Performance of DS/SSMA Systems Using TCM under Impulsive Noise Environment

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Abstract

In this paper, we investigate the effects of impulsive noise on the DS/SSMA systems using TCM. The performance of the DS/SSMA systems using TCM under impulsive noise environment is analyzed. We obtain the bound of the probability of bit error of the systems, considering both impulsive noise and Rician fading unavoidable in mobile communication environments. It turns out that we can achieve some coding gain by using TCM under impulsive noise environment. It is observed that the bit error probability is dominated by the background noise variance when the SNR is low and by the tail noise variance when the SNR is high.

1 Introduction

The developments in communications technology these days have changed our daily life profoundly and rapidly. With mobile communication, we can overcome the restrictions on time and space, which were unavoidable in traditional communications. As is evident, the demand for mobile communication is increasing rapidly. Recently, code division multiple access (CDMA) becomes an interesting research area for its potential applications in mobile communication.

To get better communication systems, a number of modulation and coding techniques have been developed. Ungerboeck [1] has shown that optimally designed rate $n/(n+1)$ trellis codes mapped into the conventional 2^{n+1} point signal sets can provide some coding gain without bandwidth expansion. Following his work, many researchers have studied various aspects of the trellis coded modulation (TCM). For example, the performance of the DS/SSMA system using TCM has been studied in [2][3]. Most of the work on TCM have been accomplished under the additive white Gaussian noise (AWGN) assumption [1][3][4].

It is well-known that sometimes the Gaussian noise assumption cannot be entirely justified [5][6]. The non-Gaussian nature of atmospheric noise, impulses caused by turning-on some electrical devices, etc, are among the typical examples. Therefore, it is worthwhile to study the effects of impulsive noise on communication systems [7][8]. In this paper we will investigate the effects of impulsive noise on the DS/SSMA system using TCM. Since fading is generally present in spread spectrum systems, it will be assumed that the channel is a slow fading Rician channel with impulsive noise. We will obtain the upper bound on the probability of bit error for the M-ary PSK trellis coded DS/SSMA systems under the impulsive noise fading environment. We will then compare the bound with that obtained for the uncoded DS/SSMA system for various channel states.

2 System Model

A block diagram of the DS/SSMA system is illustrated in Fig. 1, which has been used frequently in previous studies of SSMA systems [e.g., 3]. In Fig. 1, it should be noted that the noise $n(t)$ in this paper is impulsive noise. Each user transmits using different spreading code, and the signal $s^k(t)$ transmitted by the kth user is assumed to be delayed randomly by the delay τ^k. The received signal consists of the desired signal and interference due to the noise and the signals transmitted by other users.

The transmitter system model for the trellis-coded DS/SSMA is illustrated in Fig. 2. The information bits, k_1 and k_2, select a coset of the signal constellation through the $n/(n+1)$ convolutional code. The selected signal multiplied by the DS sequence is modulated by the carrier and transmitted.

Now, we analyze the system: the notations in complex form in the following developments are based on those in [2][3]. Let the kth user's complex baseband information signal be

$$x^k(t) = \sum_{p=-\infty}^{\infty} x_p^k P_T(t-pT), \tag{1}$$

where T is the symbol period, $P_T(\cdot)$ is a rectangular pulse with duration T, and x_p^k is the complex baseband symbol of the kth user during the pth symbol period which is determined by the coset selection. Similarly, the DS chip signal is defined as

$$a^k(t) = \sum_{m=-\infty}^{\infty} a_m^k \Psi(t-mT_c), \tag{2}$$

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where a_m^k is the mth chip of the kth user, and $\Psi(\cdot)$ is the chip waveform with duration T_c. Then, the transmitted signal modulated by a carrier with frequency f_c is

$$s^k(t) = \sqrt{\frac{2E_s}{T}} Re\{\zeta^k(t) exp[j(w_c t + \psi^k)]\}, \quad (3)$$

where $\zeta^k(t) = a^k(t)x^k(t)$ and ψ^k is the random phase of the kth carrier. The received signal can be written as

$$\begin{aligned} r(t) &= \sqrt{\frac{2E_s}{T}} Re\{\sum_{k=1}^{K}\sum_{i} \gamma_i^k(t)\zeta^k(t-\tau_i^k) \\ &\quad \times exp[j(w_c t + \beta_i^k)]\} + \eta(t), \end{aligned} \quad (4)$$

where $\gamma_i^k(t)$ is the ith attenuation factor of the kth user signal, τ_i^k is the ith random delay of the kth user at the receiver, $\beta_i^k = \psi^k - w_c\tau_i^k$ is the ith random phase of the kth user at the receiver, and $\eta(t)$ is impulsive noise.

