A Turbo-BLAST method with Non-Linear MMSE Detector for MIMO-OFDM systems

Jong Yoon Hwang and
Dong-Kyoon Cho
Telecommunication R & D Center,
Samsung Electronics,
Suwon-shi, Kyongki-do, Korea
Email: donhwang@samsung.com

Kwang Soon Kim and
Kuem-Chan Whang
Department of Electrical and
Electronic Engineering,
Yonsei University,
134 Shinchon-dong, seodaemun-gu,
Seoul, 120-749, Korea
Email:ks.kim@yonsei.ac.kr

***Abstract*— A new and improved turbo processing multi-input multi-output (MIMO) system using a non-linear minimum mean square error (MMSE) detector is proposed. Based on the non-linear mean square (MS) estimation, the *a posteriori* information of a soft-input soft-output (SISO) decoder is fed back to the MMSE detector instead of the *extrinsic* information. Unlike a MAP (Maximum a Posteriori) detector, the MMSE detector shows no error propagation caused from using the *a posteriori* as an *a priori* information. The proposed turbo process with non-linear MMSE detector needs *a posteriori* information, which is easily obtained from turbo SISO decoder without increasing receiver complexity. The performance of the proposed method is compared with conventional methods by using computer simulation in a Mobile Wimax system environment.**

I. INTRODUCTION

The turbo processing, or Turbo-BLAST (Bell Labs layered space-time) architecture in [1, 2] has been known as an effective method in utilizing the mutual effect of the soft-input soft-output (SISO) decoder and the multi-input multi-output (MIMO) detector to achieve close performance to Shannon limit. In [1], the MIMO detector with a sphere decoder based on the MAP algorithm was proposed. To reduce the high computational complexity of the optimal MAP detection, the MMSE detector was used as a MIMO detector in [2], where the *extrinsic* information of the SISO decoder is fed back to the MMSE detector as the *intrinsic* information for turbo processing. Then, the MMSE detector estimates soft interference from the *extrinsic* probabilities of coded bits and performs the soft interference cancellation with the weights based on MMSE principle. In this paper, we propose a turbo processing MIMO system using a non-linear MMSE detector. Based on the non-linear MS estimation, the *a posteriori* information of the SISO decoder is fed back to the MMSE detector instead of the *extrinsic* information. The reason that we use the non-linear MS estimation is illustrated by using the estimation of random variable (RV) $\mathbf{y}$ in terms of another RV $\mathbf{x}$. If RV $\mathbf{y}$ is estimated by a constant u to minimize the MS error $E[(\mathbf{y}-u)^2]$, the optimal value of u is $E[\mathbf{y}]$. However, if we suppose that u depends on x and we can observe the value x of the RV $\mathbf{x}$ at each time, the optimal value of $u(x)$ is $E[\mathbf{y}|x]$. The information of x can reduce the uncertainty region in estimating $\mathbf{y}$ [3]. This paper is organized as follows. Section 2 describes the improved turbo processing system with the non-linear MMSE detector. Section 3 provides simulation model and results. Finally, section 4 presents a conclusion.

II. IMPROVED TURBO PROCESSING SYSTEM

We consider a turbo processing system of N transmitting and M receiving antennas with a tail-biting turbo code of rate $1/2$ duo-binary recursive systematic convolutional (RSC) code as shown in Fig. 1 and Fig. 2. $b(i)$, $i \in \{1,\ldots,L\}$, means the information bits, and L is the encoding packet length. The coded bits $c(j)$, $j \in \{1,\ldots,2L\}$, are interleaved, divided into each substream, quaternary phase shift keying (QPSK) modulated, Inverse Fast Fourier Transformed (IFFT) and transmitted through MIMO channels. As is shown in Fig. 2, the turbo processing system of a MIMO receiver is composed of two modules: the MMSE detector and the turbo decoder. $\lambda_l(c;p)$ and $\lambda_l(c;e)$ at the output of SISO module $l(l=1,2)$ are the *a posteriori* and the *extrinsic* information of log-likelihood ratio (LLR), respectively.
Assuming proper cyclic prefix operation and discrete Fourier transform (DFT), the received signal vector $\mathbf{r}[k]$ at the kth subcarrier is given by

