Throughput analysis of MIMO cooperative decode-and-forward HARQ protocols

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***Abstract*—We consider MIMO-HARQ protocols with/without relay under block fading channels with the assumption that the source, destination, and relay are half-duplex nodes with two antennas. The achievable rate of 2×2 MIMO-HARQ protocols are obtained when the number of retransmissions is unlimited and the optimum ratio between the two streams of two antennas is obtained. The MIMO-HARQ protocol without relay can achieve the ergodic capacity and the proposed MIMO-HARQ protocol with relay is a near capacity approaching scheme.**

I. INTRODUCTION

In wireless systems, spatial diversity techniques such as the multiple input multiple output (MIMO) techniques and cooperation techniques have been investigated to achieve spectral efficient and reliable communications over fading channels. In addition to those techniques, hybrid automatic repeat request (HARQ) protocols also have been studied to improve the reliability of communications. Thus, there have been researches about the MIMO-HARQ protocols with/wihout relay [1]-[4]. In [1], an MIMO-HARQ protocol with basis hopping was proposed to obtain further diversity gain over slow fading channels. In [2], an Alamouti based MIMO-HARQ protocol was proposed for 2×2 MIMO channels. In [3], the incremental redundancy lattice space time (IR-LAST) coding was proposed and it was shown that the protocol can achieve the optimal diversity-multiplexing-delay (DMD) tradeoff. In [4], an MIMO-HARQ protocol with relay, which can obtain the optimal DMD tradeoff, was proposed for two-hop communications. However, the DMD tradeoff only gives the fundamental and asymptotic performance and a scheme having the optimal DMD tradeoff could not be a capacity approaching scheme.

In this paper, we consider MIMO-HARQ protocols with/without relay which is comprised of the IR-HARQ and decode-and-forward relaying. The achievable rates of 2×2 MIMO-HARQ protocols without/with relay are obtained when the number of retransmissions is unlimited. Over block fading channels, we show that the 2×2 MIMO-HARQ protocol without relay can achieve the ergodic capacity. In the single relay channels, the upper and lower bounds of the capacity was investigated in [5] but the capacity is still an open problem. Thus, we compare the achievable rate of the 2×2 MIMO-HARQ protocol with the cut-set bound and the protocol shown in [4]. The proposed protocol outperforms the protocol in [4] but smaller than the cut-set bound.

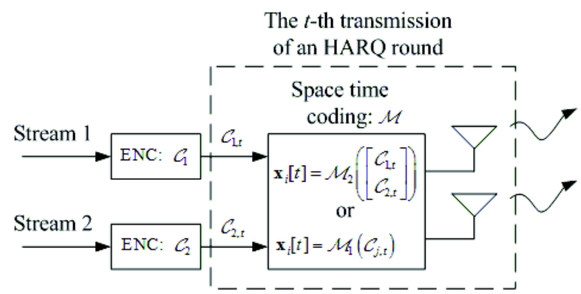

Fig. 1. The block diagram of the transmitting part of $\mathcal{S}$ or $\mathcal{R}$ at the t-th slot

II. SYSTEM MODEL

A. Protocol Description

In this paper, we consider 2×2 MIMO cooperative HARQ protocols for the single relay channel with three half duplex terminals having two antennas. It consists of the source $\mathcal{S}$, destination $\mathcal{D}$, and relay $\mathcal{R}$ and three links between them: the $\mathcal{S}$-to-$\mathcal{D}$ link (link 1), $\mathcal{S}$-to-$\mathcal{R}$ link (link 2), and $\mathcal{R}$-to-$\mathcal{D}$ link (link 3). The block diagram of the transmitting part of the source and relay is shown in Figure 1. The source has two data streams for the destination. The two streams are encoded separately and the two encoded streams are jointly coded by a space time code $\mathcal{M}_2$. If the source transmits only a single stream, the stream is coded by a space time code $\mathcal{M}_1$. The relay has the same encoding procedure with the source. If the destination decodes one of the two streams, the interferences from the decoded streams are canceled out by the interference cancelation method. At every transmissions, the degree of freedom per antenna is $L \approx \omega\tau$ where τ is the slot duration and ω is the slot bandwidth. If stream j, $j = 1, 2$, has b_j information bits, the transmission rate of stream j of the first transmission is given by $r_j = b_j/L(b/s/Hz)$.

