An LDPC-Coded Spatial Multiplexing OFDMA System with Iterative Demodulation and Decoding

Yun Hee Kim, Kwang Soon Kim, Sang Hyun Lee, and KyungHi Chang

Electronics and Telecommunications Research Institute (ETRI)
161 Gajeong Dong, Yuseong Gu, Daejeon 305-350, Korea
E-mail: yheekim@etri.re.kr

Abstract— **A suboptimal iterative demodulation and decoding receiver is presented for an LDPC coded spatial multiplexing OFDMA system. The proposed receiver is based on spatial demultiplexing at the first iteration and the interference cancellation with soft decoding outputs at the following iterations. It is shown that the proposed receiver can reduce the complexity of the optimal iterative demodulation and decoding with a slight performance degradation. Moreover, we can achieve a performance gain over the non-iterative receivers if more latency is acceptable at the receiver.**

I. Introduction

In future wireless communication systems, high data rate and quality close to the wired environments are required to support increasing demands on various services such as video and audio streaming, file transfer, internet access, and so forth. Especially demands on packet data service are increasing for ubiquitous internet access. Orthogonal frequency division multiplexing frequency division multiple access (OFDMA) provides an efficient platform with its advantages of the robustness to multipath fading, granular resource allocation capability, and no intracell interference. Thus, it has been adopted for wireless metropolitan area networks standards [1] and has been considered as a successful candidate for future multiple access schemes.

To support high data rate services, multiple input and multiple output (MIMO) links are very attractive due to their potential of high spectral efficiency. One of such techniques is spatial multiplexing which divides the input streams into multiple streams for transmission over multiple antennas. Various techniques have been proposed to extract the transmitted symbols from the received signals [2]-[4]: Maximum likelihood (ML) detection is optimal, but the complexity is unacceptably high for large number of transmit and receiver antennas and high order modulation. A linear receiver such the minimum mean square error (MMSE) or zero forcing (ZF) equalizer is more practical while performance degradation is large. The ordered successive interference cancellation (OSIC) lies between the optimal detector and linear receiver in performance and complexity.

On the other hand, low density parity check (LDPC) codes have drawn much attention since they can be constructed to have near Shannon limit performance [5][6]. They also possess advantages of decoder complexity lower than turbo codes, error detection capability, no requirement of interleaving, and no error floor. In this reason, LDPC coding was recently combined with spatial multiplexing in [7] and iterative information exchange among demodulation, decoding, and channel estimation is performed for more robust performance. However, in the paper, only the optimal demodulation based on the maximum *a posteriori* (MAP) was considered for the iterative process in the receiver.

In this paper, we focus on a suboptimal iterative demodulation and decoding (IDD) scheme for an LDPC coded spatial multiplexing OFDMA system. Since the MAP demodulator jointly computes the *a posteriori* probability (APP) of transmitted bits with the symbols from all transmit antennas, the complexity is exponentially increasing with the number of transmit antennas and the constellation size. Instead of MAP demodulation, we perform a spatial demultiplexing process such as a linear equalization or OSIC to compute the APP of the transmitted bits with only one transmitted symbol at the first iteration. For the following iterations, the soft interference cancellation is performed with decoding outputs to extract the transmitted antenna symbols. With the proposed receiver, we can obtain a performance gain over the conventional non-iterative receivers with slight increase in complexity and delay.

The system model of an LDPC coded spatial multiplexing OFDMA system is described in section II and a suboptimal IDD receiver is proposed in Section III. Monte Carlo simulations of the proposed and conventional receivers are provided in Section IV and conclusions follow.

II. System Model

In this paper, we consider an LDPC coded spatial multiplexing OFDMA (LDPC-SM-OFDMA) system with M_t transmit antennas and M_r receive antennas for downlink, where a data packet is assigned with orthogonal sets of time and frequency resources. The system model of the LDPC-SM-OFDMA is depicted in Fig. 1.