$$\mathbf{r}[k] = \mathbf{H}[k]\mathbf{a}[k] + \mathbf{v}[k], \qquad k = 1,\ldots,K, \tag{1}$$

where $\mathbf{r}[k] = [r_1[k]r_2[k]\ldots r_M[k]]^T, \mathbf{H}[k]$ is the channel impulse response matrix of size $M \times N$, $\mathbf{a}[k]$ is the transmitted symbol vector as $\mathbf{a}[k] = [a_1[k]a_2[k]\ldots a_N[k]]^T$, and $\mathbf{v}[k]$ is a vector of independent zero-mean complex Gaussian noise with variance σ^2 as $\mathbf{v}[k] = [v_1[k]v_2[k]...v_M[k]]^T$.
Let $\mathbf{a}[k]$ be the desired symbol. We may write (1) as

$$\mathbf{r}[k] = \mathbf{h}_n[k]a_n[k] + \mathbf{H}_n[k]\mathbf{a}_n[k] + \mathbf{v}[k], \tag{2}$$

where $\mathbf{H}_n[k] = [\mathbf{h}_1[k], \mathbf{h}_2[k], \ldots, \mathbf{h}_{n-1}[k], \mathbf{h}_{n+1}[k], \ldots, \mathbf{h}_N[k]]$ and $\mathbf{a}_n[k] = [a_1[k], a_2[k], \ldots, a_{n-1}[k], a_{n+1}[k], \ldots, a_N[k]]^T$ are the interfering channel matrix and the vector of interfering symbol for the nth substream, respectively. The decision statistic of the nth substream using a linear filter $\mathbf{w}_n$ is

$$\begin{aligned} y_n[k] &= \mathbf{w}_n^H[k]\mathbf{h}_n[k]a_n[k] + \mathbf{w}_n^H[k]\mathbf{H}_n[k]\mathbf{a}_n[k] + \mathbf{w}_n^H[k]\mathbf{v}[k] \\ &= d_n[k] + u_n[k] + n_n[k], \end{aligned} \tag{3}$$

where $d_n[k], u_n[k]$ and $n_n[k]$ are the desired response, the co-antenna interference (CAI) and phase-rotated noise, respectively. As in [2], we remove CAI from the linear beamformer output $y_n[k]$ and write

$$x_n[k] = \mathbf{w}_n^H \mathbf{r}[k] - u_n[k], \quad (4)$$

where $x_n[k]$ is the estimate of the transmitted symbol $a_n[k]$. In this paper, we propose to use nonlinear MS estimation so as to find the vector $\mathbf{w}_n(\mathbf{x}[k])$ and $u_n[k](\mathbf{x}[k])$ by minimizing the cost function

$$\begin{aligned}&(\hat{\mathbf{w}}_n(\mathbf{x}[k]), \hat{u}_n[k](\mathbf{x}[k])) \\ &= \arg\min_{(\mathbf{w}_n, u_n[k])} E\left[\|\mathbf{w}_n^H(\mathbf{x}[k])\mathbf{r}[k] - u_n[k](\mathbf{x}[k]) - a_n[k]\|^2\right] \quad (5)\end{aligned}$$

where $\mathbf{x}[k] = [x_1[k], x_2[k], \ldots, x_N[k]]^T$ and the expectation is over noise, the statistics of the data sequence and RV of the observation $\mathbf{x}[k]$. The cost function C can be written as

