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DS/SSMA Systems Using TCM With Asymmetric PSK Signal Constellation Under Impulsive Noise Environment

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Abstract

In this paper, we investigate the effects of impulsive noise on the DS/SSMA systems using TCM. The performance of the DS/SSMA systems using TCM under impulsive noise environment is analyzed. We obtain a bound on the probability of bit error of the systems, considering both impulsive noise and Rician fading unavoidable in mobile communication environments. It is observed that the bit error probability is dominated by the background noise variance when the SNR is low and by the tail noise variance when the SNR is high. We also see that we can get some gain by using optimally designed asymmetric PSK signal constellation.

1 Introduction

The developments in communications technology these days have changed our daily life profoundly and rapidly. With mobile communication, we can overcome the restrictions on time and space, which were unavoidable in traditional communications. As is evident, the demand for mobile communication is increasing rapidly. Recently, code division multiple access (CDMA) becomes an interesting research area for its potential applications in mobile communication.

To get better communication systems, a number of modulation and coding techniques have been developed. Ungerboeck [1] has shown that optimally designed rate $n/(n+1)$ trellis codes mapped into the conventional 2^{n+1} point signal sets can provide some coding gain without bandwidth expansion. Following his work, many researchers have studied various aspects of the trellis coded modulation (TCM). For example, the performance of the DS/SSMA system using TCM has been studied in [2][3]. Most of the work on TCM have been accomplished under the additive white Gaussian noise (AWGN) assumption [1][3][4][5].

It is well-known that sometimes the Gaussian noise assumption cannot be entirely justified [6]. The non-Gaussian nature of atmospheric noise, impulses caused by turning-on some electrical devices, etc, are among the typical examples. Therefore, it is worthwhile to study the effects of impulsive noise on communication systems [7]. In this paper we will investigate the effects of impulsive noise on the DS/SSMA system using TCM. Since fading is generally present in spread spectrum systems, it will be assumed that the channel is a slow fading Rician channel with impulsive noise. We will obtain the upper bound on the probability of bit error for the M-ary PSK trellis coded DS/SSMA systems under the impulsive noise fading environment. Optimum asymmetric PSK constellation will be investigated in bit error probability sense.

2 System Model

The transmitter system model for the trellis-coded DS/SSMA is illustrated in Fig. 1. The information bits, k_1 and k_2, select a coset of the signal constellation through the $n/(n+1)$ convolutional code. The selected signal multiplied by the DS sequence is modulated by the carrier and transmitted. The signal constellation considered in this paper is the antipodal 2^n-PSK, and, as an example, the symmetric and asymmetric 4-PSK signal constellations are illustrated in Fig. 2.

Now, we analyze the system: the notations in complex form in the following developments are based on those in [2][3]. Let the kth user's complex baseband information signal be

$$x^k(t) = \sum_{p=-\infty}^{\infty} x_p^k P_T(t-pT), \qquad (1)$$

where T is the symbol period, $P_T(\cdot)$ is a rectangular pulse with duration T, and x_p^k is the complex baseband symbol of the kth user during the pth symbol period which is determined by the coset selection. Similarly, the DS chip signal is defined as

$$a^k(t) = \sum_{m=-\infty}^{\infty} a_m^k \Psi(t-mT_c), \qquad (2)$$

where a_m^k is the mth chip of the kth user, and $\Psi(\cdot)$ is the chip waveform with duration T_c. Then, the transmitted signal modulated by a carrier with frequency f_c is

$$s^k(t) = \sqrt{\frac{2E_s}{T}} Re\{\zeta^k(t) exp[j(w_c t + \psi^k)]\}, \quad (3)$$

where $\zeta^k(t) = a^k(t)x^k(t)$ and ψ^k is the random phase of the kth carrier. The received signal can be written as

$$r(t) = \sqrt{\frac{2E_s}{T}} Re\{\sum_{k=1}^{K}\sum_{i} \gamma_i^k(t)\zeta^k(t-\tau_i^k) \times exp[j(w_c t + \beta_i^k)]\} + \eta(t), \quad (4)$$

where $\gamma_i^k(t)$ is the ith attenuation factor of the kth user signal, τ_i^k is the ith random delay of the kth user at the receiver, $\beta_i^k = \psi^k - w_c\tau_i^k$ is the ith random phase of the kth user at the receiver, and $\eta(t)$ is impulsive noise.

After the demodulation through the I and Q demodulators, the sampled received sequence Y_p can be written as [2]

$$Y_p = \rho_p X_p + Z_p + \eta_p, \quad (5)$$

where ρ_p is the fading envelope, $X_p = \sqrt{E_s}x_p$, Z_p is the inter-user interference, and η_p is the impulsive noise.

3 Impulsive Noise Environment

In some communication channels, noise usually has impulsiveness. One typical example is lightening: in addition, we are nowadays surrounded by many electrical devices, which also produce impulsive noise when turned on. In this paper, we model the impulsive noise with the ϵ-contaminated noise model [6][7], for which the pdf is

$$f(x) = (1-\epsilon)f_B(x) + \epsilon f_T(x). \quad (6)$$

In (6), the background noise pdf f_B is a Gaussian pdf with mean zero and variance ${\sigma_B}^2$, and the tail noise pdf f_T is in general a zero-mean pdf which has a variance σ_T^2 larger than ${\sigma_B}^2$. In this paper, we assume that f_B and f_T are Gaussian pdfs of equal mean, in which case we use the notation $\alpha(x;\epsilon, m_f, \sigma_B^2, \sigma_T^2)$ to denote the ϵ-contaminated pdf, with m_f the mean of f_B and f_T. The mean and variance of the impulsive noise with pdf (6) can be obtained as

$$E\{\eta\} = 0 \quad (7)$$

and

$$\begin{aligned} E\{\eta^2\} &= \int_{-\infty}^{\infty} x^2 f(x)dx \\ &= (1+(M-1)\epsilon){\sigma_B}^2, \end{aligned} \quad (8)$$

where $M = \frac{{\sigma_T}^2}{{\sigma_B}^2}$. We now define the signal to noise ratio as

$$\begin{aligned} SNR &= \frac{E_s}{2E\{\eta^2\}} \\ &= \frac{nE_b}{2(1+(M-1)\epsilon){\sigma_B}^2}. \end{aligned} \quad (9)$$

