Iterative Estimation and Decoding for an LDPC-Coded OFDMA System in Uplink Environments

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Abstract— **In this paper, we propose an LDPC-coded OFDMA system with a resource allocation method suitable for the uplink of mobile cellular environments. At the receiver of the proposed system, an iterative channel estimation and decoding (IED) method with a Wiener filter adapting to the channel variation rate is employed to support high mobility without boosting the pilot power. For practical applications, the channel variation rate is also estimated with pilot symbols to select Wiener filter coefficients for every received packet dynamically allocated. It is shown that the proposed system supports the mobility up to 250 km/h and exhibits more than 1 dB performance gain over the system with conventional non-iterative receiver.**

I. INTRODUCTION

In future wireless communication systems, high data rate and quality, close to the wired environments, are required to support increasing demands on various services such as video and audio streaming, file transfer, internet access, and so forth. Especially, demands on packet data service are increasing for ubiquitous internet access. Orthogonal frequency division multiplexing (OFDM) has been widely considered as a promising technique for future mobile radio access air interface due to its successful deployment in various standards and commercial systems for broadband wireless transmission [1]-[3]. Among the multiple access schemes based on OFDM, OFDM frequency division multiple access (OFDMA) provides us with an efficient platform for high speed packet transmission in cellular environments: high flexibility in subcarrier allocation for data rate and channel adaptation, no intracell interference, and intercell interference averaging or avoidance capability. Especially, with sophisticatedly designed frequency hopping methods for OFDMA, we can obtain both the intercell interference averaging effect and the full exploitation of the available frequency diversity.

On the other hand, coherent demodulation is essential for future applications to support high order modulation and to lower the operating signal-to-noise power ratio (SNR). Especially, for the OFDMA systems, the low operating SNR results in transmit power reduction, which also lowers the intercell interference level. However, the channel estimation in the uplink using a frequency hopping OFDMA is challengeable since different channel responses from different users to a base station should be simultaneously tracked. In fact, the pilot design for the packet-based OFDMA systems in the uplink is different from those for the OFDMA systems in the downlink and OFDM systems assigning all subcarriers to one user as in [1][2]: pilot symbols can not be shared among multiple users in the uplink and the frequency interpolation might not be possible due to the discontiguous resource allocation. Moreover, the active users transmitting the signals can be varied slot-by-slot through dynamic packet allocation. Thus, it is essential to design a pilot-and-data arrangement and a channel estimation method which reduce the overhead in power and spectral efficiency induced by the pilot symbols as well as track the multi-user channel responses dynamically varied.

For the channel estimation methods utilized for continuous or packet-based OFDM systems, pilot-aided channel estimation approaches [4]-[6] and (pilot-assisted) decision-directed channel estimation approaches [7]-[9] have been proposed. A pilot-aided channel estimator is shown to be more robust to the channel variation than a decision-directed channel estimator although the former exhibits a performance loss in low mobility [6]. Thus, it is expected that the performance improvement will be obtained with a reduced number of pilots by properly combining the pilot-aided approach and the decision-directed approach. One of the pilot-assisted decision-directed channel estimation methods utilized the decision feedback symbols to improve the frequency domain interpolation by selecting the most significant paths through the FFT and the IFFT [9]. However, the method can not be applied for the uplink OFDMA system, where each user is assigned with subcarriers which have little correlation in the frequency domain.

In this paper, we propose an OFDMA system with a resource allocation method suitable for the uplink of mobile cellular environments. For the proposed system, an iterative channel estimation and decoding (IED) scheme is investigated with a reliable channel coding scheme and a pilot-aided decision-directed channel estimation. As a channel coding scheme, low density parity check (LDPC) codes with near Shannon-limit performance [10][11] are applied to obtain a robust performance with high-speed decoding. In the channel

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estimation method, Wiener filtering is applied for both the initial estimation utilizing only the pilot symbols and the subsequent decision-directed channel estimation utilizing the decoder outputs and pilot symbols to support the mobility. The channel variation rate is also estimated with the pilot symbols in a packet to select Wiener filter coefficients for the dynamically allocated packets. The outline of the paper is as follows. The system model of the proposed LDPC-coded OFDMA system is described in Section II and the IED receiver is proposed in Section III. Simulation results are provided in Section IV followed by conclusions.