At the transmitter, a data packet of information bits b_k is encoded by an LDPC encoder with a source block length K and a codeword length N and then mapped to the modulation symbols $s(l)$ such as QPSK or QAM with the constellation size of Q. The modulation symbols are spatially multiplexed to produce the $M_t \times 1$ antenna symbol vector $\mathbf{x}(l)$ $= (x_1(l)x_2(l)\cdots x_{M_t}(l))^T$ at the lth allocated resource unit. The antenna symbol vectors are then mapped to the assigned resource positions by the time/frequency mapping and OFDM symbols are generated for each transmit antenna. To exploit the

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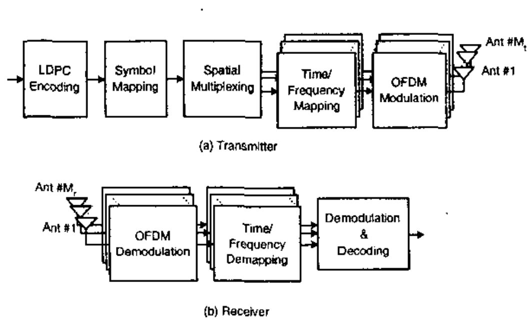

Fig. 1. System Model.

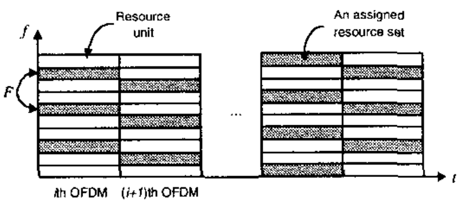

Fig. 2. A resource allocation method for the OFDMA.

frequency diversity, equally spaced subcarriers with F spacing are assigned by the time/frequency mapping as illustrated in Fig. 2.

The channel is assumed to be quasi-static so that it is time invariant over a data packet transmission and yet can vary over packet by packet. The channel is further assumed to be frequency selective fading but the fading on each subcarrier is flat with well designed OFDM parameters. Thus the received vector from M_r receive antennas at the lth allocated resource unit after OFDM demodulation and time/frequency demapping is given by

$$\mathbf{r}(n) = \mathbf{H}(n)\mathbf{x}(n) + \mathbf{w}(n), \tag{1}$$

where $\mathbf{r}(n)$ is the $M_r \times 1$ received vector, $\mathbf{H}(n)$ is the $M_r \times M_t$ complex matrix of the channel frequency response, and $\mathbf{w}(n)$ is the $M_r \times 1$ additive white Gaussian noise (AWGN) vector with $E\{\mathbf{w}(n)\mathbf{w}(n)^H\} = \frac{M_t N_0}{E_s}\mathbf{I}_{M_r}$. Here, the symbol energy is normalized to be 1 for a transmitted symbol per resource unit. The channel matrix is given by

$$\mathbf{H}(n) = [\mathbf{h}_1 \; \mathbf{h}_2 \; \cdots \; \mathbf{h}_{M_t}] \tag{2}$$

with $\mathbf{h}_q = (h_{1,q} \; h_{2,q} \; \cdots \; h_{M_r,q})^T$ and assumed to be perfectly estimated at the receiver. With the received vector and channel matrix, the transmitted information bits are recovered through demodulation and decoding process.

III. Suboptimal IDD Receiver

Fig. 3 illustrates the proposed IDD receiver. At the first iteration, spatial demultiplexing such as linear equalization or OSIC is performed to separate the transmitted symbols and the log-likelihood ratios (LLRs) of the APPs for transmitted bits are computed considering only one transmitted symbol. From

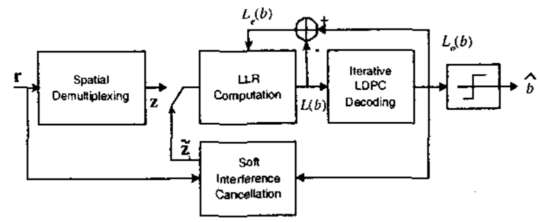

Fig. 3. The structure of the proposed receiver.

the second iteration on, the transmitted symbols are estimated with the LLRs at the decoder output, and cancelled from the received vector to compute new LLRs for LDPC decoding.