The characteristics of the impulsive noise are different for different values of ϵ and M even when the SNR's are equal. This implies that the values of ϵ and M should be specified to investigate the effects of the impulsive noise.

Next, we define the signal to background noise ratio (SBR) as

$$SBR = \frac{nE_b}{2{\sigma_B}^2}. \quad (10)$$

If ϵ is very small ($(M-1)\epsilon \ll 1$), the SBR can be a good approximation to the SNR. It is easy to see that the SBR is useful when we want to know the effects of the contamination ratio (ϵ) on the performance of the system.

4 Performance Analysis

In slow fading channels, the received sequence can be written as

$$Y_p = \rho_p X_p + Z_p + \eta_p. \quad (11)$$

Now, we assume [2] that the inter-user interference Z_p is a Gaussian random variable independent of η_p by central limit theorem. Then

$$\bar{\eta}_p = Z_p + \eta_p \quad (12)$$

is also a random variable with an ϵ-contaminated pdf. From now on, we drop the bar of $\bar{\eta}_p$ for convenience.

In (11), ρ_p is the fading envelope which has the pdf

$$f_\rho(x) = 2x(1+K)exp[-\{K+x^2(1+K)\}] \times I_0(2x\sqrt{K(K+1)}), \quad (13)$$

where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind, and K is the Rician parameter defined as the ratio of the energy of the direct component to the energy of the diffused multipath components.

First, we assume that we have an information about the channel state. This assumption is valid when we use a pilot tone and the estimation of the fading parameter is assumed to be perfect. Then the pairwise error probability given $\rho = (\rho_1, \rho_2, \cdots, \rho_N)$ can be obtained as

$$\begin{aligned} P(x \rightarrow \tilde{x}|\rho) &= Pr\{\sum_{p\in\nu}(\|Y_p - \rho_p X_p\|^2 \\ &\quad -\|Y_p - \rho_p \tilde{X}_p\|^2) \geq 0\} \\ &= Pr\{\sum_{p\in\nu}(-E_s\rho_p{}^2\|d_p\|^2 - 2\sqrt{E_s} \\ &\quad \times\rho_p Re\{\eta_p d_p^*\}) \geq 0\}. \quad (14) \end{aligned}$$

where $d_p = x_p - \tilde{x}_p$. We can simplify (14) as

$$P(x \rightarrow \tilde{x}|\rho) = Pr\{-E_s\bar{d}^2 + 2\sqrt{E_s}\bar{u} \geq 0\}, \quad (15)$$

where $\bar{d}^2 = \sum_{p\in\nu}\rho_p{}^2\|d_p\|^2$ and $\bar{u} = -\sum_{p\in\nu}\rho_p Re\{\eta_p d_p^*\}$. By using properties in [8], we get

$$\begin{aligned} &P(x \rightarrow \tilde{x}|\rho) \\ \leq\ & Pr\{\hat{u} \geq 0\} \\ =\ & (1-\epsilon)\int_0^\infty \phi(x; -E_s\bar{d}^2, 4E_s d^2\sigma_B{}^2)dx \\ &+\epsilon\int_0^\infty \phi(x; -E_s\bar{d}^2, 4E_s d^2\sigma_T{}^2)dx, \quad (16) \end{aligned}$$

where $\hat{u}$ has the pdf $f_{\hat{u}}(x) = \alpha(x; \epsilon, -E_s\bar{d}^2, 4E_s d^2\sigma_B{}^2, 4E_s d^2\sigma_T{}^2)$ and $\phi(x; m_\phi, \sigma^2)$ is the Gaussian pdf with mean m_ϕ and variance σ^2. Applying Chernoff bound to (16), we get

$$P(x \rightarrow \tilde{x}|\rho) \leq (1-\epsilon)D_B^{\bar{d}^2} + \epsilon D_T^{\bar{d}^2}. \quad (17)$$

where $D_B = exp[-\frac{E_s}{8\sigma_B{}^2}]$ and $D_T = exp[-\frac{E_s}{8\sigma_T{}^2}]$. The unconditioned pairwise error probability bound is then obtained by taking the expectation of (17) over ρ as

$$P(x \rightarrow \tilde{x}) \leq (1-\epsilon)E\{D_B^{\bar{d}^2}\} + \epsilon E\{D_T^{\bar{d}^2}\}, \quad (18)$$

and the bit error probability bound is obtained as

$$\begin{aligned} P_b \ \leq\ & (1-\epsilon)\cdot\frac{1}{2n}\frac{\partial\bar{T}(D,I)}{\partial I}|_{D=D_B,I=1} \\ &+\epsilon\cdot\frac{1}{2n}\frac{\partial\bar{T}(D,I)}{\partial I}|_{D=D_T,I=1}, \quad (19) \end{aligned}$$

where $\bar{T}(D,I)$ is the transfer function of the super state diagram whose branch label gains are modified based on the discussion in [5]. It should be noted that for $\epsilon = 0$, the impulsive noise becomes Gaussian noise and (19) becomes the same result as that shown in [5].