II. SYSTEM MODEL

In packet-based uplink OFDMA systems, time and frequency resources are divided into multiple data pipelines to be shared by active users in a cell. The data pipelines are not overlapped one another in a cell, but collide with the data pipelines used in the other cells, which results in the intercell interference. To mitigate the intercell interference and exploit the full frequency diversity, OFDMA systems with frequency hopping over the overall frequency band can be a good multiple access scheme in cellular environments. However, symbol-by-symbol hopping methods which can average the intercell interference better are not possible for the frequency selective fading uplink channels from the viewpoint of channel estimation.

In the proposed OFDMA system for the uplink, we consider a resource block (RB), consisting of F adjacent subcarriers of N consecutive OFDM symbols, as a unit of frequency hopping and channel estimation as shown in Fig. 1. A data pipeline consists of multiple RBs distributed over the overall band according to a frequency hopping pattern. The size of the RB should be minimized as much as possible to enhance the intercell interference average performance, while it should be large enough for a reliable channel estimation. In the proposed system, the number of adjacent subcarriers in an RB, F, is selected by a minimum number of subcarriers of which the channels are highly correlated, so that one pilot position in the frequency domain is enough for channel estimation. On the other hand, at least two positions in the time domain are assigned to the pilot symbols for filtering or interpolation to support high mobility. The pilot spacing P in the multiples of the OFDM symbol duration T_s should be less than $\frac{1}{2 f_D T_s}$, to support the maximum Doppler frequency f_D of the channel. Even though one data packet can be composed of multiple RBs in the time domain, we will assume that the time duration of a data packet is that of an RB in the sequel since the results can be easily extended for the multiple RB case.

The system model of the proposed LDPC coded OFDMA system is depicted in Fig. 2 for the uplink of mobile radio access. At the transmitter of a user side, a data packet of information bits $\{b_i, i = 0, 1, ..., K_L - 1\}$ is encoded by an LDPC encoder with a source block length K_L and a codeword length N_L. The coded bits, c_k, are then mapped to the modulated symbols, s_j, such as QPSK or QAM and the pilot symbols are inserted at the pilot locations in the modulated symbol streams. The pilot inserted symbol streams are then mapped to the pre-assigned locations of time and frequency resources according to the frame format for OFDM symbol generation.

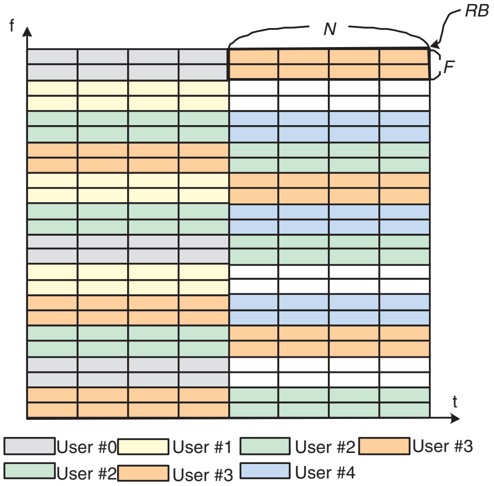

Fig. 1. A resource allocation example of the proposed OFDMA system.

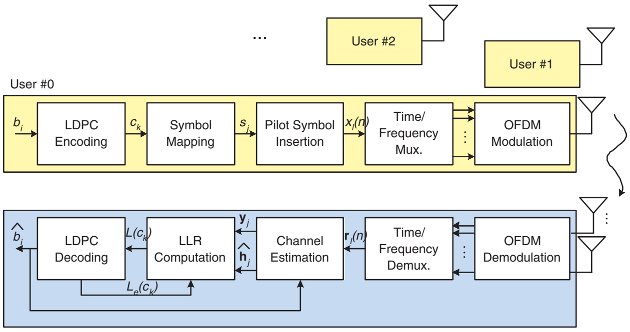

Fig. 2. The system model.

