Locally Optimum User Detection in Nakagami Interference

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Abstract— **Detection of the existence of a desired user is considered in this paper. We assume that the signal to noise ratio is high enough to ignore the effects of noise compared with those of the interference by other users. The inter-user interference and user signals are modeled by the Nakagami model. The observation model for this situation is proposed, the locally optimum test statistic is derived under the model, and the asymptotic performance of the test statistic is compared with that of the envelope detector. We show that the locally optimum detector has performance better than the conventional envelope detector.**

Keywords— **locally optimum detection, Nakagami fading**

I. Introduction

As mobile communications get in much use, multiple access (MA) techniques become more important, because the frequency resources are quite restricted. In order to accommodate more users in the restricted frequency band, many MA techniques are emerging, e.g., the frequency hopped (FH) and direct sequence (DS) spread spectrum (SS) communication systems [1]. In these MA communication systems, however, the inter-user interference (IUI) problem is unavoidable. Because modern commercial communication guarantees quite high signal to noise ratio (SNR), it can be justified to assume that the IUI is the only disturbance [2]. Actually, the IUI affects the performance of cellular mobile communication systems much more than noise does [3].

In MA communication systems, we are to detect the presence of the desired user in the interference by neighboring other users. Especially, we focus on the situation where the desired user signal is very weak when compared with the IUI. This problem is quite important to get higher connection probability in various mobile circumstances. The locally optimum (LO) detection technique has been shown to be useful when the SNR is very low. The LO detector maximizes the slope of the power function when the SNR is zero, and hence is expected to show the best performance when the SNR is low. It is also noteworthy that the LO detector can always be acquired unlike the uniformly most powerful (UMP) or optimum detector. The LO detector has also been shown to be the power-series expansion of the likelihood function truncated at the first nonzero term [4].

The main goal of this paper is to get a better user signal detector than the conventional envelope detector [5]. For this purpose, we propose an observation model for the detection of the desired user in the IUI. The Nakagami m-distribution [6] is used for modeling both the user signal and the IUI. This model assume the Nakagami fading environment. This LO detector can be used to improve the connection probability and lessen the power consumption.

II. The Observation Model

Let us consider the observation model

$$X_i = \theta S_i + W_i = \theta S_i + \sum_{j=1}^{m_i} I_{i,j}, \quad i = 1, 2, \cdots, n, \qquad (1)$$

where X_i is the envelope of the receiver output at the i-th sampling instant, m_i is the number of neighboring users at the i-th instant, $\theta \geq 0$ is the signal strength parameter, and n is the sample size. Here, S_i is from the desired user and $I_{i,j}$ is the interference term from the j-th neighboring user at the i-th sampling instant. Let us denote $X = (X_1, X_2, \cdots, X_n)$, $S = (S_1, S_2, \cdots, S_n)$, and $W = (W_1, W_2, \cdots, W_n)$.

Based on the fact that nonselective fading of radio signal envelope is adequately represented by the Nakagami m-distribution, we assume that S_i and $I_{i,j}$ are Nakagami random variables. It is not possible, however, to get the exact distribution of W_i, a sum of Nakagami random variables $I_{i,j}$. Note that we cannot derive the LO test statistic without the interference distribution. It is reasonable to assume that the interference of the nearest user is dominant, that is,

$$W_i = \sum_{j=1}^{m_i} I_{i,j} \cong I_{i,max}, \qquad (2)$$

where $I_{i,max}$ is the most powerful interference. With this approximation, we can assume that W_i is also a Nakagami random variable.

The probability density function (pdf) of the Nakagami m-distribution is

$$f(x|m, \sigma^2) = \frac{2}{\Gamma(m)} \left(\frac{m}{\sigma^2}\right)^m x^{2m-1} \exp\left(-\frac{m}{\sigma^2} x^2\right) \qquad (3)$$

where $x \geq 0$, $m \geq 1/2$ is the fading depth parameter, and σ^2 is the mean power parameter. Note that the larger m

means the more constrained fluctuation, and the case $m = 1$ subsumes the Rayleigh fading.