The MIMO-HARQ protocol with relay is as follows. Each round of the protocol starts from Phase 1 and moves to the next round when $\mathcal{D}$ decodes the two streams or the number of transmissions reaches the maximum allowable number of transmissions for a packet M. In Phase 1, $\mathcal{S}$ broadcasts two streams to $\mathcal{D}$ and $\mathcal{R}$. If $\mathcal{R}$ decodes one of the two streams, $\mathcal{S}$ sends the other stream using a space time code. Phase 2 begins only if $\mathcal{R}$ decodes the two streams but $\mathcal{D}$ does not decodes one or both of them. In Phase 2, $\mathcal{R}$ sends streams which are not decoded at $\mathcal{D}$. If $\mathcal{D}$ decodes one of the two streams, $\mathcal{R}$

sends the other stream using a space time code. The MIMO-HARQ protocol without relay has only phase 1 since it does not have a relay. In the protocol, if the destination decodes one of the two streams, the other stream transmitted using a space time code. Note that, in this paper, we focused on the synchronous MIMO-HARQ protocol, i.e. a decoded stream is waited until the other stream is successfully decoded. The analysis of the asynchronous MIMO-HARQ protocols remains as a future work.

B. Signal Model

The IR-HARQ operation is similar to that in [6] as follows. The b_j bits information of a stream is encoded using the channel code with codebook $\mathcal{C}_j \in \mathbb{C}_j{}^{LM}$ over the complex numbers, $j = 1, 2$. Then, the overall codeword $\mathcal{C}_j$ is divided into M blocks of length L, i.e., $\{\mathcal{C}_{j,m} | m = 1, 2, \cdots, M\}$ where $\mathcal{C}_{j,m}$ denotes the m-th code block of stream j. If the destination sends NACK signal for stream j after receiving $\{\mathcal{C}_{j,1}, \cdots, \mathcal{C}_{j,m}\}$, the source or relay sends $\mathcal{C}_{j,m+1}$ to the destination for the next retransmission.

In the MIMO-HARQ protocols, the transmitted symbols at slot t of link i is given by a $2 \times L$ matrix, $i = 1, 2, 3$. The t-th code blocks of the two streams, $\mathcal{C}_{1,t}$ and $\mathcal{C}_{2,t}$, are mapped to a $2 \times L$ space time coded symbol $\mathbf{x}_i[t]$: $\mathcal{M}_2 : C_{1,t}, C_{2,t} \rightarrow \mathbf{x}_i[t]$ when the source has two streams to transmit and $\mathcal{M}_1 : C_{j,t} \rightarrow \mathbf{x}_i[t]$ when the source has a single stream j to transmit, where $\mathcal{M}_a$ denotes the mapping function of a space time code $a = 1, 2$. For example, $\mathbf{x}_i[t] = \left[C_{1,t}^T \; C_{2,t}^T\right]^T$ if the bell lab layered space time (BLAST) code is used. The symbol energy is normalized as $E[\mathbf{x}_i^H[t]\mathbf{x}_i[t]]/L = 1$ and the total transmit energy of symbol is E_s. The 2x2 channel of link i is given by

$$\mathbf{H_i[t]} = \sqrt{g_i} \begin{bmatrix} h_{11}^i[t] & h_{12}^i[t] \\ h_{21}^i[t] & h_{22}^i[t] \end{bmatrix}, \quad (1)$$

where g_i is the path loss gain of link i and $h_{mn}^i[t]$, $m = 1, 2$ and $n = 1, 2$, is a circularly Gaussian random variable with $\mathcal{CN}(0, 1)$ which remains constant over a slot but varies independently from one slot to another (i.e. block fading channel). Then, the received signal of the receiver of link i can be written as

$$\mathbf{y}_i[t] = \sqrt{\frac{E_s}{2}} \mathbf{H_i}[t] \mathbf{x}_i[t] + \mathbf{n}_i[t], \quad (2)$$

where $\mathbf{n}_i[t]$ is additive white Gaussian noise with $E[\mathbf{n}_i[t]] = \mathbf{0}$ and $E[\mathbf{n}_i[t]\mathbf{n}_i^H[t]]/L = N_0 I_2$.

C. The achievable rate of 2×2 MIMO-HARQ protocols

In [6], the throughput of an HARQ protocol was given by

$$\eta = \frac{E[\lambda]}{E[\mathcal{T}]} \; (b/s/Hz), \quad (3)$$

where λ denotes the rate successfully decoded at the destination and $\mathcal{T}$ denotes the number of transmissions required to finish one round of the HARQ protocol. Let $T_j(r_j) \in \mathbb{Z}^+$ be the random variable denoting the number of transmissions for successfully decoding of stream j at the destination, $j = 1, 2$ when the transmission rate of stream j is r_j. For simplification, we abbreviate $T_j(r_j)$ as T_j whenever it does not cause any confusion. In the proposed MIMO-HARQ protocols, each round is ended if the destination decodes the both streams. Thus, $\mathcal{T} = \max(T_1, T_2)$ is given by the maximum value between T_1 and T_2. Also, the destination can decode the two streams separately. Thus, $\lambda = r_j$ if the destination decodes only stream j and $\lambda = r_1 + r_2$ if the destination decodes the both streams. From the above aspect, the throughput of the 2×2 MIMO-HARQ protocols can be rewritten as