Now, we consider the asymmetric signal constellation. In [4], it has been shown that TCM with optimally designed asymmetric PSK (APSK) signal constellation in bit error probability sense has some gain over that with symmetric PSK (SPSK) signal constellation. We can expect that TCM with optimally designed APSK signal constellation will have better performance than that with SPSK under impulsive noise environment also. With the APSK constellation in Fig. 2(b), the transfer function is a function of θ as well as D and I. We will find the optimum value θ_{opt} of θ in the bit error probability sense and the corresponding minimum bit error probability.

We have

$$\begin{aligned} P_b \ \leq\ & \min_{0\leq\theta\leq\pi}\ (1-\epsilon)\cdot\frac{1}{2n}\frac{\partial\bar{T}(D,I|\theta)}{\partial I}|_{D=D_B,I=1} \\ &+\epsilon\cdot\frac{1}{2n}\frac{\partial\bar{T}(D,I|\theta)}{\partial I}|_{D=D_T,I=1} \\ =\ & (1-\epsilon)\cdot\frac{1}{2n}\frac{\partial\bar{T}(D,I|\theta_{opt})}{\partial I}|_{D=D_B,I=1} \\ &+\epsilon\cdot\frac{1}{2n}\frac{\partial\bar{T}(D,I|\theta_{opt})}{\partial I}|_{D=D_T,I=1} \quad (20) \end{aligned}$$

where $\bar{T}(D,I|\theta)$ is the transfer function of the TCM scheme with APSK signal constellation for fading channel.

Now, let us analyze (20) asymptotically. At very low SNR, the bit error probability becomes much higher than ϵ, and the first term in (20) will dominate the bit error probability since the second term cannot be larger than ϵ. At very high SNR, the bit error probability becomes much lower than ϵ, and the second term in (20) will dominate the bit error probability since the effect of the fact that D_B is smaller than D_T is considerable at high SNR. Since the bit error probability is dominated by the first term in (20) at low SNR and by the second term at high SNR, we can guess that the values of θ_{opt} are equal to those under AWGN of variances $\sigma_B{}^2$ and $\sigma_T{}^2$, respectively. Thus we can use the results in [4][5], where the values of θ_{opt} under AWGN environment are obtained. For intermediate values of the SNR, there exists a transition from one asymptote to the other, and the value of SNR at which the transition begins will become large as ϵ decreases.

5 Numerical Results

Now we consider the performance of the 2-state 1/2 rate 4-PSK trellis coded DS/SSMA system under the impulsive noise environment. The comparisons between the probabilities of bit error with SPSK constellation and that with optimum APSK constellation under various environments are shown in Figs. 3-5. It is clearly observed that we can achieve some SNR gain with optimally designed APSK constellation. We can also observe that the performance gain of the APSK system over the SPSK system becomes larger as the channel becomes more favorable (SNR gets higher, ϵ gets smaller, and K gets larger). Some optimum angles of the optimum APSK constellation are shown in Figs. 6-8, from which we can easily check the asymptotic analysis at the end of the last section.

6 Conclusion

In this paper we have studied the effects of impulsive noise on the performance of 2^{n+1}-PSK trellis coded DS/SSMA systems. We analyzed the system performance and obtained the bit error probability bound under the impulsive noise environment, where we also considered the effects of Rician fading which is inevitable in mobile communication systems.

From the results, We see that the system performance can be improved by using optimally designed APSK signal constellation as in the AWGN case. It is also observed that the bit error probability is dominated by background noise variance at low SNR and by tail noise variance at high SNR.

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References

[1] G. Ungerboeck, "Channel Coding with Multilevel Phase Signals," *IEEE Trans. Inform. Theory*, Vol. IT-28, pp. 55-67, Jan. 1982.

[2] G.D. Boudreau, D.D. Falconer, and S.A. Mahmoud, "A Comparison of Trellis Coded Versus Convolutionally Coded Spread-Spectrum Multiple-Access Systems," *IEEE J. Select. Areas Comm.*, Vol. JSAC-8, pp. 628-640, May 1990.

[3] M.B. Pursley, "Performance Evaluation for Phase-Coded Spread Spectrum Multiple Access Communications - Part I: System analysis," *IEEE Trans. Comm.*, Vol. COM-25, pp. 795-799, Aug. 1977.

[4] D. Divsalar and M.K. Simon, "Trellis Coding with Asymmetric Modulations," *IEEE Trans. Comm.*, Vol. COM-35, pp. 130-141, Feb. 1987.

[5] E. Biglieri, D. Divsalar, P.J. McLane, and M.K. Simon, *Introduction to Trellis-Coded Modulation with Applications*. New York: Macmillan Publishing Company, 1991.

[6] S.A. Kassam and H.V. Poor, "Robust Techniques for Signal Processing: A Survey," *IEEE Proc.*, Vol. 73, pp. 433-481, Mar. 1985.

[7] S.I. Park, K.Y. Lee, and I. Song, "Performance Analysis of FHSS BFSK Systems with Nonlinear Detectors in Selective Fading Impulsive Noise Environment", *Signal Proc.*, Vol. 45, pp. 275-292, Sep. 1995.

[8] K.S. Kim, I. Song, S.I. Park, and J. Bae, "The Effects of Impulsive Noise on the Performance of DS/SSMA Systems using TCM," *IEE Proc. Comm.* (submitted for publication).

