enhancement and correlation of noise can be reduced.

In DS/CDMA systems with antenna arrays, there are two factors that can be used to discriminate user signals. One is the user pseudonoise code, and the other is the channel vector. The conventional decorrelating approach uses the user pseudonoise code only. The system proposed in this paper, on the other hand, uses both of them to discriminate user signals: it is therefore expected that the performance of the proposed system is better than that of the conventional decorrelating approach.

As a simple example, consider a system with two users where the channel is a single path channel. Then the asymptotic efficiency of the conventional decorrelating detector is $1 - \gamma_{1,2}\gamma_{2,1}$ [6], while the asymptotic efficiency of the proposed system is $1 - \gamma_{1,2}\gamma_{2,1} \|a_1^H a_2\|^2 / (\|a_1\|^2 \|a_2\|^2)$. Since $\|a_1^H a_2\| \leq \|a_1\| \|a_2\|$, the asymptotic efficiency of the proposed system is clearly higher than the conventional one.

5 Performance Analysis

In this section, we analyze the performance of the proposed system in the sense of bit error probability. Without loss of generality, we assume that the first user signal is the desired signal. The decorrelator output of the first user is

$$v_1(n) = \sqrt{P_1 a_1^H a_1} \alpha_1 e^{j\phi_1} x_1(n) + n'_1(n), \quad (21)$$

where $E\{n'_1(n)n_1'^*(n)\} = [\Lambda^{-1}]_{1,1} \sigma_n'^2$. If we assume coherent reception, we can get the decision variable $\rho_1(n)$ as

$$\begin{aligned} \rho_1(n) &= \sqrt{P_1 a_1^H a_1} \alpha_1 e^{-j\phi_1} v_1(n) \\ &= P_1 a_1^H a_1 \alpha_1^2 x_1(n) + \sqrt{P_1 a_1^H a_1} \alpha_1 e^{-j\phi_1} n'_1(n). \end{aligned} \quad (22)$$

The instantaneous SNR ν_1 is

$$\begin{aligned} \nu_1 &= \frac{P_1^2 (a_1^H a_1)^2 \alpha_1^4}{P_1 a_1^H a_1 \alpha_1^2 [(\Lambda^{-1})^H]_{1,1} \sigma_n'^2} \\ &= \frac{E_1 a_1^H a_1 \alpha_1^2}{[\Lambda^{-1}]_{1,1} \sigma_n'^2}, \end{aligned} \quad (23)$$

where $E_1 = P_1 T_s$ is the transmitted symbol energy of the first user. Then we can get the conditional bit error probability as

$$P_{b,1}(\nu_1) = \frac{1}{2} \operatorname{erfc}(\sqrt{\nu_1}). \quad (24)$$

Since α_1 is a Rayleigh distributed random variable, ν_1 is a chi-square random variable with 2 degrees of freedom, and the bit error probability is

$$\begin{aligned} P_b &= \int_0^\infty P_{b,1}(x) f_{\nu_1}(x) dx \\ &= \frac{1}{2} \left[1 - \sqrt{\frac{\kappa_1}{1 + \kappa_1}} \right], \end{aligned} \quad (25)$$

where $\kappa_1 = \frac{E_1 \mathbf{a}_1^H \mathbf{a}_1 E\{\alpha_1^2\}}{\sigma_n^2 [\mathbf{A}^{-1}]_{1,1}}$.

6 Numerical Examples and Simulation Results

In this section, we consider some examples using randomly generated channel vectors. For each user, we set the channel response vector as the linear combination of 10 array response vectors whose angles are uniformly distributed over $[0, 2\pi]$, and whose coefficients are randomly generated with a complex Gaussian pdf and then normalized, i.e. the sum of the squares of the absolute values of the coefficients is equal to 1. The time delays are uniformly distributed over $[-3T_c, 3T_c]$.

In Figs. 2-4, the bit error probabilities of the two systems are plotted when the number of active users is 20, 40, and 60, respectively. In each figure, we have three solid lines and three dotted lines. The solid lines represent the bit error probability curves of System B (the proposed system) and the dotted lines represent the bit error probability curves of System A (the conventional decorrelator system that uses antenna arrays), when the number M of antenna arrays is 2, 3, and 4. We can see clearly that we can get more gain over the conventional system as the number of antenna arrays and active users increase.

7 Concluding Remarks

In this paper, we proposed a quasi-ML detector employing antenna arrays in quasi-synchronous channels. We analyzed the performance of the proposed systems and showed that the enhancement and correlation of noise, which was the major drawback of the conventional decorrelator system, could be reduced. The proposed quasi-ML detector can be considered as a system which performs beamforming first and decorrelating later, while the decorrelator precedes the beamformer in the conventional system.