We assume that both W_i and S_i, $i = 1, 2, \cdots, n$, are independent and identically distributed (i.i.d.) random variables, and W_i and S_j are independent of each other. We also assume that W_i and S_i follow the Nakagami m-distributions with parameter sets (m, σ^2) and (m_K, σ_K^2), respectively. Let us denote the pdf's of W_i and S_i by f_W and f_S, respectively.

Let us now consider a binary hypothesis testing problem. Under the null hypothesis H, X_i consists only of the IUI term W_i $(\theta = 0)$, and under the alternative hypothesis K, X_i consists of the desired user signal S_i and the IUI W_i $(\theta > 0)$. Based on this consideration, the hypothesis problem

$$H : X_i = W_i, \quad i = 1, 2, \cdots, n, \tag{4}$$

versus

$$K : X_i = \theta S_i + W_i, \quad i = 1, 2, \cdots, n, \tag{5}$$

can be considered as a parameter test

$$H : \theta = 0, \tag{6}$$

versus

$$K : \theta > 0. \tag{7}$$

The joint pdf of X_i, $i = 1, 2, \cdots, n$, under H and K are

$$H : \phi(x) = \prod_{i=1}^{n} f_W(x_i) \tag{8}$$

and

$$K : \phi(x) = \int_{R^n} \prod_{i=1}^{n} f_W(x_i - \theta s_i) f_S(s_i) ds_i. \tag{9}$$

For later use, let us define the LO nonlinearity

$$g(x) = -f_W'(x)/f_W(x) = \frac{2m}{\sigma^2}x - (2m-1)\frac{1}{x} \tag{10}$$

and the *Fisher's information* of f_W

$$I(f_W) = \int \{g(x)\}^2 f_W(x) dx. \tag{11}$$

III. Locally Optimum Detector Test Statistic

Since it is generally not possible to obtain uniformly most powerful (UMP) detectors, we can concentrate on the problem of designing detectors for weak signals, which results in LO detectors. Having a basis in the generalized Neyman-Pearson lemma, an LO detector has maximum slope for the detector power function at the origin $(\theta = 0)$ in the class of all detectors that have its false-alarm probability. The power of an LO detector is guaranteed under mild conditions to be no smaller than that of other detectors at least for θ in some nonnull interval $(0, \theta_M)$ [4] [7] [8].

From the generalized Neyman-Pearson lemma, the LO detector test statistic may be obtained as

$$\begin{aligned} T_{LO}(X) &= \frac{1}{\phi(X|H)} \lim_{\theta \to 0} \frac{d}{d\theta} \phi(X|K) \\ &= \sum_{i=1}^{n} E\{S_i\} g(X_i). \end{aligned} \tag{12}$$

Noting that $E\{S_i\}$ is the same for all $i = 1, 2, \cdots, n$, the LO detector statistic can be represented by

$$\begin{aligned} T_{LO}(X) &= \sum_{i=1}^{n} g(X_i) \\ &= \sum_{i=1}^{n} \frac{1}{\sigma^2} X_i \\ &\quad + (2m-1) \sum_{i=1}^{n} \left\{ \frac{1}{\sigma^2} X_i - \frac{1}{X_i} \right\}. \end{aligned} \tag{13}$$

Noting that the envelope (EV) detector test statistic is

$$T_{EV}(X) = \sum_{i=1}^{n} \frac{1}{\sigma^2} X_i, \tag{14}$$

we have

$$\begin{aligned} T_{LO}(X) &= T_{EV}(X) + (2m-1) \sum_{i=1}^{n} \left\{ \frac{1}{\sigma^2} X_i - \frac{1}{X_i} \right\} \\ &= 2m T_{EV}(X) + (1-2m) \sum_{i=1}^{n} \frac{1}{X_i}. \end{aligned} \tag{15}$$

That is, $T_{LO}(X)$ has an additional term containing (X_i and) $1/X_i$, and $T_{LO}(X) = T_{EV}(X)$ when $m = 1/2$. It is noteworthy that $m = 1/2$ means the most fluctuated fading environment. It is easy to expect that the additional term would improve the performance of the LO detector over the EV detector when $m > 1/2$. This LO detector test statistic depends only on the parameters of W_i, m and σ^2, not on those of S_i, m_K and σ_K^2: what we need to know to construct the LO detector is thus information only about the IUI, and we need not know the statistics of the desired user signal. Figure 1 shows the LO nonlinearity $g(x)$ for various values of m. It is interesting to see that the shape of the LO nonlinearity does not change with m: only the slope changes.