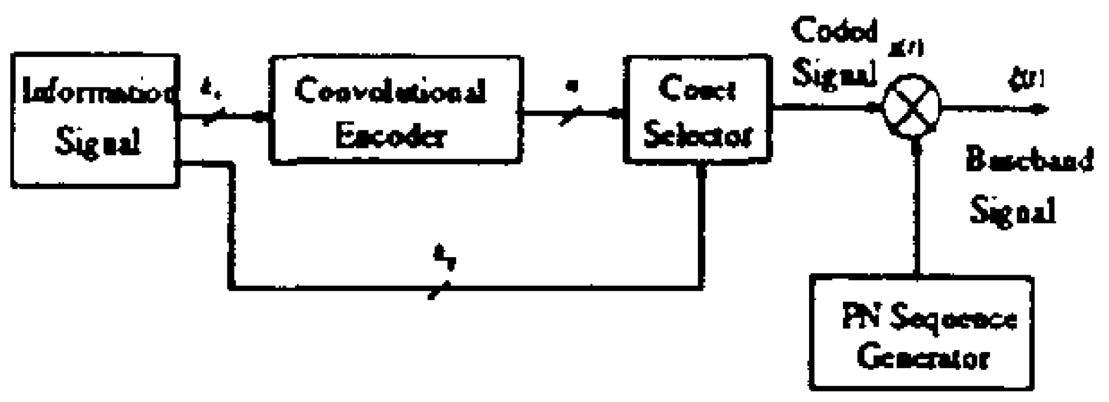

Figure 1: The transmitter system model for the trellis-coded DS/SSMA.

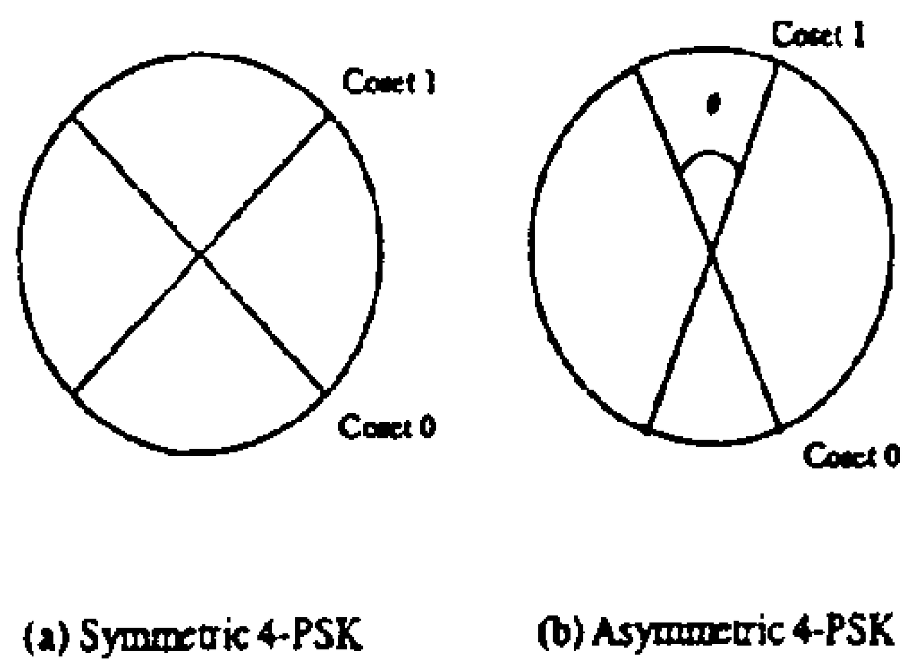

Figure 2: Symmetric and asymmetric 4-PSK signal constellations.