After the demodulation through the I and Q demodulators, the sampled received sequence Y_p can be written as [2]

$$Y_p = \rho_p X_p + Z_p + \eta_p, \quad (5)$$

where ρ_p is the fading envelope, $X_p = \sqrt{E_s}x_p$, Z_p is the inter-user interference, and η_p is the impulsive noise.

3 Impulsive Noise Environment

In communication channels, noise usually has impulsiveness. One typical example is lightening: in addition, we are nowadays surrounded by many electrical devices, which also produce impulsive noise when turned on. In this paper, we model the impulsive noise with the ϵ-contaminated noise model [5][6][7][8], for which the pdf is

$$f(x) = (1-\epsilon)f_B(x) + \epsilon f_T(x). \quad (6)$$

In (6), the background noise pdf f_B is a Gaussian pdf with mean zero and variance ${\sigma_B}^2$, and the tail noise pdf f_T is in general a zero-mean pdf which has a variance σ_T^2 larger than ${\sigma_B}^2$. In this paper, we assume that f_B and f_T are Gaussian pdfs of equal mean, in which case we use the notation $\alpha(x;\epsilon, m_f, \sigma_B^2, \sigma_T^2)$ to denote the ϵ-contaminated pdf, with m_f the mean of f_B and f_T. The mean and variance of the impulsive noise with pdf (6) can be obtained as

$$E\{\eta\} = 0 \quad (7)$$

and

$$\begin{aligned} E\{\eta^2\} &= \int_{-\infty}^{\infty} x^2 f(x)dx \\ &= (1+(M-1)\epsilon){\sigma_B}^2, \end{aligned} \quad (8)$$

where $M = {\sigma_T}^2/{\sigma_B}^2$. We now define the signal to noise ratio as

$$\begin{aligned} SNR &= \frac{E_s}{2E\{\eta^2\}} \\ &= \frac{nE_b}{2(1+(M-1)\epsilon){\sigma_B}^2}. \end{aligned} \quad (9)$$

The characteristics of the impulsive noise are different for different values of ϵ and M even when the SNR's are equal. This implies that the values of ϵ and M should be specified to investigate the effects of the impulsive noise.

Next, we define the signal to background noise ratio (SBR) as

$$SBR = \frac{nE_b}{2{\sigma_B}^2}. \quad (10)$$

If ϵ is very small ($(M-1)\epsilon \ll 1$), the SBR can be a good approximation to the SNR. It is easy to see that the SBR is useful when we want to know the effects of the contamination ratio (ϵ) on the performance of the system.

4 Performance Analysis for Rician Fading Channels

In slow fading channels, the received sequence can be written as

$$Y_p = \rho_p X_p + Z_p + \eta_p. \quad (11)$$

Now, we assume [2] that the inter-user interference Z_p is a Gaussian random variable independent of η_p by central limit theorem. Then

$$\bar{\eta_p} = Z_p + \eta_p \quad (12)$$

is also a random variable with an ϵ-contaminated pdf. From now on, we drop the bar of $\bar{\eta_p}$ for convenience.

In (11), ρ_p is the fading envelope which has the pdf

$$\begin{aligned} f_\rho(x) &= 2x(1+K)exp[-\{K + x^2(1+K)\}] \\ &\quad \times I_0(2x\sqrt{K(K+1)}), \end{aligned} \quad (13)$$

where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind, and K is the Rician parameter defined as the ratio of the energy of the direct component to the energy of the diffused multipath components.