$$\eta(\bar{r}; M) = \frac{r_1(1 - \Pr\{T_1 > M\}) + r_2(1 - \Pr\{T_2 > M\})}{1 + \sum_{t=1}^{M-1} \Pr\{\max(T_1, T_2) > t\}}, \quad (4)$$

since $E[\mathcal{T}] = 1 + \sum_1^{M-1} \Pr\{\mathcal{T} > t\}$. Here, $\Pr\{T_j > M\}$ denotes the outage probability of stream j under given M. Then, the achievable rate of the MIMO-HARQ protocols, $R(M)$, is given by

$$R(M) = \max_{\bar{r}} \eta(\bar{r}, M). \quad (5)$$

In this paper, we compute the asymptotic achievable rate of the MIMO-HARQ protocols when M goes to infinity for the case of $r_1 \leq r_2$. This is because, in MIMO systems, the streams transmitted by different antennas have the same statistical characteristic. Thus, the analysis for $r_1 \leq r_2$ is enough to understand the 2×2 MIMO systems.

III. Analysis of the MIMO-HARQ protocols with/without relay

In this section, we obtain the asymptotic achievable rate of the MIMO-HARQ protocols without/with relay when $\mathcal{M}_2$ is a BLAST with linear receiver and $\mathcal{M}_1$ is a space time coding. It is well known that the virtual parallel channels in MIMO systems with the linear receiver are *strongly* statistically dependent [8] when r is not low. That is, if a virtual channel falls into a deep fade, then the other channels also falls into a deep fade with very high probability. Thus, we can suppose that $\Pr\{T_2 > t | T_1 > t\} \simeq 1$ from the assumption of $r_1 \leq r_2$. Then, the throughput of the MIMO-HARQ protocol can be approximated as

$$\eta(\bar{r}, M) \simeq \frac{r_1(1 - \Pr\{T_1 > M\}) + r_2(1 - \Pr\{T_2 > M\})}{1 + \sum_{t=1}^{M-1} \Pr\{T_2 > M\}}, \quad (6)$$

since $\Pr\{\max(T_1, T_2) > t\} \simeq \Pr\{T_2 > t\}$.

A. Complementary Cumulative distribution functions

In this subsection, we define two functions to denote the conditional complementary Cumulative distribution functions (ccdf) and the conditional probability mass function (pmf) of T_j, for $j = 1, 2$. One of the functions is $f_{i,j}^{(a_1, a_2, \cdots, a_K)}(t_1, t_2, \cdots, t_K)$, which denotes the probability of decoding failure of stream j at the receiver of link i when stream j is received t_k times in the k-th virtual parallel channel with diversity order a_k, for $i = 1, 2, 3$, $j = 1, 2$, and $k = 1, 2, \cdots, K$. The other is $g_{i,j}^{(a_1, a_2, \cdots, a_K)}(t_1, t_2, \cdots, t_K)$, which denotes the probability of successful decoding of stream j at the receiver of link i when the t_K-th signal of stream j is received in the K-th virtual parallel channel.

For circularly symmetric complex Gaussian inputs, $f_{i,j}^{(a_1,a_2,\cdots,a_K)}(t_1,t_2,\cdots,t_K)$ is given by

$$\begin{aligned} f_{i,j}^{(a_1,a_2,\cdots,a_K)}&(t_1,t_2,\cdots,t_K) \\ &= \Pr\left\{\sum_{k=1}^{K}\sum_{n=1}^{t_k}\log_2(1+\gamma_{i,j}^{(a_k)}Y_{2a_k}[n]) < r_j\right\} \quad (7)\\ &\simeq 1-Q\left(\frac{r_j-\sum_{k=1}^{K}t_k u_{i,j}^{(a_k)}}{\sqrt{\sum_{k=1}^{K}t_k v_{i,j}^{(a_k)}}}\right), \quad (8)\end{aligned}$$

where $\gamma_{i,j}^{(a_k)}$ denotes the average received SNR of stream j at the receiver of link i in the k-th virtual channel and $Y_{2a_k}[n]$ denotes a random variable with diversity order a_k. For example, if zero-forcing receiver is used, $Y_{2a_k}[n]$ is the Chi-squared random variable with the degree of $2a_k$ and the variance of 0.5 [7]. Here, the approximation in the third line comes from the central limit theorem since generally $\sum_{k=1}^{K}t_k$ is typically not a small number. In the approximation, *the mean of the mutual information* can be obtained as

$$u_{i,j}^{(a_k)} = \int_0^\infty \log_2(1+\gamma_{i,j}^{(a_k)}y)p_Y(y)dy, \quad (9)$$

where $p_Y(y)$ denotes the pdf of $Y_{2a_k}[n]$. Also, *the variance of the mutual information* can be obtained as

$$\begin{aligned} v_{i,j}^{(a_k)} = &\int_0^\infty \left(\log_2(1+\gamma_{i,j}^{(a_k)}y)\right)^2 p_Y(y)dy \\ &-\left(u_{i,j}^{(a_k)}\right)^2. \quad (10)\end{aligned}$$