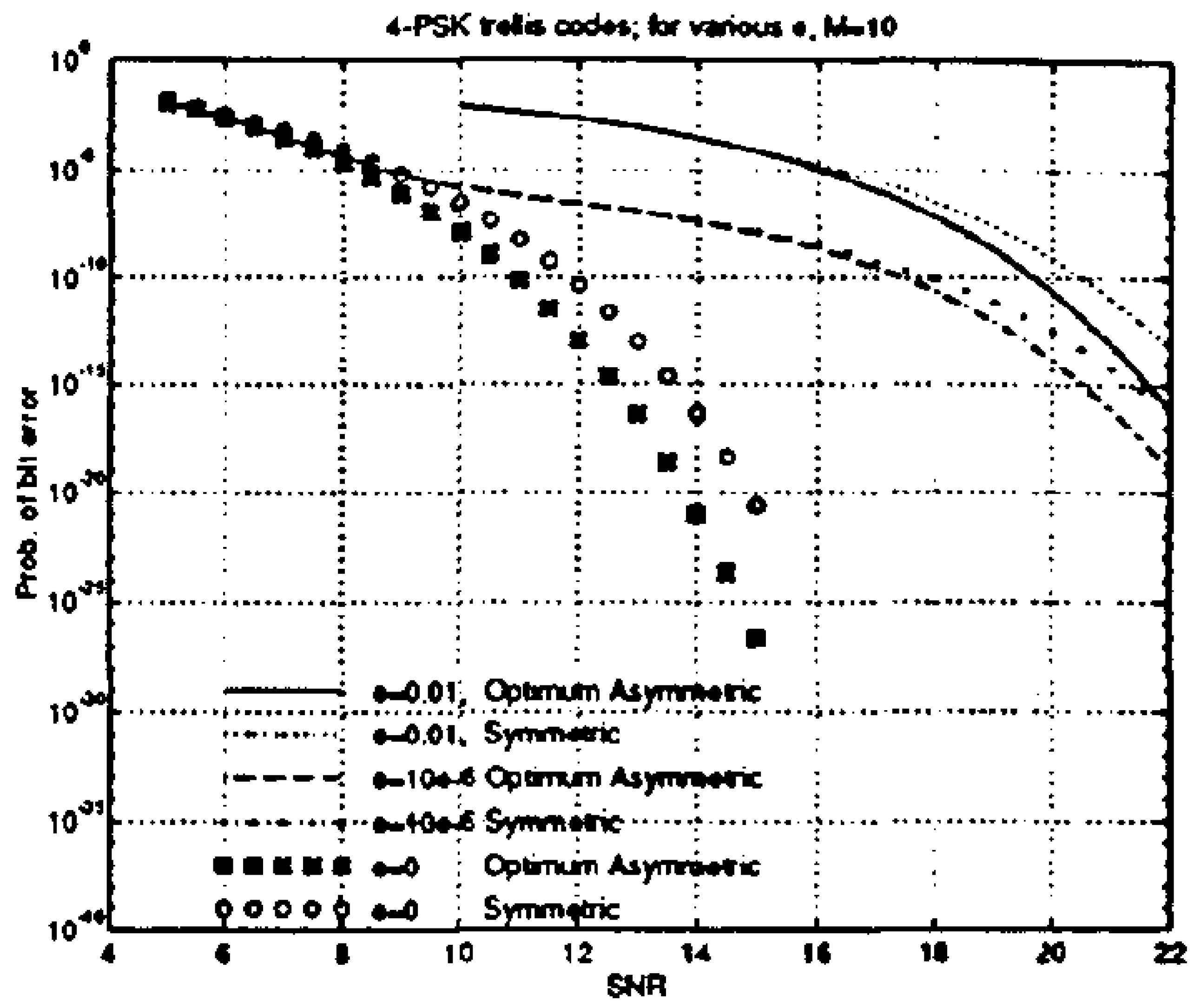

Figure 3: The bit error probability for optimum APSK and SPSK constellations for various ϵ when $M = 10$.

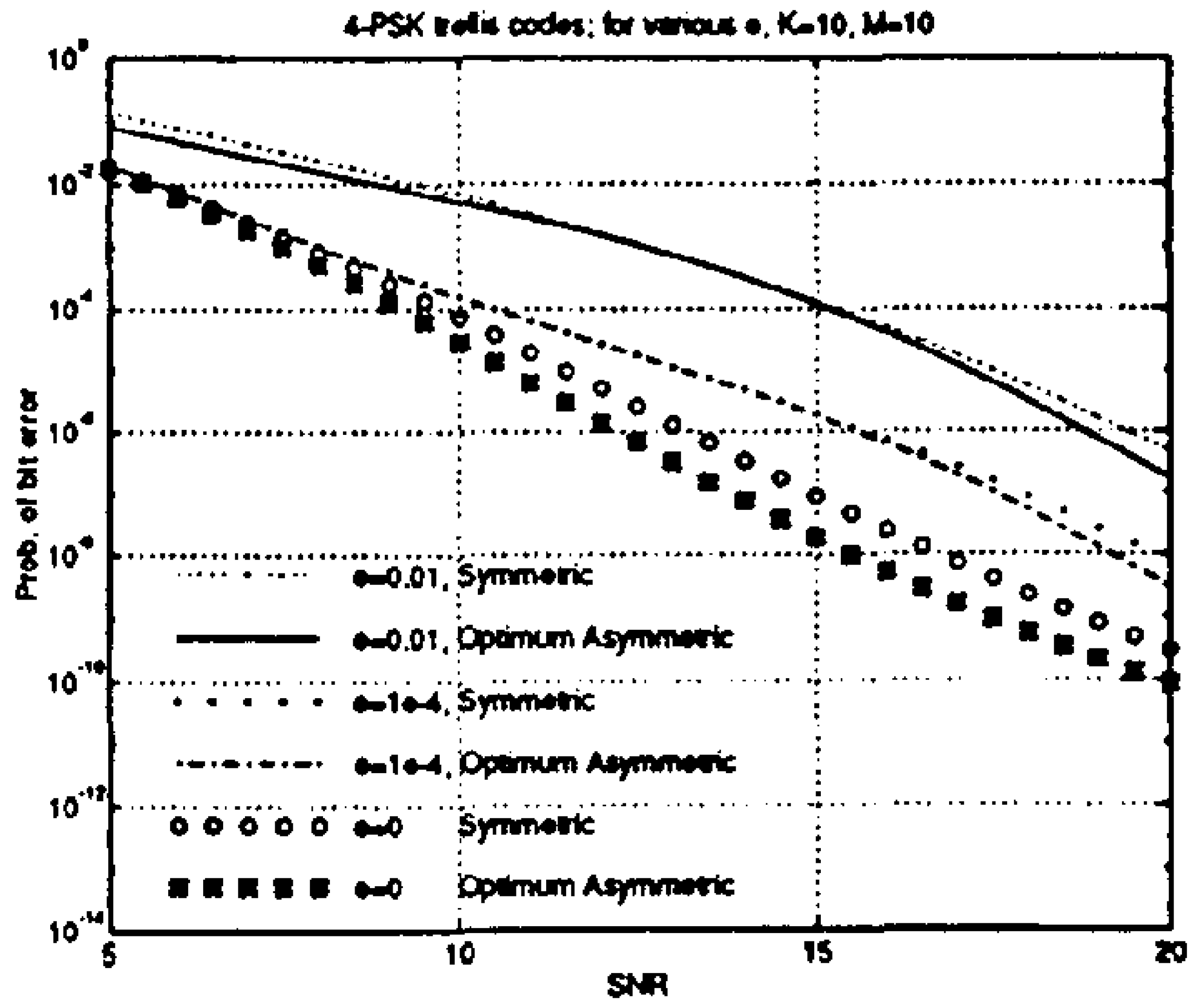

Figure 4: The bit error probability for optimum APSK and SPSK constellations for various ϵ when $K = 10$ and $M = 10$.