First, we assume that we have an information about the channel state. This assumption is valid when we use a pilot tone and the estimation of the fading parameter is assumed to be perfect. Then the pairwise error probability given $\rho = (\rho_1, \rho_2, \cdots, \rho_N)$ can be obtained as

$$\begin{aligned} P(x \to \tilde{x}|\rho) &= Pr\{\sum_p (\|Y_p - \rho_p X_p\|^2 \\ &\quad -\|Y_p - \rho_p \tilde{X}_p\|^2) \geq 0\} \\ &= Pr\{\sum_p [-E_s \rho_p{}^2 \|d_p\|^2 - 2\sqrt{E_s} \\ &\quad \times \rho_p Re\{\eta_p d_p^*\}] \geq 0\}. \quad (14) \end{aligned}$$

where $d_p = x_p - \tilde{x_p}$. We can simplify (14) as

$$P(x \to \tilde{x}|\rho) = Pr\{-E_s \bar{d}^2 + 2\sqrt{E_s}\bar{u} \geq 0\}, \quad (15)$$

where $\bar{d}^2 = \sum_p \rho_p{}^2 \|d_p\|^2$ and $\bar{u} = -\sum_p \rho_p Re\{\eta_p d_p^*\}$. By using properties in [9], we get

$$\begin{aligned} & P(x \to \tilde{x}|\rho) \\ \leq\ & Pr\{\hat{u} \geq 0\} \\ =\ & (1-\epsilon)\int_0^\infty \phi(x; -E_s \bar{d}^2, 4E_s d^2 \sigma_B{}^2)dx \\ & +\epsilon \int_0^\infty \phi(x; -E_s \bar{d}^2, 4E_s d^2 \sigma_T{}^2)dx, \quad (16) \end{aligned}$$

where $\hat{u}$ has the pdf $f_{\hat{u}}(x) = \alpha(x; \epsilon, -E_s \bar{d}^2, 4E_s d^2 \sigma_B{}^2, 4E_s d^2 \sigma_T{}^2)$ and $\phi(x; m, \sigma^2)$ is the Gaussian pdf with mean m and variance σ^2. Applying Chernoff bound to (16), we get

$$P(x \to \tilde{x}|\rho) \leq (1-\epsilon) D_B{}^{\bar{d}^2}(\lambda_1) + \epsilon D_T{}^{\bar{d}^2}(\lambda_2). \quad (17)$$

where $D_B(\lambda_1) = exp[-E_s\lambda_1 + 2E_s\sigma_B{}^2\lambda_1{}^2]$, $D_T(\lambda_2) = exp[-E_s\lambda_2 + 2E_s\sigma_T{}^2\lambda_2{}^2]$, and λ_1 and λ_2 are Chernoff parameters. The optimum values of λ_1 and λ_2 and the minimum bounds D_B and D_T can be obtained to be

$$\lambda_{1,opt} = \frac{1}{4\sigma_B{}^2}, \quad (18)$$

$$\lambda_{2,opt} = \frac{1}{4\sigma_T{}^2}, \quad (19)$$

$$D_B = exp[-\frac{E_s}{8\sigma_B{}^2}], \quad (20)$$

and

$$D_T = exp[-\frac{E_s}{8\sigma_T{}^2}]. \quad (21)$$

Thus, substituting (18)-(21) into (17), we get

$$P(x \to \tilde{x}|\rho) \leq (1-\epsilon) D_B^{\bar{d}^2} + \epsilon D_T^{\bar{d}^2}. \quad (22)$$

The unconditioned pairwise error probability bound is then obtained by taking the expectation of (22) over ρ as

$$P(x \to \tilde{x}) \leq (1-\epsilon) E\{D_B^{\bar{d}^2}\} + \epsilon E\{D_T^{\bar{d}^2}\}, \quad (23)$$

and the bit error probability bound is obtained as

$$\begin{aligned} P_b \ \leq\ & (1-\epsilon) \cdot \frac{1}{2n} \frac{\partial \bar{T}(D,I)}{\partial I}|_{D=D_B, I=1} \\ & +\epsilon \cdot \frac{1}{2n} \frac{\partial \bar{T}(D,I)}{\partial I}|_{D=D_T, I=1}, \quad (24) \end{aligned}$$

where $\bar{T}(D,I)$ is the transfer function of the super state diagram whose branch label gains are modified based on the discussion in [4], and the factor $\frac{1}{2}$ is inserted based on the discussion on the error bound in [10]. It should be noted that for $\epsilon = 0$, the impulsive noise becomes Gaussian noise and (24) becomes the same result as that shown in [4].