In case of a linear receiver employed for spatial demultiplexing, the received signals are equalized with an $M_t \times M_r$ matrix $\mathbf{G}$ such that

$$\mathbf{z} = \mathbf{G}\mathbf{H}\mathbf{x} + \hat{\mathbf{w}}, \tag{3}$$

where $\hat{\mathbf{w}} = \mathbf{G}\mathbf{w}$ and the resource index n is omitted for brevity. When the MMSE criterion is used, $\mathbf{G}$ is given by

$$\mathbf{G} = (\mathbf{H}^H\mathbf{H} + \frac{MN_0}{E_s}\mathbf{I}_{M_t})^{-1}\mathbf{H}^H, \tag{4}$$

and, for the ZF receiver, it is given by

$$\mathbf{G} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H. \tag{5}$$

With the equalized outputs, the LLR values are computed independently for each transmit symbol. Let $b_{i,j}$ be the jth constituent bit of the ith symbol x_i of the antenna symbol vector $\mathbf{x}$. The LLR for $b_{i,j}$ is computed with the equalized output such that

$$L(b_{i,j}) = \ln\left(\frac{\sum_{x_i \in A_j^0} \exp(-\frac{|z_i - a_{i,i}x_i|^2}{\sigma_{z_i}^2})}{\sum_{x_i \in A_j^1} \exp(-\frac{|z_i - a_{i,i}x_i|^2}{\sigma_{z}^2})}\right), \tag{6}$$

where z_i is the ith element of $\mathbf{z}$, $a_{i,j}$ is the (i,j)th element of $\mathbf{GH}$, and $\sigma_{z_i}^2 = \sum_{q=1, q\neq i}^{M_t} |a_{i,q}|^2 + \frac{M_t N_0}{E_s}\sum_{q=1}^{M_r} |g_{i,q}|^2$. Here, A_j^c is the set of $Q/2$ modulation symbols of x_i under the constraint of $b_{i,j}$ to be c ($\in \{0,1\}$). The *a priori* probability of $b_{i,j}$ is initially set to be $1/2$ assuming random source generation. For the OSIC, a similar process is performed with the interference cancelled output following the detection algorithm as in [3]. Since the antenna symbol is separated, only Q conditional probabilities are required to compute an LLR while Q^{M_t} probabilities are required to compute an LLR in the optimal demodulation.

After first iteration, each symbol on a transmit antenna is estimated such that

$$\hat{x}_i = \sum_{s \in A} s P_{x_i}(s), \tag{7}$$

where A is the set of modulation symbols and $P_{x_i}(s)$ is the probability of $x_i = s$ computed with the decoder outputs

$L_o(b_{i,j})$. With the estimated soft symbols, the interfering parts are cancelled from the received vector such that

$$\tilde{\mathbf{r}}_i = \mathbf{r} - \sum_{\substack{q=1 \\ q \neq i}}^{M_t} \mathbf{h}_q \hat{x}_q \tag{8}$$

and the interference cancelled outputs are diversity combined as

$$\tilde{z}_i = \mathbf{h}_i^H \tilde{\mathbf{r}}_i. \tag{9}$$

Due to the antenna symbol separation with interference cancellation, the LLRs are updated respectively for each antenna symbol as follows.

$$L(b_{i,j}) = \ln \left(\frac{\sum\limits_{x_i \in A_j^0} \exp(-\frac{|\tilde{z}_i - \tilde{a}_i x_i|^2}{\sigma_{\tilde{z}_i}^2}) P(x_i | b_{i,j}^0)}{\sum\limits_{x_i \in A_j^1} \exp(-\frac{|\tilde{z}_i - \tilde{a}_i x_i|^2}{\sigma_{\tilde{z}_i}^2}) P(x_i | b_{i,j}^1)} \right), \tag{10}$$

where $\tilde{a}_i = \mathbf{h}_i^H \mathbf{h}_i$, $\sigma_{\tilde{z}_i}^2 = \frac{M_t N_0}{E_s} \tilde{a}_i$, and $P(x_i | b_{i,j}^c) = \prod_{\substack{m=1 \\ m \neq j}}^{\log_2(Q)} P(b_{i,m})$, $c \in \{0, 1\}$, is the *a priori* probability of a symbol x_i with its constituent bits $b_{i,1}, b_{i,2}, \cdots, b_{i,\log_2(Q)}$, conditioned on $b_{i,j} = c$. The *a priori* probability is estimated with the extrinsic information of the LDPC decoder, $L_e(b_{i,j}) = L_o(b_{i,j}) - L(b_{i,j})$. For the following iterations, the soft interference cancellation is performed to update the decoding input LLRs using the decoder outputs.