The other function, $g_{i,j}^{(a_1,a_2,\cdots,a_K)}(t_1,t_2,\cdots,t_K)$, is given by

$$\begin{aligned} g_{i,j}^{(a_1,a_2,\cdots,a_K)}&(t_1,t_2,\cdots,t_K) = \\ &f_{i,j}^{(a_1,a_2,\cdots,a_K)}(t_1,t_2,\cdots,t_{K-1},t_K-1) \\ &-f_{i,j}^{(a_1,a_2,\cdots,a_K)}(t_1,t_2,\cdots,t_{K-1},t_K). \quad (11)\end{aligned}$$

B. MIMO-HARQ protocol without relay

In this subsection, we compute the approximated throughput of the MIMO-HARQ protocol without relay and obtain the asymptotic achievable rate of it when M goes to infinity.

From the approximation of $\Pr\{T_2 > t | T_1 > t\} \simeq 1$, we can see that stream 1 is decoded at the destination using a linear receiver before stream 2 is decoded with very high probability. Thus, the diversity order of stream 1 is 1 and the ccdf of T_1 is given by

$$\Pr\{T_1 > t\} = f_{1,1}^{(1)}(t), \quad (12)$$

and $\gamma_{1,1}^{(1)} = g_1 E_s/2$. Stream 2 is very likely to be decoded after than stream 1, since $\Pr\{T_2 > t|T_1 > t\} \simeq 1$. If stream 1 is decoded, stream 2 is decoded after the interference of stream 1 is canceled out. Thus, the diversity order of the virtual channel path for stream 2 becomes 2. After stream 1 is successfully decoded, stream 2 is transmitted using the space time code $\mathcal{M}_1$. In that case, the diversity order of the channel is 4. Thus, the conditional ccdf of T_2 is given by $\Pr\{T_2 > t|T_1 = n\} = f_{1,2}^{(2,4)}(n, t-n)$ and $\Pr\{T_2 > t\}$ is given by

$$\begin{aligned} \Pr\{T_2 > t\} &= \Pr\{T_1 > t, T_2 > t\} + \Pr\{T_1 \le t, T_2 > t\} \\ &\simeq \Pr\{T_1 > t\} + \sum_{n=1}^{t}\Pr\{T_2 > t|T_1 = n\}\Pr\{T_1 = n\}. \\ &= f_{1,1}^{(1)}(t) + \sum_{n=1}^{t} f_{1,2}^{(2,4)}(n, t-n) g_{1,1}^{(1)}(n), \quad (13)\end{aligned}$$

and $\gamma_{1,2}^{(2)} = \gamma_{1,2}^{(4)} = g_1 E_s/2$. Here, the approximation comes from $\Pr\{T_2 > t|T_1 > t\}\Pr\{T_1 > t\} \simeq \Pr\{T_1 > t\}$. By substituting (12) and (13) into (6), the asymptotic throughput is given by

$$\begin{aligned} \eta(\bar{r}, M) &\simeq \\ &= \frac{r_1\left(1-f_{1,1}^{(1)}(M)\right)}{1+\sum_{t=1}^{M-1}\left(f_{1,1}^{(1)}(t)+\sum_{n=1}^{t}f_{1,2}^{(2,4)}(n,t-n)g_{1,1}^{(1)}(n)\right)} \\ &+\frac{r_2\left(1-f_{1,1}^{(1)}(M)-\sum_{n=1}^{M}f_{1,2}^{(2,4)}(n,M-n)g_{1,1}^{(1)}(n)\right)}{1+\sum_{t=1}^{M-1}\left(f_{1,1}^{(1)}(t)+\sum_{n=1}^{t}f_{1,2}^{(2,4)}(n,t-n)g_{1,1}^{(1)}(n)\right)}. \end{aligned} \quad (14)$$

If M goes to infinity, the outage probabilities goes to 0 and a stream can obtain large amount of diversity if its transmission rate is very high. Let $\lim_{M\to\infty}\eta(\bar{r}, M) = \eta(\bar{r}, \infty)$ and $\lim_{M\to\infty}R(M) = R(\infty)$. The throughput of the 2×2 MIMO-HARQ protocol without relay is given as follows.