$$\begin{aligned}C =\ & E\left[\|\mathbf{w}_n^H(\mathbf{x}[k])\mathbf{r}[k] - u_n[k](\mathbf{x}[k]) - a_n[k]\|^2\right] \\ =& \iiint \left(\mathbf{w}_n^H(\mathbf{x}[k])\mathbf{r}[k] - u_n[k](\mathbf{x}[k]) - a_n[k]\right)^2 \\ & \cdot f\left(\mathbf{a}[k], \mathbf{v}[k], \mathbf{x}[k]\right) d\mathbf{a}[k] d\mathbf{v}[k] d\mathbf{x}[k] \\ =& \int f\left(\mathbf{x}[k]\right) \iint \left(\mathbf{w}_n^H\left(\mathbf{x}[k]\right)\mathbf{r}[k] - u_n[k]\left(\mathbf{x}[k]\right) - a_n[k]\right)^2 \\ & \cdot f\left(\mathbf{a}[k], \mathbf{v}[k] | \mathbf{x}[k]\right) d\mathbf{a}[k] d\mathbf{v}[k] d\mathbf{x}[k] \quad (6)\end{aligned}$$

The integrands above are positive. Hence C is minimum if the inner integral is minimum for every observation $\mathbf{x}[k]$. The inner integral can be written as

$$\begin{aligned}&\iint \left(\mathbf{w}_n^H\left(\mathbf{x}[k]\right)\mathbf{r}[k] - u_n[k]\left(\mathbf{x}[k]\right) - a_n[k]\right)^2 \\ &\quad \cdot f\left(\mathbf{a}[k], \mathbf{v}[k] | \mathbf{x}[k]\right) d\mathbf{a}[k] d\mathbf{v}[k] \\ &= E\left[\|\mathbf{w}_n^H(\mathbf{x}[k])\mathbf{r}[k] - u_n[k](\mathbf{x}[k]) - a_n[k]\|^2 | \mathbf{x}[k]\right] \quad (7)\end{aligned}$$

The weight vector $\mathbf{w}_n(\mathbf{x}[k])$ and $u_n(\mathbf{x}[k])$ can be found by minimizing the conditional cost function in (7). This expectation is of the form in [2] if $u_n[k]$ is changed to $u_n[k](\mathbf{x}[k]), \mathbf{w}_n$ is changed to $\mathbf{w}_n(\mathbf{x}[k])$, and $f\left(\mathbf{a}[k], \mathbf{v}[k]\right)$is changed to $f\left(\mathbf{a}[k], \mathbf{v}[k] | \mathbf{x}[k]\right)$. Hence it is minimum if $u_n[k](\mathbf{x}[k])$ and $\mathbf{w}_n(\mathbf{x}[k])$ equal to the expectation in [2], provided that $E\left[\mathbf{a}_n[k]\right]$ is changed to $E\left[\mathbf{a}_n[k] | \mathbf{x}[k]\right]$. The result is as follow.

$$\begin{aligned}\hat{u}_n[k](\mathbf{x}[k]) &= \mathbf{w}_n^H(\mathbf{x}[k])\mathbf{H}_n[k] E\left[\mathbf{a}_n | \mathbf{x}[k]\right] \\ \hat{\mathbf{w}}_n(\mathbf{x}[k]) &= \Bigg(\mathbf{h}_n[k]\mathbf{h}_n^H[k] + \mathbf{H}_n[k]\Big[\mathbf{I}_{N-1} \\ &\quad - \ Diag\Big[E\big[\mathbf{a}_n[k]|\mathbf{x}[k]\big] E\big[\mathbf{a}_n[k]|\mathbf{x}[k]\big]^H\Big]\Big]\mathbf{H}_n^H[k] \\ &\quad + \ \sigma^2 \mathbf{I}_M\Bigg)^{-1} \mathbf{h}_n[k]. \quad (8)\end{aligned}$$

By assuming that the coded bits of transmitted symbol $a_n[k]$ are independent each other, the conditional expectation of transmitted symbol $a_n[k]$ can be modulated as like QPSK modulation from the conditional expectation of coded bits as

$$\begin{aligned}&E\left[a_n[k] | \mathbf{x}[k]\right] \\ &= \frac{1}{\sqrt{2}}\left(E\left[Re\left(a_n[k]\right) | \mathbf{x}[k]\right] + jE\left[Im(a_n[k]) | \mathbf{x}[k]\right]\right). \quad (9)\end{aligned}$$