IV. Asymptotic Performance of the Locally Optimum Detector

One of the most commonly-used measures of relative asymptotic performance of detectors is the asymptotic relative efficiency (ARE) [4]. The $\text{ARE}_{1,2}$ of two detectors D_1 and D_2 based on test statistics T_1 and T_2 can be expressed under some regularity conditions as the ratio

$$\text{ARE}_{1,2} = \frac{\xi_1}{\xi_2}, \tag{16}$$

where

$$\xi_i = \lim_{n \to \infty} \frac{\left[\frac{dE\{T_i|\theta\}}{d\theta} \Big|_{\theta=0} \right]^2}{nV\{T_i|\theta = 0\}}, \quad i = 1, 2, \tag{17}$$

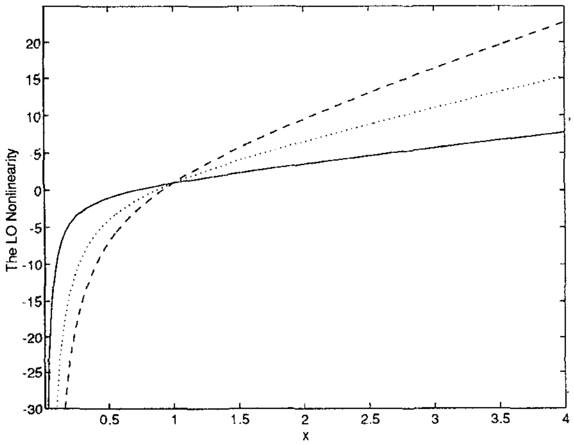

Fig. 1. The LO nonlinearity when $\sigma^2 = 1$, $m = 1$ (solid), $m = 2$ (dotted), and $m = 3$ (dashed).

are the efficacies of the detectors. The detectors we will consider here are the LO and EV detectors.

The efficacy of the LO detector can be derived via the steps similar to those in [4]. Here we will assume that $m > 1/2$ for the LO detector, because $m = 1/2$ means $T_{LO}(X) = T_{EV}(X)$.

Noting that

$$\lim_{\theta \to 0} \frac{d}{d\theta} E_K\{T_{LO}\} = E_H\{T_{LO}^2\} = nI(f_W) \tag{18}$$

and

$$E_H\{T_{LO}\} = n \int_0^\infty g(x) f_W(x) dx = 0, \quad m > 1/2, \tag{19}$$

the efficacy of the LO detector is

$$\xi_{LO} = \frac{E_H^2\{T_{LO}^2\}}{nV_H\{T_{LO}\}} = \frac{V_H\{T_{LO}\}}{n} = I(f_W). \tag{20}$$

Similarly, the efficacy of the EV detector can be derived. It is easy to show that

$$\lim_{\theta \to 0} \frac{d}{d\theta} E_K\{T_{EV}\} = E_H\{T_{EV} T_{LO}\} = \frac{n}{\sigma^2} \tag{21}$$

and

$$V_H\{T_{EV}\} = E_H\{T_{EV}^2\} - E_H^2\{T_{EV}\} = \frac{n}{\sigma^4} V_H\{X_i\}. \tag{22}$$