Theorem 1. If $M \to \infty$, r_1, and r_2 are sufficiently large and $\alpha = r_1/r_2$ where $0 \le \alpha \le 1$, the throughput of the 2×2 MIMO-HARQ protocol is given by

$$\eta(\bar{r}, \infty) \simeq \frac{\alpha+1}{\frac{\alpha}{u_{1,1}^{(1)}} + \left[\frac{1-\alpha u_{1,2}^{(2)}/u_{1,1}^{(1)}}{u_{1,2}^{(4)}}\right]^+}, \quad (15)$$

where $[\beta]^+ = \max(\beta, 0)$. Also, the optimum ratio α^* is given by

$$\alpha^* = \begin{cases} \frac{u_{1,1}^{(1)}}{u_{1,2}^{(2)}}, & \text{if } u_{1,1}^{(1)} + u_{1,2}^{(2)} \ge u_{1,2}^{(4)}, \\ 0, & \text{if } u_{1,1}^{(1)} + u_{1,2}^{(2)} < u_{1,2}^{(4)}. \end{cases} \quad (16)$$

Proof. See Appendix A. □

Note that, if r_1 and r_2 are sufficiently large, the values of them are not important but α is a dominant factor to obtain the achievable rate. If $\alpha = u_{1,1}^{(1)}/u_{1,2}^{(2)}$, it means that the two streams are transmitted with the relation of $r_1 = \alpha r_2$ and the achievable rate is given by $R(\infty) = u_{1,1}^{(1)} + u_{1,2}^{(2)}$. If $\alpha = 0$, it means that only stream 2 is transmitted and the achievable rate is given by $u_{1,2}^{(4)}$. Thus, the achievable rate is given by

$$R(\infty) \simeq \max(u_{1,1}^{(1)} + u_{1,2}^{(2)}, u_{1,2}^{(4)}). \quad (17)$$

In (17), $u_{1,1}^{(1)} + u_{1,2}^{(2)}$ is the ergodic capacity of $\mathcal{M}_2$. Thus, the MIMO-HARQ without relay can achieve the ergodic capacity when $\mathcal{M}_2$ is the BLAST with MMSE-SIC since it is known as the capacity approaching scheme.

Figure 2 shows the throughput of MIMO-HARQ protocol without relay. In this figure, 'anal.' and 'simul.' respectively denote the throughput obtained by the analysis in (14) and

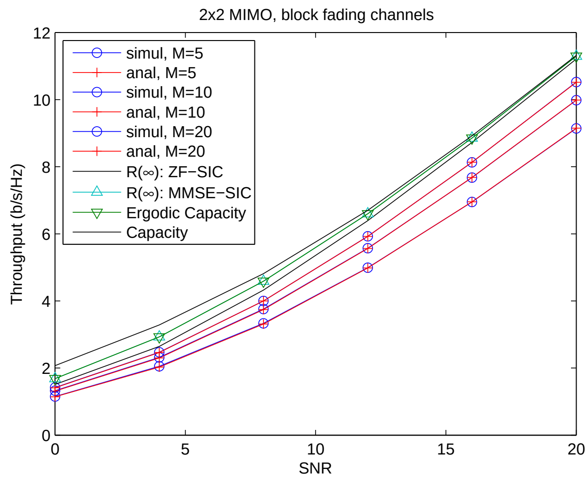

Fig. 2. The throughput of MIMO-HARQ without relay versus SNR

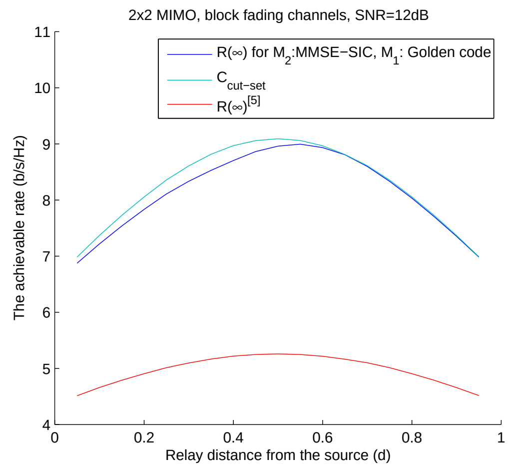

Fig. 3. The achievable rate of MIMO-HARQ protocol with relay and the cut set bound of the ergodic capacity

simulation when $\mathcal{M}_2$ is the BLAST with ZF-SIC receiver and $\mathcal{M}_1$ is the Alamouti. From the figure, we can see that the analysis is quite accurate with the simulations and the throughput increases as M increases and converges to $R(\infty)$. Also, we can see that the MIMO-HARQ protocol can achieve the ergodic capacity if $\mathcal{M}_2$ is spatial multiplexing and the MMSE-SIC receiver is used.

C. MIMO-HARQ protocol with relay

In this subsection, we compute the approximated throughput of the MIMO-HARQ protocol with relay and obtain the asymptotic achievable rate of it when M is sufficiently large.

The throughput of MIMO-HARQ with relay is calculated in Appendix B. From the throughput, we can obtain the asymptotic throughput when $M \rightarrow \infty$, r_1, and r_2 are sufficiently large as follows.