For the exact LLR computation, the noise variance should be estimated for $\mathbf{z}$ and $\tilde{\mathbf{z}}_i$. When the cancellation is employed, the noise variance might be changed for each symbol due to imperfect cancellation and it is hardly tractable since the correct symbols are unknown. However, the probability of imperfect cancellation is relatively low in the soft interference cancellation of the proposed receiver since the cancellation is performed with LDPC decoding outputs. On the other hand, when the OSIC is used for spatial demultiplexing, cancellation is performed with less reliable signals compared to the cancellation after LDPC decoding. Thus it might result in an adverse effect on LDPC decoding.

IV. Numerical Results

In this section, we investigate the performance of the proposed receiver via Monte Carlo simulations with following parameters. The source block length of a data packet is 768 and a codeword length is 1536. A binary (1536, 768) irregular LDPC code is constructed using the method by [5] with 2 and 3 column weights to produce an average column weight of 2.7. For symbol mapping, QPSK modulation is employed and a subset of total 1536 used subcarriers is allocated. A data packet is transmitted over 384 subcarrires with $F = 4$ subcarrier spacing when $M_t = 2$, and over 192 subcarrires with $F = 8$ subcarrier spacing when $M_t = 4$. Each branch of the MIMO channel, from a transmit antenna to a receive antenna, is generated by a 6-tap delayed line model with independent complex Gaussian channel gains. The power delay profile is exponential with delays providing a rms delay spread of about 0.05 fraction of the OFDM symbol duration. Furthermore, there is no correlation among transmit antennas and among receive antennas. In the receiver, the MIMO channel is perfectly estimated and the sum-product decoding is employed for LDPC decoding with 50 decoding iterations.

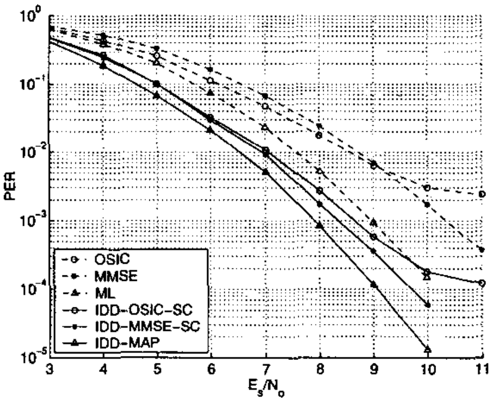

Fig. 4. The PERs of the proposed and conventional receivers when $M_t = 2$ and $M_r = 2$.

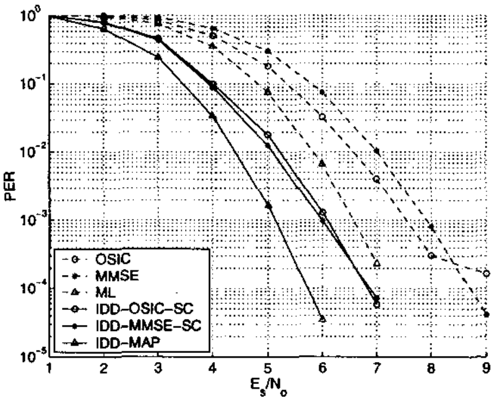

Fig. 5. The PERs of the proposed and conventional receivers when $M_t = 4$ and $M_r = 4$.