We use the turbo decoder to provide the *a posteriori* probability of the coded bit *c* as

$$\begin{aligned}&E\left[c | \mathbf{x}[k]\right] \\ &= E\left[Re\left(a_n[k]\right) | \mathbf{x}[k]\right] \text{ or } E\left[Im\left(a_n[k]\right) | \mathbf{x}[k]\right] \\ &= (+1)p\left(c = +1 | \mathbf{x}[k]\right) + (-1)p\left(c = -1 | \mathbf{x}[k]\right) \\ &= \frac{(+1)\lambda_2(c;p)}{1 + exp(\lambda_2(c;p))} + \frac{(-1)}{1 + exp(\lambda_2(c;p))} \\ &= tanh\left(\frac{\lambda_2(c;p)}{2}\right) \quad (10)\end{aligned}$$

The proposed turbo processing system using the non-linear MMSE detector is depicted in Fig. 3. Based on the non-linear MMSE detector, the *a posteriori* information $\lambda_2(c;p)$ of the turbo decoder is used for the feedback information to MMSE detector, instead of the *extrinsic* information $\lambda_2(c;e)$. The *extrinsic* information $\lambda_1(c;e)$ to be delivered to the turbo decoder can be derived by approximating the error at the MMSE detector output by a Gaussian process without regarding the feedback information [4].

III. Simulation Model and Results

We compare the performance of the proposed method with the conventional turbo-BLAST method in [2] and a non-turbo MMSE method in the IEEE 802.16e Mobile Wimax environment [5]. The performance measure is packet error rate (PER) over received Signal-to-Noise Ratio (SNR). The RSC encoder with octal generators (13, 15, 11) is used, namely the generating polynomials are $1 + D + D^3$ for feedback loop, $1 + D^2 + D^3$ for Y parity bits and $1 + D^3$ for W parity bits. We assume the number of transmitting antenna is same to the number of receiving antenna and the receiver performs real channel estimation of 2×2 MIMO channels based on the Mobile Wimax system. Used Channel models are Veh. A (60km/h) and Ped. B (10km/h) of ITU-R Recommendations. Used data burst profile is as follows. FFT size is 1024, Modulation is QPSK with 1/2 coding rate and the number of subchannels (each subchannel has 48 subcarriers) is 5. Each encoding packet contains 480 information bits and is composed of 480×2 coded bits. For each encoding packet, the receiver performs six iterations over MIMO detection loop and six iterations within the turbo decoder of MAX logMAP algorithm. In Fig. 4 and Fig. 5, the PER performances of the turbo processing system are shown. MMSE shows the performance without turbo processing. The proposed system with a non-linear MMSE detector can improve the PER performance by more than 0.6dB in the channel of Veh. A and 0.3dB in the channel of Ped. B over the conventional system with a linear MMSE detector at 1% PER in MIMO channels.

IV. Conclusion

In this paper, we described and evaluated the turbo processing system with a non-linear MMSE detector. We propose to use the *a posteriori* information of the turbo decoder as a feedback information to MMSE detector instead of the

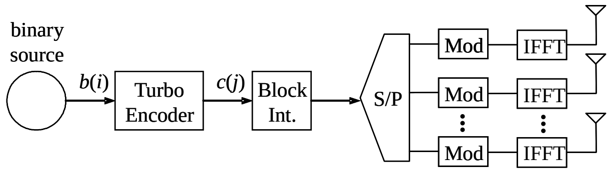

Fig. 1. Transmitter of turbo processing system

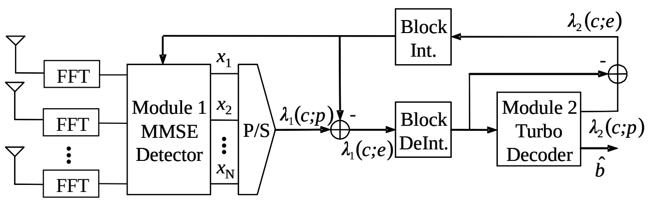

Fig. 2. Conventional turbo processing system with linear MMSE detector

extrinsic information. The simulation results show that the PER performance of the proposed system is better than that of the conventional system with the MMSE detector based on a linear MS estimation.