To eliminate or minimize the intracell interference in the uplink, the signals from multiple users in the same cell are controlled to be arrived at the base station within an acceptable timing error range through a timing control feedback loop. Then the received vector from the M receiving antennas at the lth subcarrier of the nth OFDM symbol for the desired user is given by

$$\mathbf{r}_l(n) = \mathbf{h}_l(n) x_l(n) + \mathbf{w}_l(n), \tag{1}$$

where $\mathbf{r}_l(n) = [r_{l,1}(n) r_{l,2}(n) \cdots r_{l,M}(n)]^H$ is the $M \times 1$ received vector, $\mathbf{h}_l(n) = [h_{l,1}(n) h_{l,2}(n) \cdots h_{l,M}(n)]^H$ is the $M \times 1$ vector of the channel frequency response, $x_l(n)$ is the modulated symbol (data or pilot), and $\mathbf{w}_l(n) = [w_{l,1}(n) w_{l,2}(n) \cdots w_{l,M}(n)]^H$ is the $M \times 1$ additive white Gaussian noise (AWGN) vector with $E\{\mathbf{w}_l(n)\mathbf{w}_l(n)^H\} = \sigma_w^2 \mathbf{I}_M$.

With the received vectors, channel estimation, demodulation, and LDPC decoding are performed iteratively.

III. ITERATIVE ESTIMATION AND DECODING

The proposed IED method is based on the initial pilot-aided channel estimation and the subsequent channel estimation refinement with the tentative decoded bits during LDPC decoding.

At the first iteration prior to decoding, the channel estimation is performed with pilot symbols such that

$$\hat{\mathbf{h}}_l^{(1)}(n) = \sum_{p \in I_{T_p}} g_i(n,p)\tilde{\mathbf{h}}_{l_p}(p), n = 0, 1, ..., N-1, \qquad (2)$$

where $\tilde{\mathbf{h}}_{l_p}(p) = \mathbf{r}_{l_p}(p)/x_{l_p}(p)$ is the least square estimation of the channel at the pilot position of time p and frequency l_p, I_{T_p} is the time index set of pilots for the desired packet, and $g_i(n,p)$ is the initial Wiener filter coefficients obtained by channel estimates on the pilot positions.

After channel decoding, the channel estimation is refined through the tentatively decided symbols from LDPC decoding. The channel estimates with the decision feedback symbols and pilot symbols are $\tilde{\mathbf{h}}_l^{(q)}(n)$ at the $q(>1)$th iteration, with which the time varying channel response is estimated through Wiener filtering as follows.

$$\hat{\mathbf{h}}_l^{(q)}(n) = \sum_{n'=0}^{N-1} g(n,n') \left[\frac{1}{F} \sum_{l' \in I_{F_{RB(l)}}} \tilde{\mathbf{h}}_{l'}^{(q)}(n') \right], \qquad (3)$$

where $I_{F_{RB(l)}}$ is the frequency index set of the RBs to which the lth subcarrier belongs and $g(n,n')$ is the Wiener filter coefficients obtained from the channel estimates with both pilot and data symbols.

In practice, we should know the channel statistics and the maximum Doppler frequency to apply Wiener filtering. Here, we assume that, at least, it is known that the channel has the classical Doppler spectrum with autocorrelation of $E\{h_{l,m}(n_1)h_{l,m}^*(n_2)\} = J_0(2\pi f_D T_s(n_1 - n_2))$, where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. To apply the proper filter gains, the channel variation rate of the current data packet is coarsely estimated by estimating the autocorrelation with the pilot symbols inserted in the RBs as follows.

$$E\{h_{l,m}(n)h_{l,m}^*(n+P)\} \approx \qquad (4)$$

$$\frac{\frac{1}{|I_{T_p}|-1} \sum_{l' \in I_{F_p}} \sum_{n' \in \tilde{I}_{T_p}} \sum_{m=1}^{M} h_{l',m}(n')h_{l',m}(n'+P)^*}{\frac{1}{|I_{T_p}|} \sum_{l' \in I_{F_p}} \sum_{n' \in I_{T_p}} \sum_{m=1}^{M} (|h_{l',m}(n')|^2 - \sigma_{e,p}^2)},$$

where I_{F_p} is the frequency index set of the pilots located in the desired data packet, $|I|$ is the cardinality of a set I, $\tilde{I}_{T_p}$ is the set formed from I_{T_p} excluding the rightmost time index of the pilots, and $\sigma_{e,p}^2$ is the variance of the pilot channel estimation error.