Figs. 4 and 5 show the packet error rates (PERs) of the proposed receiver with different spatial demultiplexing methods when $M_t = M_r = 2$ and $M_t = M_r = 4$, respectively. The 5 iterations are performed for the IDD receivers and the OSIC in the receivers is based on the MMSE criterion for interference nulling. In the figures, *MMSE*, *OSIC*, and *ML* denote non-iterative case of the proposed receivers (*IDD-MMSE(OSIC)-SC*) and the optimal IDD receiver (*IDD-MAP*). Without iterations between demodulation and decoding, the performance of *OSIC* lies between that of *MMSE* and *ML*. However, the proposed IDD receiver using OSIC (*IDD-OSIC-SC*) exhibits similar performance with that using MMSE (*IDD-MMSE-SC*) after IDD iterations employing the identical soft interference cancellation. Moreover, the proposed IDD

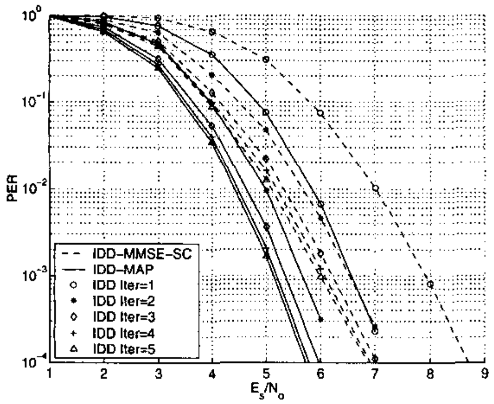

Fig. 6. The PERs of the IDD receivers with the number of IDD iterations when M_t=4 and M_r = 4.

receiver with OSIC exhibits an error floor at the high SNR region which is caused by imperfect interference cancellation in the OSIC at the first iteration. While imperfect cancellation has no effect in the low SNR region where the AWGN is dominant, it provides wrong LLRs by ignoring the interference term which is dominant in the high SNR region. Thus, the MMSE equalizer is more preferable for spatial demultiplexing of the proposed receiver in the viewpoint of complexity.

From the results, it is observed that the proposed receiver with MMSE provides us with 2dB SNR gain over the non-iterative MMSE receiver and it lies between the performance of the IDD receiver with MAP and the non-iterative receiver with ML. In Fig. 4, the proposed IDD receiver with MMSE exhibits a performance gain of 0.6dB over the non-iterative receiver with ML at the PER of 10^{-2}. At the same PER value, the performance degradation over the optimal IDD receiver (*IDD-MAP*) is only about 0.3dB. In case of more transmit and receive antennas as in Fig. 5, the performance tendency is similar except that the slope of PER curves is steeper due to the increased antenna diversity and more performance improvement is achieved through IDD receivers. In case of two transmit antennas, the complexity of the IDD receiver with MAP is manageable and thus the optimal IDD receiver can be employed. However, with the increased number of transmit antennas and receive antennas, the proposed IDD receiver can reduce the complexity of the IDD receiver without severe performance degradation.

The performance of the proposed receiver with MMSE and the IDD receiver with MAP is shown with the number of IDD iterations in Fig. 6, when $M_t = 4$ and $M_r = 4$. In the figure, *IDD Iter*= 1 denotes non-iterative cases of the proposed receiver and the IDD receiver with MAP. With *IDD Iter* = 1, the performance degradation by employing a linear receiver instead of the ML receiver is more than 1dB at the PER of 10^{-2}. However, with 5 IDD iterations employing soft interference cancellation, the performance degradation of the proposed receiver over the *IDD-MAP* is reduced to be about 0.7dB. Another observation is that the performance is drastically improved at the second iteration and further improvement is negligible after the third iteration. Thus it is likely to achieve enough performance with only three iterations. This implies that the proposed receiver with reduced complexity can provide better performance than the non-iterative ML receiver if more processing delay is allowed in the receiver.

V. Conclusions

In this paper, a suboptimal IDD receiver based on the spatial demultiplexing and post soft cancellation was presented for an LDPC-SM-OFDMA system. First observation was that the MMSE equalization is more suitable than the OSIC for spatial demultiplexing in the proposed receiver and another observation was that the performance can be considerably improved through iterations between the demodulation and decoding. While the the proposed receiver has some performance degradation over the optimal IDD receiver, it is more practical for the system with large number of transmit and receive antennas. Especially, for non-real time services, we expect a performance gain by replacing non-iterative receivers with the proposed receiver.

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