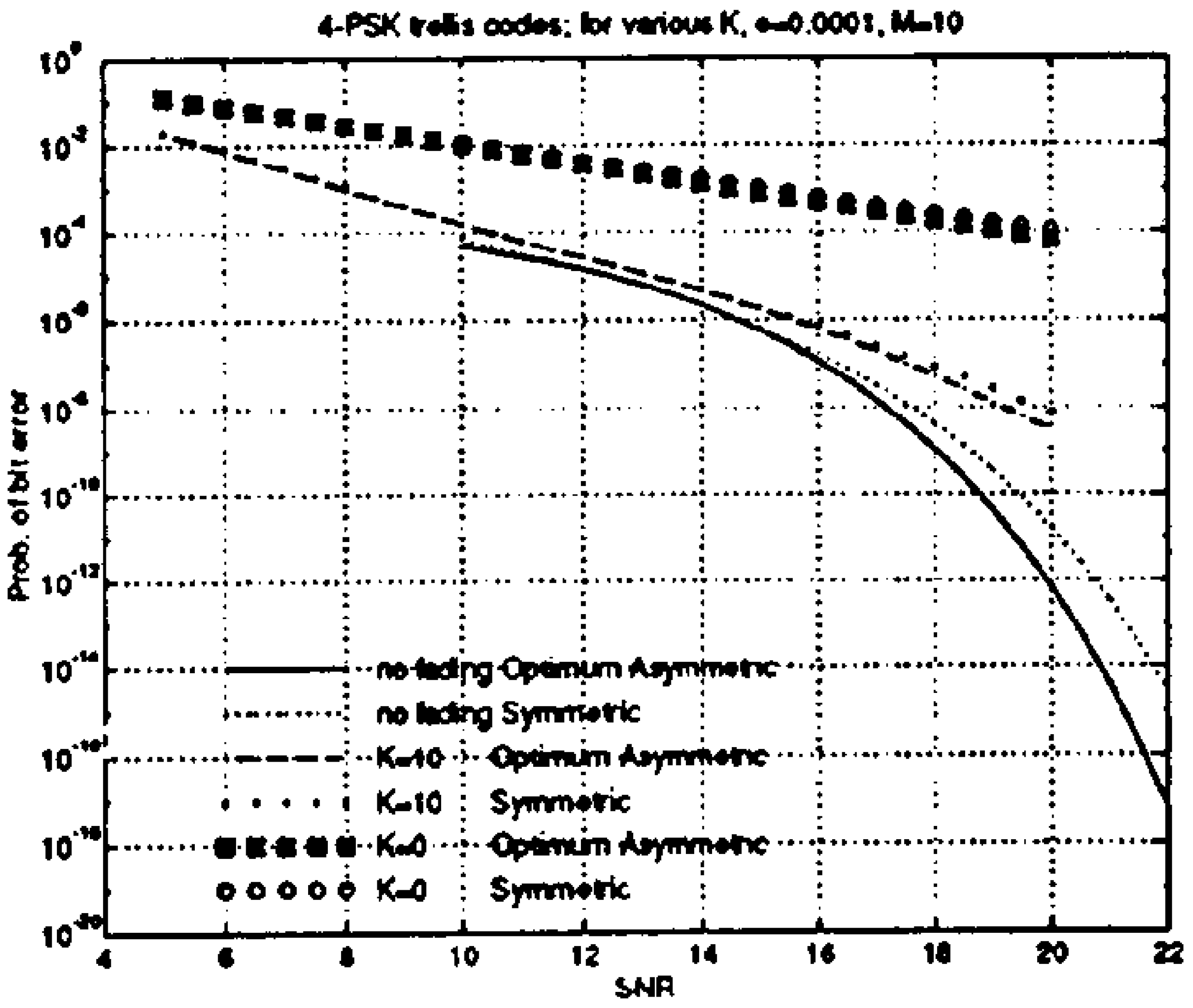

Figure 5: The bit error probability for optimum APSK and SPSK constellations for various K when $\epsilon = 0.0001$ and $M = 10$.