Once the channel estimation is performed, the log-likelihood ratios (LLRs) for the LDPC decoder are computed after pilot symbol removal. Let us define $\mathbf{y}_j$ and $\hat{\mathbf{h}}_j$ as the jth received vector and channel estimate after pilot symbol removal, respectively. Then the LLR of $c_{j,i}$, the ith constituent bit of the jth symbol s_j, is computed as

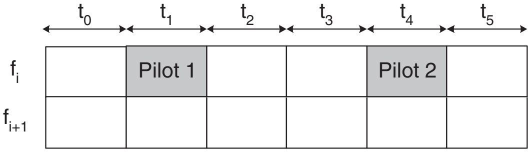

Fig. 3. The RB structure used in simulation.

$$L(c_{j,i}) = \log \left(\frac{\sum_{s_j \in A_i^0} \exp\left(-\frac{|\mathbf{y}_j - \hat{\mathbf{h}}_j s_j|^2}{\sigma_{e,j}^2 |s_j|^2 + \sigma_w^2}\right) \prod_{k \neq i} p_e(c_{j,k})}{\sum_{s_j \in A_i^1} \exp\left(-\frac{|\mathbf{y}_j - \hat{\mathbf{h}}_j s_j|^2}{\sigma_{e,j}^2 |s_j|^2 + \sigma_w^2}\right) \prod_{k \neq i} p_e(c_{j,k})} \right), \qquad (5)$$

where A_i^b is the set of possible modulated symbols given that the ith constituent bit is $b(\in \{0,1\})$ and $\sigma_{e,j}^2 = E\{|\hat{h}_{j,m} - h_{j,m}|^2\}$. At the first iteration prior to decoding, there exists no prior information on the transmitted bits and thus a priori probability of each bit, $p_e(\cdot)$, is 0.5. However, after LDPC decoding, the extrinsic information from the LDPC decoder is utilized for LLR computation as $p_e(c_{j,k} = b) = (1 + e^{\{(-1)^{1-b} L_e(c_{j,k})\}})^{-1}$ for $b = 0, 1$. Furthermore, we incorporate the variance of channel estimation errors, $\sigma_{e,j}^2$, into the calculation of the LLRs for more exact evaluation as proposed in [12].

For every IED iteration over the channel estimation, demodulation, and decoding, the LDPC decoder also iterates maximum D times until the decoded results meet a stop condition. If the stop condition is not satisfied at the maximum number of decoding iterations, the IED receiver begins new IED iteration by refining the channel estimates and it is performed up to R times. In addition, the internal messages of the LDPC decoder at the end of the current IED iteration are used for the next IED iteration to speed up the convergence rate.

IV. NUMERICAL RESULTS

In this section, we investigate the performance of the proposed system via Monte Carlo simulations with following parameters. For a data packet transmission, 6 consecutive OFDM symbols with 256 subcarriers distributed over 20 MHz bandwidth are assigned. The RB consists of 2 adjacent subcarriers of 6 consecutive OFDM symbols with pilot position as shown in Fig. 3. In the case, 17% of the given time and frequency resources are used for the pilot symbols with a power equal to the data symbols. A binary $(2560, 1440)$ irregular LDPC code is constructed with 2, 3, and 6 column weights of which the density is optimized by the density evolution based on the Gaussian approximation as in [13]. At the receiver side, the sum-product decoding algorithm is employed for LDPC decoding with the stop criterion based on the syndrome check.

The performance is evaluated under the ITU-R pedestrian A channel model with each tap experiencing independent

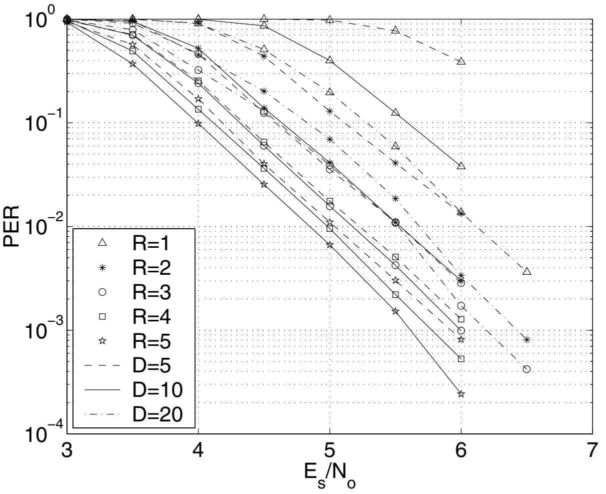

Fig. 4. The PERs of the proposed IED receiver with different maximum decoding iterations (D) and and maximum IED iterations (R) when the vehicular speed is 3 km/h.