5 Numerical Results

Now we consider the performance of the 2-state 1/2 rate 4-PSK trellis coded DS/SSMA system under the impulsive noise environment.

The bit error probabilities for various K when $M = 10$ and $\epsilon = 0.01$ are shown in Fig. 3. It is natural that the performance gets worse as the fading gets more severe. The bit error probability for various ϵ when $M = 10$ and $K = 0, 10, 50$ are shown in Figs. 4-6, respectively. We can see that the bit error probability increases as ϵ increases for fixed M and K. We can also see that the bit error probability is dominated by the first term of (24) at low SNR and by the second term at high SNR : the transition point is around the SNR which makes the bit error probability ϵ.

It should be noted that as the fading becomes less severe (K gets larger) the performance depends more on the value of ϵ.

6 Conclusion

In this paper we have studied the effects of impulsive noise on the performance of 2^{n+1}-PSK trellis coded DS/SSMA systems. We analyzed the system performance and obtained the bit error probability bound under the impulsive noise environment considering the effects of Rician fading which was unavoidable in mobile communication systems.

From the results, it was observed that some SNR gain could be obtained by using TCM scheme in DS/SSMA system under the impulsive noise environment as in the AWGN case. It was also observed that the bit error probability is dominated by background noise variance at low SNR and by tail noise variance at high SNR. For the 2 state 1/2 rate 4-PSK TCM scheme, approximately 2dB gain could be obtained using TCM.

Acknowledgements

This research was supported in part by the Ministry of Information and Communications under a Grant from the University Basic Research Fund, for which the authors would like to express their thanks.

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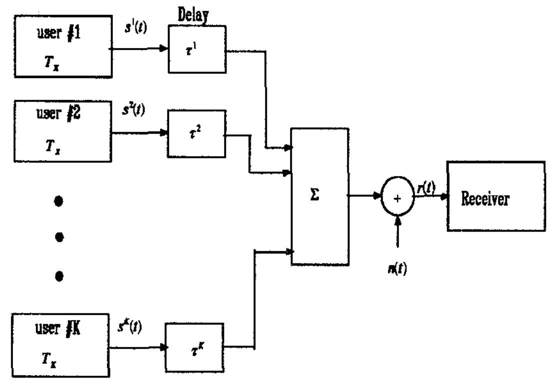

Figure 1: A block diagram of the general system architecture.

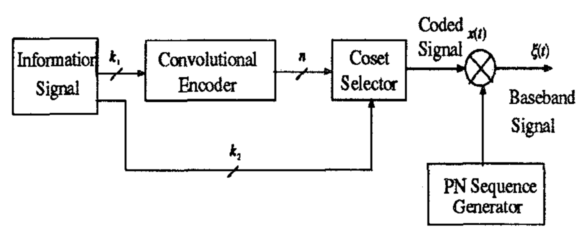

Figure 2: The transmitter system model for the trellis-coded DS/SSMA.

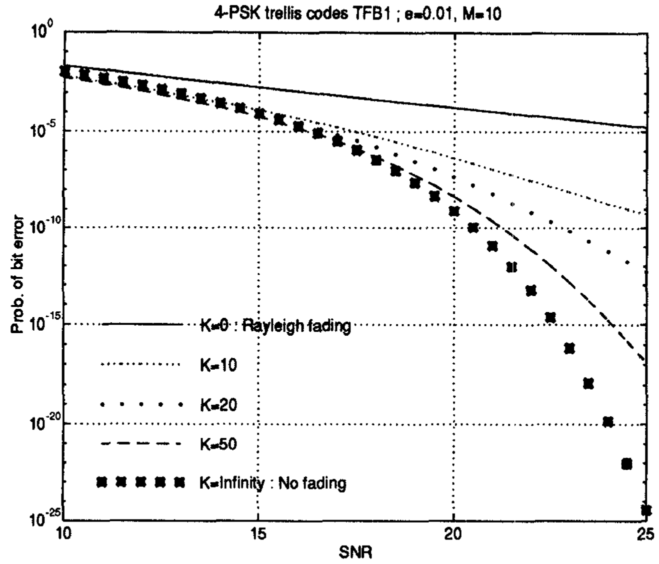

Figure 3: The bit error probability for various K when $M = 10$ and $\epsilon = 0.01$.

