Joint Feedback Design for Multiple Access Channel

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***Abstract*—We study joint feedback design for multiple access channel (MAC) over fading Gaussian Channel. The transmitters use partial channel-state information (CSI), which is obtained via an optimized feedback link. We develop a simple iterative algorithm for the design of CSI quantization to maximize the expected rate. Also, we show the performance by simulation result.**

I. Introduction

There has recently been significant interest in feedback design for wireless communication. When available, channel-state information at the transmitter (CSIT) can be used to adapt resources and transmission strategy, and can greatly improve the performance over a fading channel. Since it is necessary to use the limited feedback due to theresource limitation, there are many studies related to the feedback design in point-to-point (P2P) communication [1]-[4]. [1] considers a fixed-rate system and deals with feedback design and power control to minimize the outage probability based on the partial CSIT. Also, feedback design for an ergodic channel is in [2] and [3] where there is only a single centroid, namely the power, associated with each quantization region. Furthermore, [4] studied the feedback design method and transmission strategy in the multiple-layer variable-rate system. But, in case of multiple access channel (MAC), these conventional feedback designs are not optimal since all transmitter average powers are not considered. If we perform feedback design jointly considering each transmitter average powers, the performance gain is obtained. In this paper, we consider simple MAC which consists of two transmitters and one receiver with perfect CSI at the receiver (CSIR). In such scenario, our aim is to construct joint feedback design to maximize the expected rate [5] for MAC with due regard to 2 transmitter's average powers.

II. System model

Consider the discrete-time complex baseband model of MAC described in Fig. 1, where the complex-valued channel gain is assume to be random but constant during one fading block consisting of N channel uses (i.e., block-fading channel model [6]). The received signal at time instant t within fading block m, $m = 1, 2, ...,$ can be

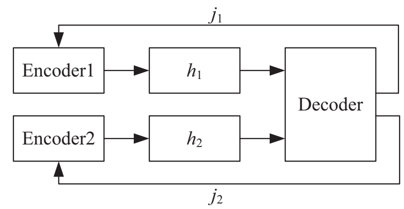

Fig. 1. System model

written as

$$y_m(t) = \sum_{i=1}^{2} h_i^m(t) s_i^m(t) + w^m(t), t = 1, ..., N \tag{1}$$

where h_i^m, i denote the channel gain and the ith transmitter, respectively. Also, the $s_i^m(t)$'s are the transmitted symbols. The noise samples $w^m(t)$ are independent and identically distributed (i.i.d) complex Gaussian with zero mean and unit variance. We assume that the h^m's are i.i.d according to some distribution. Let $\gamma_i^m \triangleq |h_i^m|^2$, that is the resulting i.i.d. channel power. It is reasonable to study the case $N \to \infty$, modeling a scenario with very slow fading and a delay constraint on the transmitted codeword. For notation simplicity, we omit the fading index m. Denote the cumulative distribution function (cdf) and the probability density function (pdf) of γ_i as $F(\gamma)$ and $f(\gamma)$, respectively. The receiver employs a deterministic index mapping $\mathcal{I}(\gamma_i)$ that partitions the nonnegative real line into K quantization regions

$$\mathcal{I}(\gamma_i) = j_i \text{ if } \gamma_i \in [\gamma_{j_i}^{\mathrm{b}}, \gamma_{j_i+1}^{\mathrm{b}}),\ \ j_i = 0, ..., K-1 \tag{2}$$

where the $\gamma^{\mathrm{b}}(j_i)$'s denote the boundary points of the quantization regions. For convenience, we use the convention $\gamma^{\mathrm{b}}(K) = \infty$ and $\gamma^{\mathrm{b}}(-1) = 0$. Herein K is a given quatization level. The index j_i is sent to the ith transmitter via a noiseless, zero-delay feedback channel. Let the ith transmitter's power be P_i ($P_1 \geqslant P_2$), independently of the feedback index.

III. Feedback Design

With the single-layer coding approach, given indexes j_1 and j_2, each transmitter selects an operating rate

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Algorithm 1 Joint feedback design for MAC

Initialize $k = 0$, arbitrary $\{\bar{\gamma}_{j_2}(0)\}, \gamma_0(0) = \bar{\gamma}_0(0)$
repeat
Fix $\{\bar{\gamma}_{j_2}(k)\}$, solve for $\{\gamma_{j_1}(k)\}$ using (5);
$k \leftarrow k + 1$
$\{\bar{\gamma}_{j_2}(k)\} = \gamma_{j_1}(k-1)$;
until *Convergence*