Rayleigh fading generated by the Jakes model. The carrier frequency is 2 GHz and the vehicular speed is chosen from typical speeds such as 3, 60, 120, and 250 km/h. The number of receiving antennas is set to 2 for all simulation results. We assume that the transmitted power for each data packet is adjusted for the mean SNR averaged over the packet to produce a target SNR value: there exist the time selectivity and the frequency selectivity during a data packet transmission while the average received SNR over the packet is the same for all data packet transmissions by controlling the transmit power.

The packet error rates (PERs) of the proposed receiver are shown in Fig. 4 for various numbers of the maximum IED iterations (R) and maximum decoding iterations (D) when the vehicular speed is 3 km/h. In the figure, the IED receiver with $D = 20$ shows much better performance at the first IED iteration than those with $D = 5$ or $D = 10$ due to LDPC decoding capability. However, it provides us with only slight performance gain as the IED iteration increases. The reason is that the LDPC decoder already converges to a state where the incorrect decoding results can not be corrected further by the refined channel estimation and demodulation. On the other hand, the convergence rate is slow when the number of maximum LDPC decoding iterations is small. To compromise it, we choose $D = 10$ and $R = 5$ for the following simulations.

Fig. 5 shows the performance of the proposed IED receiver compared with the non-iterative receiver (*No IED*) when the total number of LDPC decoding iterations is the same: $R = 5$ and $D = 10$ for the proposed IED receiver and $R = 1$ and $D = 50$ for the non-iterative receiver. The results are obtained with Wiener filter coefficients designed for each vehicular speed assuming that the maximum Doppler frequency is known. With the identical number of maximum decoding iterations, 50, the proposed receiver shows about 1 dB performance gain over the non-iterative receiver wihtout boosting the pilot power. In fact, the performance of the IED recever is far behind what we expected since we conjecture that the good performance of the LDPC codes will much improve the refined channel estimation of the IED receiver by providing the reliable estimates of data symbols. However, error propagations among the channel estimator, the demodulator, and the LDPC decoder result in the unreducable performance gap between the IED receiver and the ideal receiver with known channel response. Thus, more study is required to reduce the error propagations to improve the performance further. On the other hand, the bright side of the results is that the gain is obtained with only the increase in the receiver complexity without any increase in the complexity and power at the transmitter. It is more desirable in the uplink where the transmitter has a limitation in the power and size.

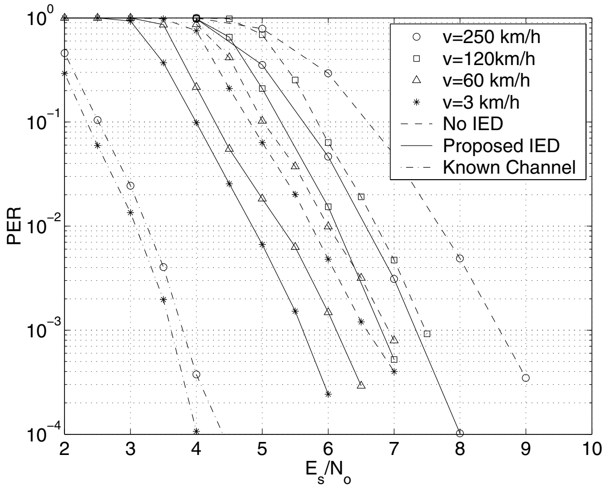

Fig. 5. The PERs of the proposed IED receiver compared with a non IED receiver when the vehicular speed is 3, 60, 120, and 250 km/h.