Theorem 2. If $M \rightarrow \infty$, r_1, and r_2 are sufficiently large and $\alpha = r_1/r_2$ where $0 \le \alpha \le 1$, the throughput of the 2×2 MIMO-HARQ protocol without relay is given by

$$\eta(\bar{r}, \infty) \simeq \frac{\alpha + 1}{\bar{\mathcal{T}}_{2,1}(\alpha) + \bar{\mathcal{T}}_{2,2}(\alpha) + \bar{\mathcal{T}}_{3,1}(\alpha) + \bar{\mathcal{T}}_{3,2}(\alpha)}, \quad (18)$$

where

$$\bar{\mathcal{T}}_{2,1}(\alpha) = \frac{\alpha}{u_{2,1}^{(1)}}, \ \ \bar{\mathcal{T}}_{2,2}(\alpha) = \left[\frac{1 - \bar{\mathcal{T}}_{2,1}(\alpha) u_{2,2}^{(2)}}{u_{2,2}^{(4)}}\right]^+,$$

$$\bar{\mathcal{T}}_{3,1}(\alpha) = \left[\frac{\alpha - \bar{\mathcal{T}}_{2,1}(\alpha) u_{1,1}^{(1)}}{u_{3,1}^{(1)}}\right]^+,$$

$$\bar{\mathcal{T}}_{3,2}(\alpha) = \left[\frac{1 - \bar{\mathcal{T}}_{2,1}(\alpha) u_{1,2}^{(2)} - \bar{\mathcal{T}}_{2,2}(\alpha) u_{1,2}^{(4)} - \bar{\mathcal{T}}_{3,1}(\alpha) u_{3,2}^{(2)}}{u_{3,2}^{(4)}}\right]^+.$$

Let $L_{\min} = \min(u_{2,1}^{(1)}/u_{2,2}^{(2)}, u_N/u_D)$ and $L_{\max} = \min(\max(u_{2,1}^{(1)}/u_{2,2}^{(2)}, u_N/u_D), 1)$, where $u_N = 1 - u_{1,2}^{(4)}/u_{2,2}^{(4)}$ and

$$u_D = \frac{u_{1,2}^{(2)}}{u_{2,1}^{(1)}} - \frac{u_{1,2}^{(4)} u_{2,2}^{(2)}}{u_{2,1}^{(1)} u_{2,2}^{(4)}} + u_{3,2}^{(2)} \frac{u_{2,1}^{(1)} - u_{1,1}^{(1)}}{u_{3,1}^{(1)} u_{2,1}^{(1)}}. \quad (19)$$

If $M \rightarrow \infty$, r_1, and r_2 are sufficiently large, the optimum ratio α^* is given by

$$\alpha^* = \begin{cases} 0, & \text{if } \frac{d\eta}{d\alpha} < 0 \text{ for } \alpha \in R_1, \ \frac{d\eta}{d\alpha} < 0 \text{ for } \alpha \in R_2, \\ L_{\min}, & \text{if } \frac{d\eta}{d\alpha} > 0 \text{ for } \alpha \in R_1, \ \frac{d\eta}{d\alpha} < 0 \text{ for } \alpha \in R_2, \\ L_{\max}, & \text{if } \frac{d\eta}{d\alpha} > 0 \text{ for } \alpha \in R_1, \ \frac{d\eta}{d\alpha} > 0 \text{ for } \alpha \in R_2, \end{cases} \quad (20)$$

where $R_1 = \{\alpha | 0 \le \alpha < L_{\min}\}$ and $R_2 = \{\alpha | L_{\min} \le \alpha \le L_{\max}\}$.

Proof. See Appendix B. □

The achievable rate $R(\infty)$ can be obtained by plugging α^* into (18). If $\alpha^* = u_{2,1}^{(1)}/u_{2,1}^{(2)}$, it means that the achievable rate is limited by the achievable rate of link 2. If $\alpha^* = u_N/u_D$, it means that the achievable rate is limited by the achievable rate of link 1 and 3. Figure 3 shows the achievable rate of the MIMO-HARQ with relay when $\mathcal{M}_2$ is the BLAST with the MMSE-SIC and $\mathcal{M}_1$ is the Golden code. In this figure, $C_{\text{cut-set}}$ denotes the the cut set bound of the ergodic capacity of the decode-and-forward protocol with a half-duplex relay (HR), which is given by

$$C_{\text{cut-set}} = \max_{0 \le \beta \le 1} \min(\beta C_2, \beta C_1 + (1 - \beta) C_3), \quad (21)$$

where $C_i = u_{i,1}^{(1)} + u_{i,2}^{(2)}$, which can be obtained from [9]. Also, $R(\infty)^{[5]}$ denotes the achievable rate with the direct link of the protocol proposed in [4], which is given by

$$R(\infty)^{[5]} = \max_{0 \le \beta \le 1} \min \beta(u_{2,2}^{(4)}, \beta u_{1,2}^{(4)} + (1 - \beta)(u_{3,2}^{(4)})). \quad (22)$$

From this figure, we can see that the achievable rate of the MIMO-HARQ with a relay is smaller than $C_{\text{cut-set}}$ but greater than $R(\infty)^{[5]}$. However, $R(\infty)$ is very similar to $C_{\text{cut-set}}$ when $d > 0.6$.