From these, the efficacy of the EV detector can be represented by

$$\xi_{EV} = \frac{1}{V_H\{X_i\}}. \tag{23}$$

Therefore,

$$\text{ARE}_{LO,EV} = I(f_W) V_H\{X_i\}. \tag{24}$$

Let us now calculate the ARE as a function of m. Noting that

$$E_H\{X_i^p\} = \left(\frac{\sigma^2}{m}\right)^{\frac{p}{2}} \frac{\Gamma(m+p/2)}{\Gamma(m)} \tag{25}$$

with $2m + p > 0$, it is easy to show that

$$\begin{aligned} I(f_W) &= \int_0^\infty g^2(x) f(x) dx \\ &= \frac{4m(1-m)}{\sigma^2} + (2m-1)^2 E_H\{\frac{1}{X_i^2}\}. \end{aligned} \tag{26}$$

Now, when $m > 1$, we have

$$E_H\{\frac{1}{X^2}\} = \frac{m}{\sigma^2(m-1)}, \tag{27}$$

and hence, the *Fisher's information* becomes

$$I(f_W) = \frac{m(4m-3)}{\sigma^2(m-1)}. \tag{28}$$

When $1/2 < m \leq 1$, $E_H\{1/X_i^2\}$ is infinite, and so is $I(f_W)$. It is also straightforward to get the variance of X_i under H,

$$V_H\{X\} = \frac{\sigma^2\{m\Gamma^2(m) - \Gamma^2(m+1/2)\}}{m\Gamma^2(m)}. \tag{29}$$

Using (24), (28), and (29), we have

$$\text{ARE}_{LO,EV} = \frac{(4m-3)\{m\Gamma^2(m) - \Gamma^2(m+1/2)\}}{(m-1)\Gamma^2(m)} \tag{30}$$

when $m > 1$. From Figure 2, we can see that $\text{ARE}_{LO,EV} > 1$ for all $m > 1$, which means that the LO detector has better asymptotic performance than the EV detector. Note that $\text{ARE}_{LO,EV}$ increases as m decreases. Note that the $\text{ARE}_{LO,EV}$ is obviously infinite when $1/2 < m \leq 1$, since ξ_{LO} is infinite while ξ_{EV} is finite.

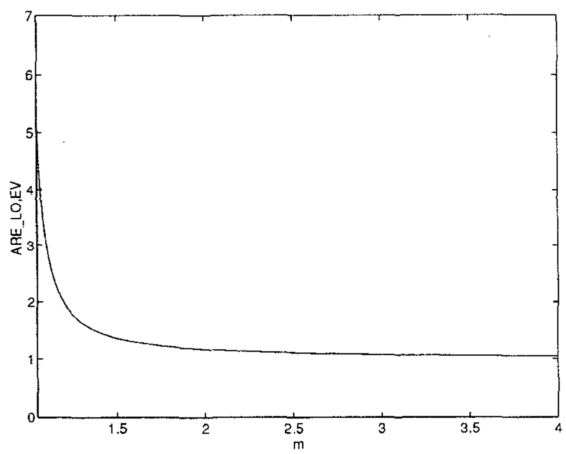

Fig. 2. The $\text{ARE}_{LO,EV}$

Because T_{LO} has a term containing $1/X_i$, T_{LO} may have large negative values in some cases, and an overflow can occur. The overflow may cause malfunctions. Therefore, we need to reduce the negative dynamic range by considering a modified version of the LO detector. It is interesting to consider a set of locally suboptimum (LS) detectors, whose test statistics are made from the LO statistic by replacing $1/X_i$

with $1/X_i^k$, $0 < k < 1$. The test statistic of an LS detector is thus

$$T_{LS}(X) = \sum_{i=1}^{n} \frac{1}{\sigma^2} X_i + (2m-1) \sum_{i=1}^{n} \left\{ \frac{1}{\sigma^2} X_i - \frac{1}{X_i^k} \right\}, \quad (31)$$

and the nonlinearity of the LS detector is

$$g_s(x) = \frac{2m}{\sigma^2} x - (2m-1) \frac{1}{x^k}, \quad (32)$$

which is shown in Figure 3. We can see that the LS nonlinearity has a shape similar to the LO nonlinearity.