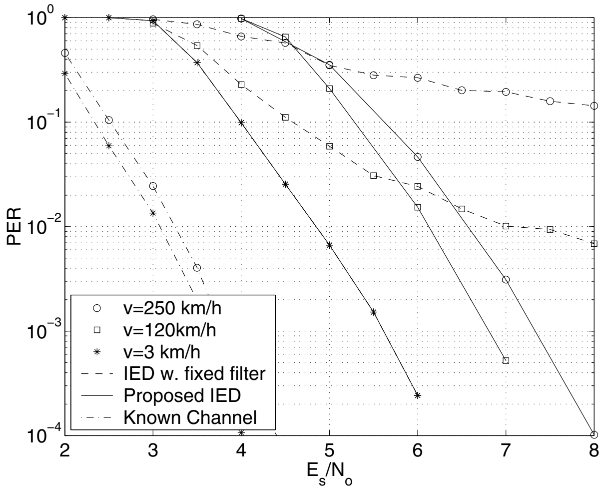

Fig. 6. The PERs of the proposed IED receiver compared with an IED receiver with the fixed filter coefficients when the vehicular speed is 3, 120, and 250 km/h.

Fig. 6 compares the performance of the proposed IED receiver with that of the IED receiver utilizing the fixed filter

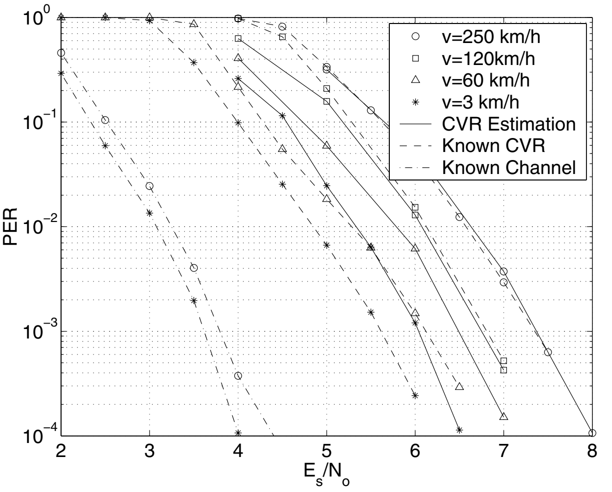

Fig. 7. The PERs of the proposed IED receiver when the channel variation rate (CVR) is estimated with pilot symbols.

coefficients designed for the vehicular speed of 3 km/h. At the low vehicular speed, the Wiener filter coefficients take the form of the average of the tentative channel estimates in the RB. While the average of the tentative channel estimates is robust to the background noise in low mobility environments, it cancels out the channel variation in high mobility environments. Thus, the IED receiver with the fixed filter coefficients exhibits error floors as the vehicular speed or the SNR increases. On the contrary, a performance loss is expected when the filter coefficients designed for the high vehicular speed is applied for the low mobile environments even though it is not shown in this paper.

Fig. 7 shows the performance of the IED receiver when the channel autocorrelation reflecting the channel variation rate is estimated with the pilots in each packet to select the proper filter coefficients. Since there are only two pilot positions in time, the accurate estimation is not possible. Thus, we utilize the real value of the estimated autocorrelation to provide a guideline whether the channel variation rate (CVR) is high or low by setting a threshold. In the figure, the threshold value is set to the auto-correlation value at the vehicular speed of 90 km/h. It is observed that the performance loss due to the estimation of the CVR is somewhat large in case of the low mobility while is is negligible for the high mobility. In case of the low mobility, the operating SNR is somewhat low and the background noise can mislead the channel estimator into believing that the CVR is high. However, even with the coarse estimation on the CVR, a performance gain is expected by adapting the filter coefficients.

V. CONCLUSIONS

We have proposed an LDPC-coded OFDMA system with an RB based resource allocation for the uplink of mobile cellular systems and have investigated an iterative estimation and decoding method for the proposed system. In the channel estimation method, Wiener filtering was applied for both the initial estimation utilizing the pilot symbols and the subsequent decision-directed channel estimation utilizing both the LDPC decoder outputs and pilot symbols. For practical applications, the channel variation rate was also estimated to select Wiener filter coefficients. The proposed IED receiver has shown a performance gain of 1 dB over the conventional non-iterative receiver in various vehicular speeds with a proper choice of the maximum numbers of LDPC decoding iterations and IED iterations. It was also shown that the performance loss due to the estimation on the channel variation rate is less than 0.5 dB at the low mobility and it is negligible at the high mobility. However, the LDPC decoder seems to be sensitive to error propagations so that the performance of the iterative receiver does not converge to that of the known channel case. Thus, furthre considerations are required to reduce the error proagations in the iterative receiver for the performance improvement.

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