IV. Conclusion

In this paper, we analyze the throughput and achievable rate of the MIMO-HARQ protocol with/without relay. From the analysis, we can see that the MIMO-HARQ without relay protocol can achieve the ergodic capacity and the ratio between two streams is a dominant factor to achieve the ergodic capacity when the number of retransmissions is unlimited. The MIMO-HARQ protocols with relay cannot achieve the ergodic capacity of the cut-set bound of the decode-and-forward protocols but it has similar performance with it where the relay is closer to the destination.

APPENDIX A

The throughput of the MIMO-HARQ without relay in *Theorem 1* can be obtained from (18) by the proposed method in [10]. Using the method, $f_{i,j}^{(a_1,\ldots,a_K)}(t_1,\ldots,t_K)$ and $g_{i,j}^{(a_1,\ldots,a_K)}(t_1,\ldots,t_K)$ can be respectively approximated as the modified unit step function (see [10]) and Kronecher delta function by the strong law of large numbers. Thus, $f_{1,1}^{(1)}(t)$, $f_{1,2}^{(2,4)}(n,t-n)$, and $g_{1,1}^{(1)}(n)$ are respectively approximated as $U(t-r_1/u_{1,1}^{(1)})$, $U(t-r_1u_{1,2}^{(2)}/u_{1,1}^{(1)}-nu_{1,2}^{(4)})$, and $\delta(n-r_1/u_{1,1}^{(1)})$. By substituting them into (18), *Theorem 1* can be obtained after some manipulations.

Corollary 1 can be obtained from *Theorem 1* as follows. As shown in *Theorem 1*, the throughput can be differentiated by α. If $\alpha > u_{1,1}^{(1)}/u_{1,2}^{(2)}$, $\frac{d\eta}{d\alpha} < 0$. Otherwise (if $\alpha \le u_{1,1}^{(1)}/u_{1,2}^{(2)}$), if $u_{1,1}^{(1)}+u_{1,2}^{(1)} \ge u_{1,2}^{(4)}$, $\frac{d\eta}{d\alpha} \ge 0$ and if $u_{1,1}^{(1)}+u_{1,2}^{(1)} < u_{1,2}^{(4)}$, $\frac{d\eta}{d\alpha} \le 0$. Thus, the achievable rate can be obtained when $\alpha = u_{1,1}^{(1)}/u_{1,2}^{(2)}$ if $u_{1,1}^{(1)}+u_{1,2}^{(1)} \ge u_{1,2}^{(4)}$ and $\alpha = 0$ if $u_{1,1}^{(1)}(\rho_1)+u_{1,2}^{(1)} > u_{1,2}^{(4)}$.

APPENDIX B

The throughput of 2×2 MIMO-HARQ with relay can be obtained as like that of without relay. However, T_1 and T_2 are dependent on Z_1 and Z_2, where $Z_j \in \mathbb{Z}^+$ denotes the number of successful decoding of stream j at the relay, $j = 1, 2$. Thus, $\Pr\{T_1 > t\}$ is given by

$$\begin{aligned}
&\Pr\{T_1 > t\} \simeq \Pr\{T_1 > t, Z_1 > t\} \\
&\quad + \sum_{m=1}^{t} \Pr\{T_1 > t, Z_1 = m, Z_2 > t\} \\
&\quad + \sum_{m=1}^{t}\sum_{n=m}^{t} \Pr\{T_1 > t, Z_1 = m, Z_2 = n\} \\
&= f_{1,1}^{(1)}(t)g_{1,1}^{(1)}(t) + \sum_{m=1}^{t} f_{1,1}^{(1)}(m) f_{2,2}^{(2,4)}(m,t-m) g_{2,1}^{(1)}(m) \\
&\quad + \sum_{m=1}^{t}\sum_{n=m}^{t} f_{3,1}^{(1,1)}(m,t-n) g_{2,2}^{(2,4)}(m,n-m) g_{2,1}^{(1)}(m),
\end{aligned}$$