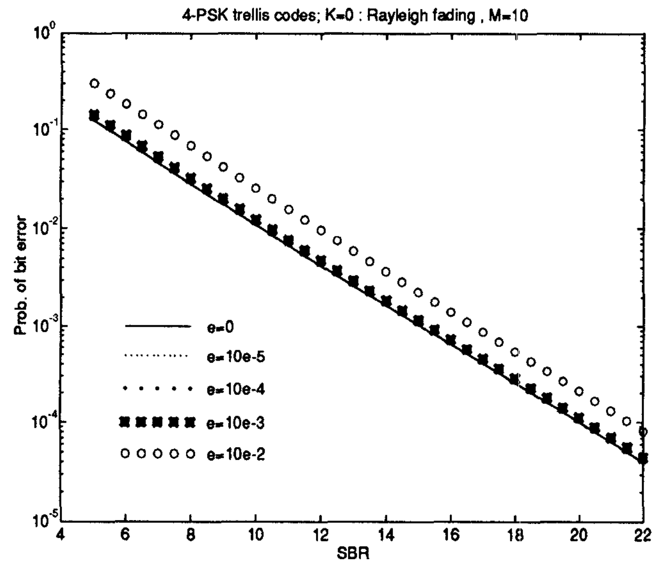

Figure 4: The bit error probability for various ϵ when $K = 0$ and $M = 10$.

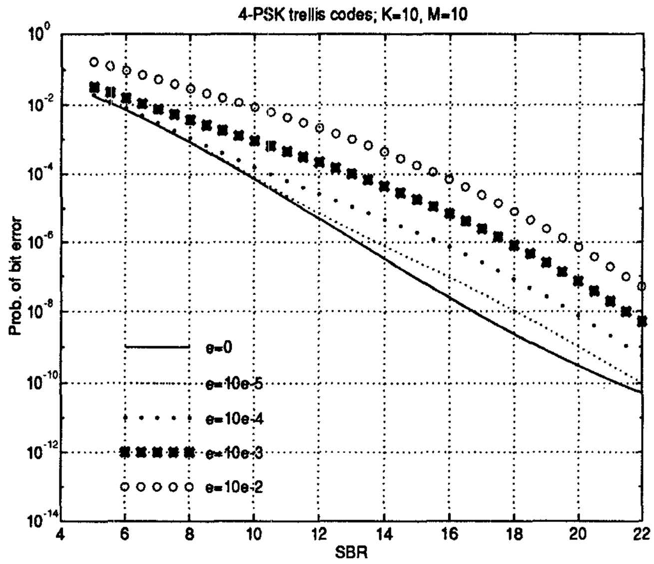

Figure 5: The bit error probability for various ϵ when $K = 10$ and $M = 10$.

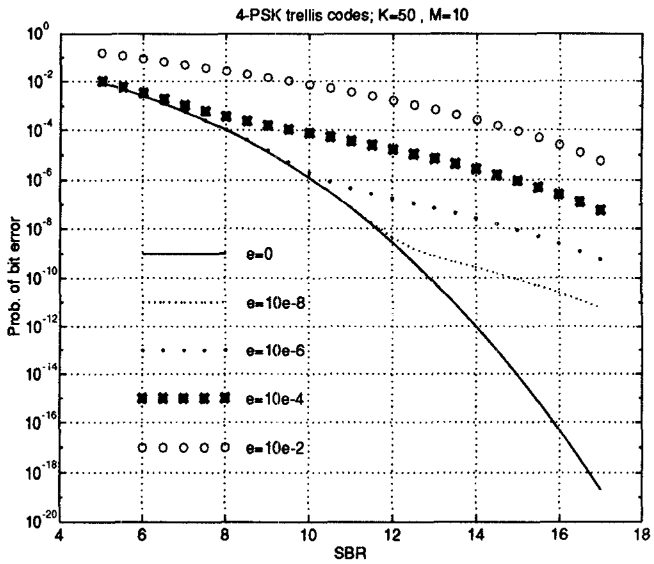

Figure 6: The bit error probability for various ϵ when $K = 50$ and $M = 10$.