It was shown that we could get some gain over the conventional decorrelator system and the gain got larger as the number of active users increased and the number of antenna arrays increased: with the proposed system, we can get a capacity considerably higher than that with the conventional decorrelator system.

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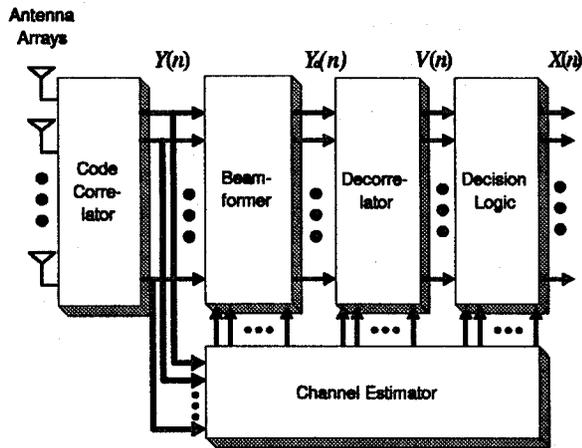


Figure 1: The receiver system architecture.

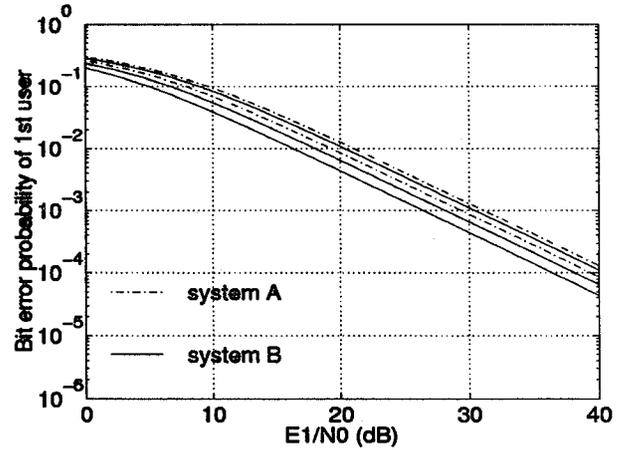


Figure 3: The bit error probabilities of the two systems when the number of active users is 40 ($K = 40$), the number (M) of antenna arrays is 2, 3, and 4, the period of Gold sequence is 63, and $E\{\alpha_1^2\} = 0.2$.

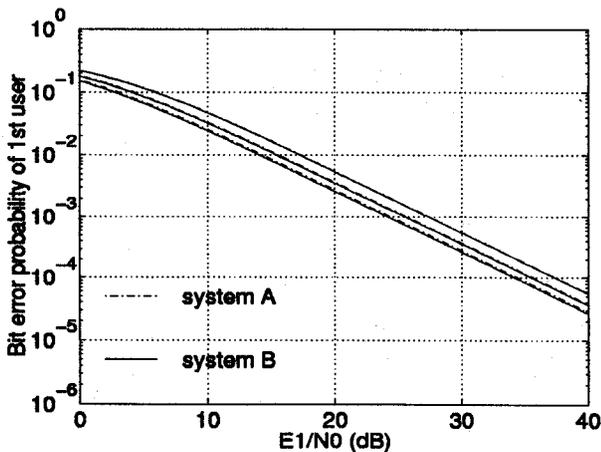


Figure 2: The bit error probabilities of the two systems when the number of active users is 20 ($K = 20$), the number (M) of antenna arrays is 2, 3, and 4, the period of Gold sequence is 63, and $E\{\alpha_1^2\} = 0.2$.

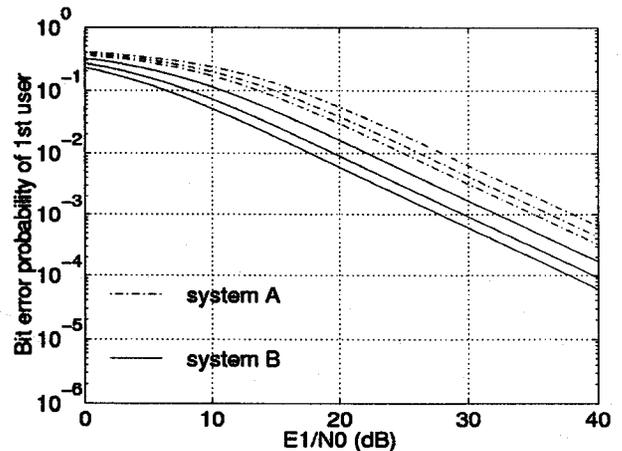


Figure 4: The bit error probabilities of the two systems when the number of active users is 60 ($K = 60$), the number (M) of antenna arrays is 2, 3, and 4, the period of Gold sequence is 63, and $E\{\alpha_1^2\} = 0.2$.