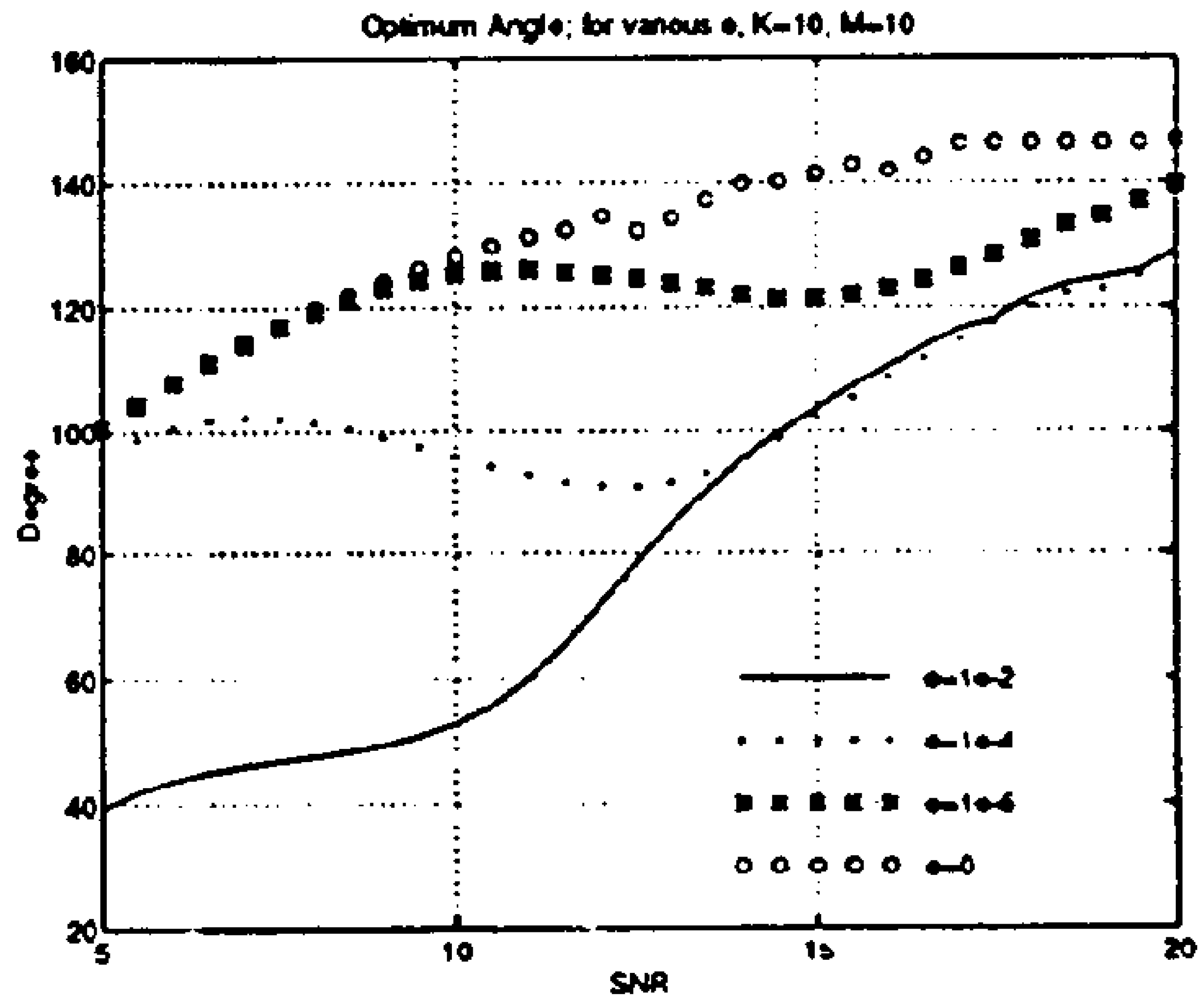

Figure 7: The optimum angle for various ϵ when $K = 10$ and $M = 10$.