and $\Pr\{T_1 \le t, T_2 > t\}$ is given by

$$\begin{aligned}
&\Pr\{T_1 \le t, T_2 > t\} \simeq \Pr\{T_1 \le t, T_2 > t, Z_1 > t, Z_2 > t\} \\
&\quad + \sum_{m=1}^{t}\sum_{l=1}^{m} \Pr\{T_1 = l, T_2 > t, Z_1 = m, Z_2 > t\} \\
&\quad + \sum_{m=1}^{t}\sum_{n=m}^{t}\sum_{l=n+1}^{t} \Pr\{T_1 = l, T_2 > t, Z_1 = m, Z_2 = n\} \\
&\quad + \sum_{m=1}^{t}\sum_{n=m}^{t}\sum_{l=1}^{m} \Pr\{T_1 = l, T_2 > t, Z_1 = m, Z_2 = n\} \\
&= \sum_{l=1}^{t} f_{1,2}^{(2,4)}(l,t-l) g_{1,1}^{(1)}(l) f_{2,1}^{(1)}(t) \\
&\quad + \sum_{m=1}^{t}\sum_{l=1}^{m} f_{1,2}^{(2,4)}(m,t-m) g_{1,1}^{(1)}(l) f_{2,2}^{(2,4)}(m,t-m) g_{2,1}^{(1)}(m) \\
&\quad + \sum_{m=1}^{t}\sum_{n=m}^{t}\sum_{l=n+1}^{t} \Big(f_{3,2}^{(2,4,2,4)}(m,n-m,l-n,t-l) \\
&\qquad \times g_{3,1}^{(1,1)}(m,l-n) g_{2,2}^{(2,4)}(m,n-m) g_{2,1}^{(1)}(m) \Big) \\
&\quad + \sum_{m=1}^{t}\sum_{n=m}^{t}\sum_{l=1}^{m} \Big(f_{3,2}^{(2,4,2,4)}(m,n-m,0,t-n) g_{1,1}^{(1)}(l) \\
&\qquad \times g_{2,2}^{(2,4)}(m,n-m) g_{2,1}^{(1)}(m) \Big).
\end{aligned}$$

By substituting them into (6), the approximated throughput of the MIMO-HARQ with relay can be obtained. The throughput when M goes to infinity and the optimum α can be obtained using the method used for the MIMO-HARQ without relay in Appendix A. Precisely, $\frac{d\eta}{d\alpha}$ is given as $\frac{B_{i(\alpha)}-A_{i(\alpha)}}{(A_{i(\alpha)}\alpha+B_{i(\alpha)})^2}$, where $A_{i(\alpha)}$ and $B_{i(\alpha)}$ are changed according to $i(\alpha)$ and $i(\alpha) = 1$ if $\alpha \in R_1$ and $i(\alpha) = 2$ if $\alpha \in R_2$. Thus, η is a monotonic increasing function if $\frac{d\eta}{d\alpha} > 0$ and decreasing function if $\frac{d\eta}{d\alpha} < 0$.

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REFERENCES

[1] E. N. Onggosanusi, A. G. Dabak, Y. Hui, G. Jeong, "Hybrid ARQ transmission and combining for MIMO systems," *Proc. IEEE Int. Commun. Confer (ICC)*, pp. 3205-3209, Seatle, WA, U.S.A., May 2003.

[2] K. Acolatse, Y. Bar-Ness, "An Alamouti-based Hybrid-ARQ scheme for MIMO systems," *Inform. Society Tech. (IST)*, Dresden, Germany, Jun, 2005.

[3] H. E. Gamal, G. Caire, and M. O. Damen, "The MIMO ARQ Channel: Diversity.Multiplexing.Delay Tradeoff," *IEEE Trans. on Inform. Theory*, vol. 52, no. 8, pp. 3601-3621, August 2006.

[4] X. Yao, D. Gunduz, and A. Goldsmith, "Multihop MIMO Relay Networks with ARQ," *Proc. IEEE Glob. Telecom. Confer. (Globecom)*, pp. 1-6, Honolulu, Hawaii, U.S.A., November 2009.

[5] B. Wang, J. Zhang, A. Høst-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. on Inform. Theory*, vol. 51, no. 1, pp. 29-42, April 2005.

[6] G. Caire and D. Tuninetti, "The throughput of hybrid ARQ protocols for the gaussian collision channel," *IEEE Trans. Inform. Theory*, vol. 47, no. 5, July 2001.

[7] J. H. Winters, J. Salz, and R. D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. on Commum.*, vol. 42, no. 2, pp. 1740-1751, April 1994.

[8] K. R. Kumar, G. Caire, and A. L. Moustakas, "Asymptotic Performance of Linear Receivers in MIMO Fading Channels," *IEEE Trans. on Inform. Theory*, vol. 55, no. 10, pp. 4398-4418, April 2010.

[9] R. Nikjah and N. C. Beaulieu, "Exact closed-form expressions for the outage probability and ergodic capacity of decode-and-forward opportunistic relaying," *Proc. IEEE Glob. Telecommun. Confer. (Globecom)*, pp. 1-8, Honolulu, Hawaii, U.S.A., October 2009.

[10] I. Byun and K.S. Kim, "The Maximum Achievable Throughput of a Decode-and-Forward Based Hybrid-ARQ Protocol," *Proc. IEEE Mil. Commun. Confer. (MILCOM)*, pp. 1-5, Boston, MA, U.S.A., October 2009.