Acknowledgment

This paper is supported by SAMSUNG Electronics.

References

[1] B. Hochwald and S. Ten Brink, "Achieving Near-Capacity on a Multiple-Antenna Channel," *IEEE Trans. Commun.*, vol. 51, no. 3, pp. 389–399, March 2003.

[2] M. Sellathurai and S. Haykin, "TURBO-BLAST for Wireless Communications: Theory and Experiments," *IEEE Trans. Commun.*, vol. 50, no. 10, pp. 2538–2546, October 2002.

[3] Athanasios Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw-Hill, New York, 3rd edition, 1991.

[4] X. Wang and H. V. Poor, "Iterative (Turbo) Soft Interference Cancellation and Decoding for Coded CDMA," *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1046–1061, 1999

[5] IEEE Std. 802.16e-2005, Part 16: Air Interface for Fixed and Mibile Broadband Wireless Access Systems - Amendment 2 for Physical and Medium Access Control Layers for Combined Fixed and Mobile operation in Licensed Bands, Feb., 2006.

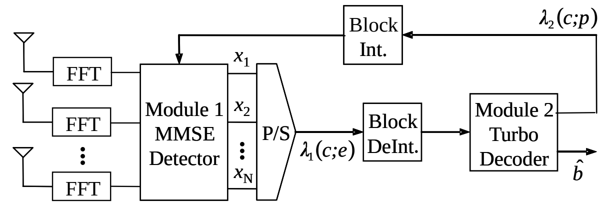

Fig. 3. Proposed turbo processing system with non-linear MMSE detector

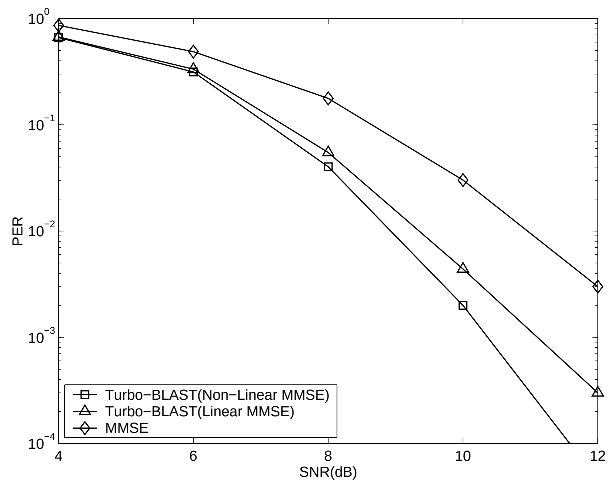

Fig. 4. PER performance of the turbo processing systems with MMSE detector in the channel of Veh. A (60km/h) of ITU-R

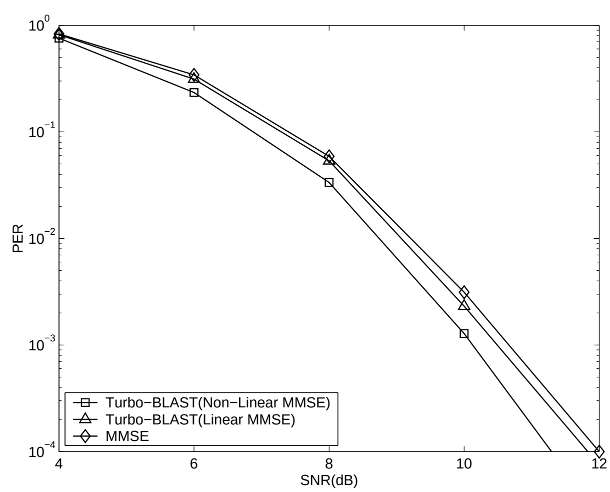

Fig. 5. PER performance of the turbo processing systems with MMSE detector in the channel of Ped. B (10km/h) of ITU-R