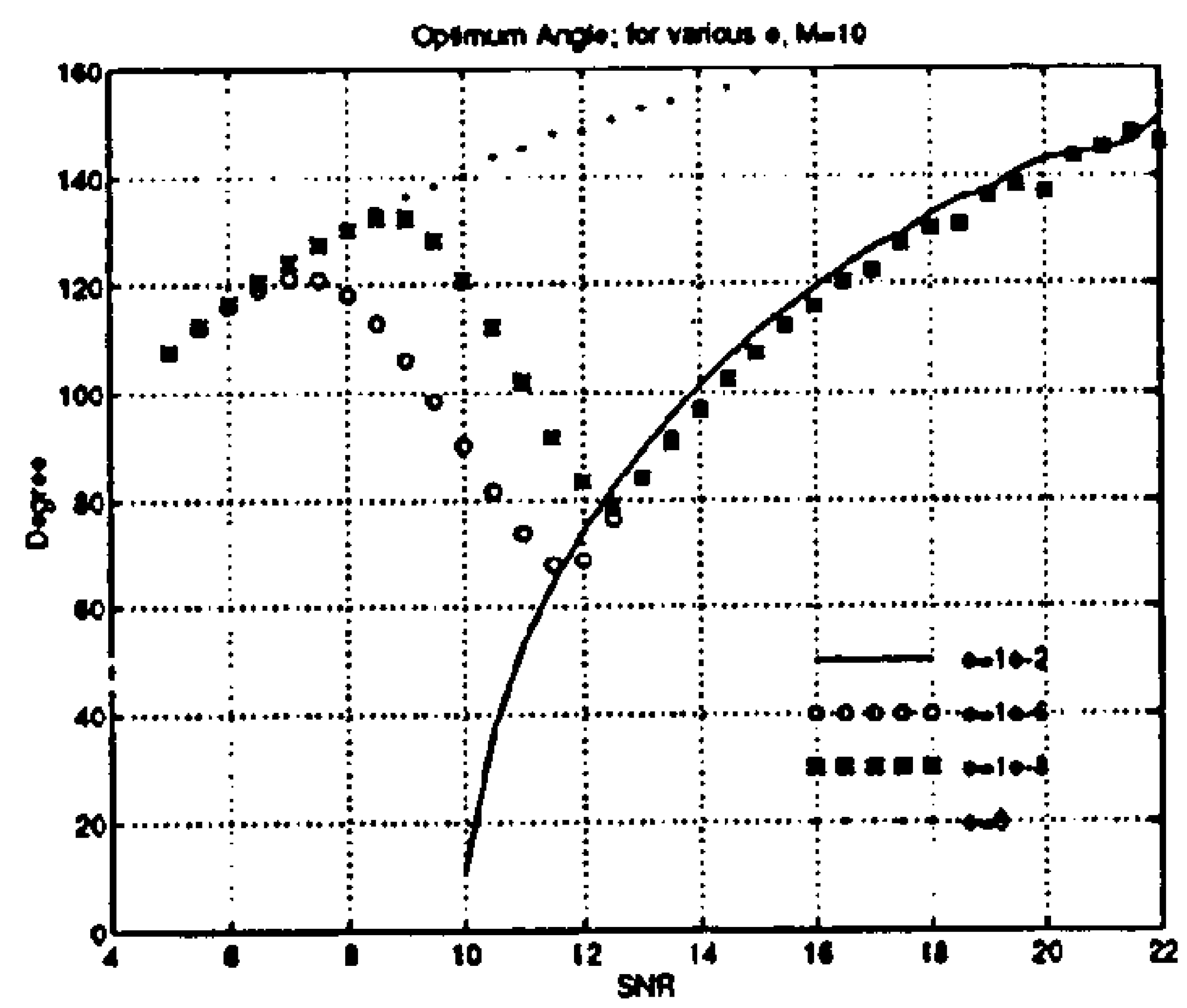

Figure 6: The optimum angle for various ϵ when $M = 10$.

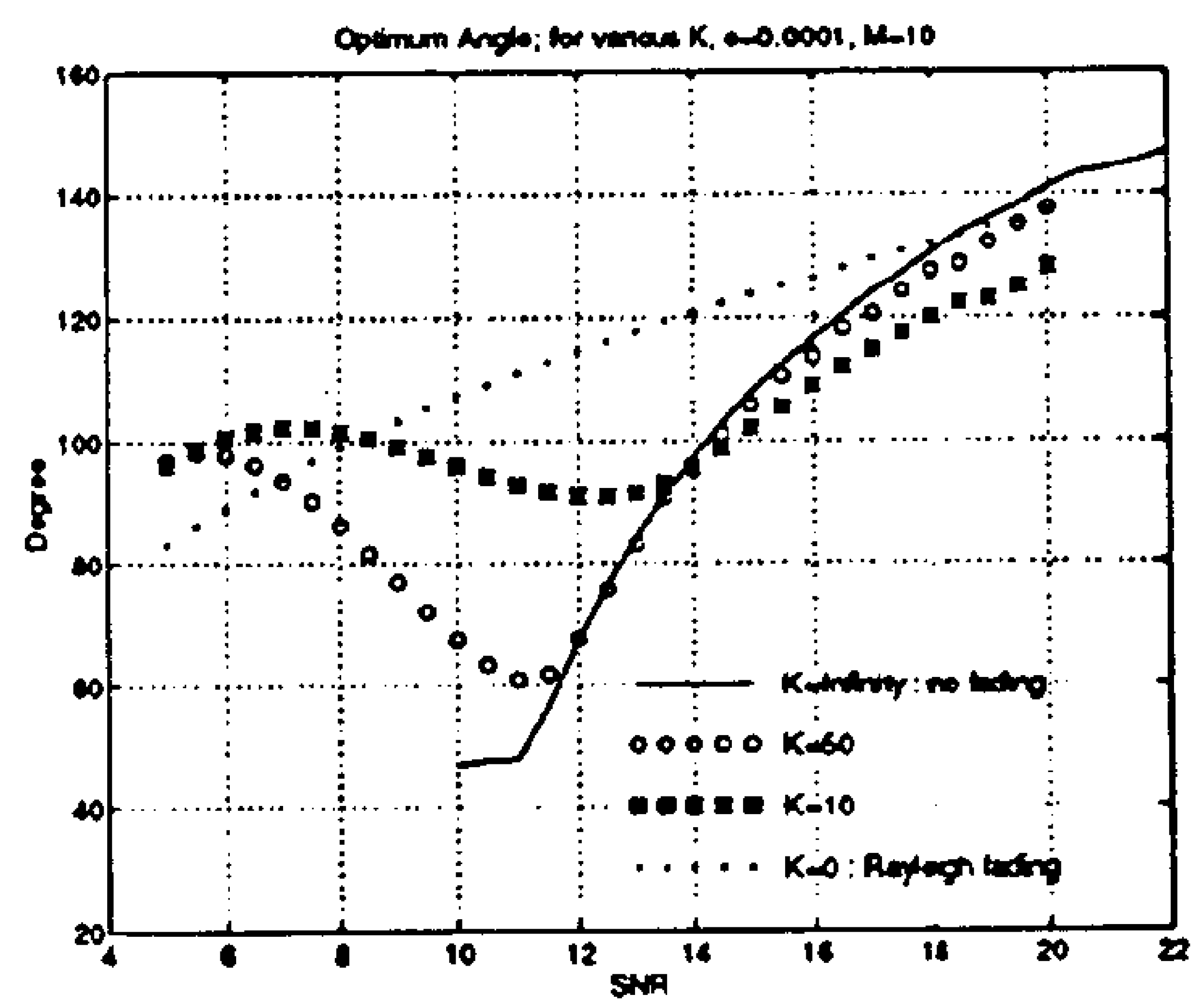

Figure 8: The optimum angle for various K when $\epsilon = 0.0001$ and $M = 10$.