R_{j_i}, which is associated with a reconstruction point γ_{j_i}. Assume that the receiver can perform powerful joint decoder. Finally the operating rate at the receiver, $R^{sum}_{j_1,j_2}$, is $\log(1 + \gamma_{j_1}P_1 + \gamma_{j_2}P_2)$, since $\gamma_{j_i} \in [\gamma^{\mathrm{b}}_{j_i}, \gamma^{\mathrm{b}}_{j_i+1})$. (All logarithms in this paper are natural.) If the actual power $\gamma_i > \gamma_{j_i}$, the codeword will be successfully decoded. But in the other case, the receiver is in outage. Designing a feedback scheme optimal in the sense of expected rate is equivalent to solve the following optimization problem:

$$\max_{\{\gamma^{\mathrm{b}}_{j_1},\gamma_{j_1}\}} \sum_{j_1=0}^{K}\sum_{j_2=0}^{K}[F(\gamma^{\mathrm{b}}_{j_1}) - F(\gamma_{j_1-1})][F(\gamma^{\mathrm{b}}_{j_2}) - F(\gamma_{j_2-1})] \cdot \log(1 + \gamma_{j_1-1}P_1 + \gamma_{j_2-1}P_2) \tag{3}$$

Given arbitrary sets $\{\gamma_{j_2}, \gamma^{\mathrm{b}}_{j_2}\}$ as rewritten $\{\bar{\gamma}_{j_2}\}$, we can simplify the Karush-Kuhn-Tuncker (KKT) conditions for a scheme $\{\gamma_{j_1}, \gamma^{\mathrm{b}}_{j_1}\}$ to be optimal to

$$\gamma^{\mathrm{b}}_{j_1} = \gamma_{j_1},\ j_1 = 0, ..., K-1. \tag{4}$$

$$F(\gamma_{j_1+1}) = F(\gamma_{j_1}) + \frac{f(\gamma_{j_1})\sum_{j_2=0}^{K}[F(\bar{\gamma}_{j_2}) - F(\bar{\gamma}_{j_2-1})]\log\left(\frac{1+\gamma_{j_1}P_1+\bar{\gamma}_{j_2-1}P_2}{1+\gamma_{j_1-1}P_1+\bar{\gamma}_{j_2-1}P_2}\right)}{\sum_{j_2=0}^{K}[F(\bar{\gamma}_{j_2}) - F(\bar{\gamma}_{j_2-1})]\left(\frac{P_1}{1+\gamma_{j_1}P_1+\bar{\gamma}_{j_2-1}P_2}\right)}. \tag{5}$$

The intuition behind (4) is in [4]. The outage event can only occur if the zero indexes ($j_1 = 0, j_2 = 0$) are received at each transmitter. Our proposed simple iterative algorithm is developed from Lloyd-like algorithm [7]. As the first step, it is relatively simple to solve for $\{\gamma_{j_1}\}$ from (5), since one can express $\gamma_1, ..., \gamma_{K-1}$ as a function of γ_0. Given γ_0 (can be solved numerically), one can successively compute $\gamma_1, ..., \gamma_{K-1}$ using (5). In the next step, we substitute the initial values of $\{\bar{\gamma}_{j_2}\}$ for the obtained values of $\{\gamma_{j_1}\}$ corresponding to iteration index k. This procedure is summarized in Algorithm 1. A natural termination condition is to check whether $\sum_{j=0}^{K-1}(\gamma_j - \bar{\gamma}_j)/\gamma_j \leqslant \varepsilon$ (ε is a small positive number).

IV. Simulation Results

In Fig. 2, we plot the expected rate achieved by feedback scheme with different numbers of quantization regions and the perfect CSIT over a Rayleigh channel with unit mean power (i.e., $F(\gamma) = 1 - \exp(-\gamma)$). The average SNR is defined as $\mathrm{SNR}_i = P_i/\sigma_n^2 = P_i$, since the noise variance σ_n^2 is assumed to be unit. The performance gap between the feedback scheme and the perfect CSIT is more smaller when the quantization level is increased. Moreover, compared to the conventional feedback design (P2P), the proposed feedback design scheme has the better performance since we jointly consider all of transmitter average powers.

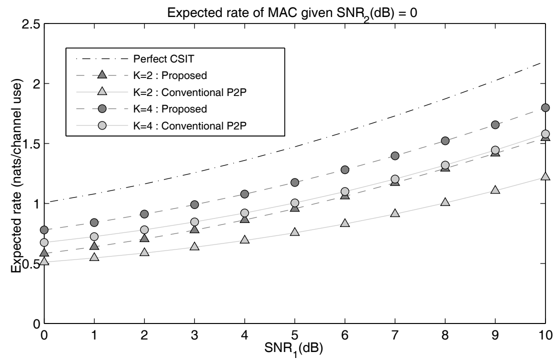

Fig. 2. Expected rate achieved with different feedback schemes over a Rayleigh channel.

V. Conclusion

We have studied joint feedback design for maximizing the expected rate over MAC. Our proposed iterative algorithm is very simple and it achieves the better performance than the conventional feedback design scheme since we jointly consider both transmitter average powers. For the future work, we extend this result for more complex MAC which consists of many transmitters.

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