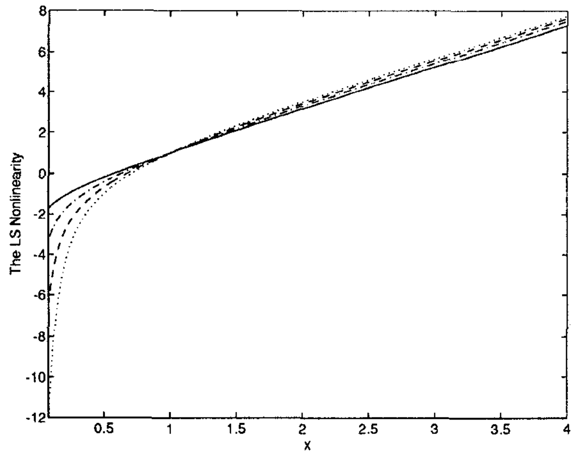

Fig. 3. The LS nonlinearity with m, $\sigma^2 = 1$ (solid: $k = 0.25$, dashdot: $k = 0.5$, dashed: $k = 0.75$, dotted: $k = 1$)

Noting that

$$V_H\{T_{LS}\} = E_H\{T_{LS}^2\} - E_H^2\{T_{LS}\} \quad (33)$$

and

$$\lim_{\theta \to 0} \frac{d}{d\theta} E_K\{T_{LS}\} = E_H\{T_{LS} T_{LO}\}, \quad (34)$$

we have

$$\xi_{LS} = \frac{\{2m\Gamma(m) + km^{(k+1)/2}(2m-1)\Gamma(m - \frac{k+1}{2})\}^2}{A\Gamma(m) - B^2}, \quad (35)$$

where

$$A = \{4m^2\Gamma(m) - 4m^{\frac{k+1}{2}}(2m-1)\Gamma(m + \frac{1-k}{2}) + (2m-1)^2 m^k \Gamma(m-k)\} \quad (36)$$

and

$$B = \{2m^{\frac{1}{2}}\Gamma(m+.5) - (2m-1)m^{k/2}\Gamma(m-.5k)\} \quad (37)$$

Note that the expression (35) for the efficacy of the LS detector is valid for all $m > (k+1)/2$. Figure 4 shows $\mathrm{ARE}_{LS,EV}$ as a function of m. We can see that the $\mathrm{ARE}_{LS,EV} > 1$. Note that ξ_{LS} is finite only for $m > (k+1)/2$.

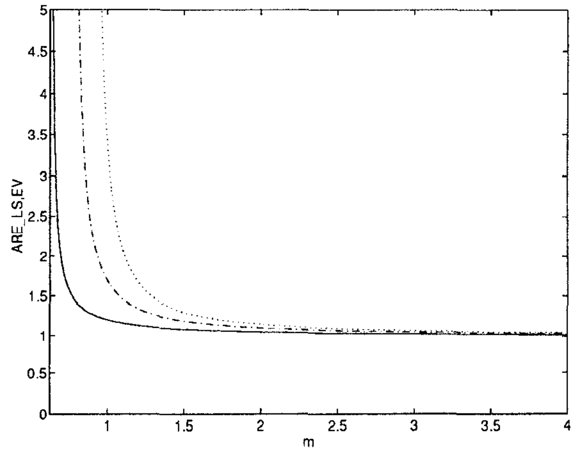

Fig. 4. The $\mathrm{ARE}_{LS,EV}$ with $\sigma^2 = 1$ (solid: $k = 0.25$, dashdot: $k = 0.5$, dotted: $k = 0.75$)

V. Concluding Remarks

In this paper, we applied the LO detection scheme to the user detection problem. This can be regraded as a special case of common random signal detection problems. We proposed an observation model, derived the LO and the LS detectors, and analyzed their asymptotic performance characteristic compared to those of the conventional EV detector. We also showed that the LO and LS detectors all outperform the EV detector asymptotically. It is noteworthy that the LS detector shows almost the same performance as the LO detector except when the fading is extremely severe.

Acknowledgements

The research was supported by Korea Research Foundation (KRF), Korea, and the Natural Science and Engineering Research Council (NSERC), Canada, for which the authors would like to express their thanks.

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