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A Turbo Coded Multicarrier DS/CDMA System with Unequal Error Protection on the Encoder Output Bits

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Abstract

This paper presents a turbo coded multicarrier direct sequence code division multiple access (DS/CDMA) system, where the outputs of a turbo encoder are repetition coded at multiple rates and transmitted in parallel over a number of subchannels. It is observed that the error floor can be lowered, with some performance loss in the low SNR region, by assigning a higher diversity order to the parity check symbols.

1 Introduction

Multicarrier DS/CDMA systems were proposed as an alternative to the classical single carrier DS/CDMA systems as a means to realize a wideband CDMA system [1] [2]. In multicarrier systems, the available bandwidth is divided into a set of disjoint equi-width subchannels, and narrowband DS/CDMA waveforms are transmitted over the subchannels. In [1], a rate $1/M$ repetition code was applied to transmit M band-limited DS/CDMA waveforms. As an extension of the work in [1], a convolutionally coded multicarrier DS/CDMA system was described and analyzed in [2], where each convolutionally encoded symbol was repetition-coded and then transmitted over independent frequency diversity branches. The systems of [1] and [2] all showed a performance similar to that of a classical single carrier DS/CDMA system which employed a RAKE receiver having a comparable receiver complexity. However, the multicarrier systems exhibited superior performance in the presence of partial-band interference, because of which it is considered in a CDMA overlay scheme [3].

In this paper, we introduce a turbo [4] coded multicarrier DS/CDMA system, where each turbo code symbol is repetition coded at a different rate and transmitted over multiple subchannels. The scheme is similar to the system in [2], except that a turbo code is used instead of a convolutional code and a different diversity order is given to each of the C encoder outputs for a rate $1/C$ turbo code. The reason for different diversity levels for different turbo code symbols is that the various code symbols do not contribute to the bit error rate in an identical manner. In [5], different energy was assigned to the two kinds of outputs of a turbo encoder, information symbols and parity symbols, for an AWGN channel, to increase the Euclidean distance and thus improve the performance. In CDMA systems, however, a different energy assignment is not desirable since more signal energy assigned to a transmitted symbol of one user would result in stronger interference to the other users. Moreover, in a fading channel, increasing channel and coding diversity is more important than increasing the Euclidean distance to combat the fading.

From this point of view, we investigate the performance of the turbo coded multicarrier DS/CDMA system when the diversity order is not the same for all the turbo code symbols, assuming the total available diversity order is fixed. It is observed that the error floor can be lowered by assigning more diversity branches to the parity check symbols, so that the minimum weight of the codewords due to the input data sequences of weight 2 increases after repetition coding. It is also observed, however, that assigning more diversity branches to the parity check symbols leads to some performance loss in the low SNR region.

2 System Model

We consider an asynchronous multicarrier DS/CDMA system with K active users. Fig. 1 shows the transmitter of the turbo coded multicarrier DS/CDMA system for the kth user. The system of Fig. 1 is based on the system described in [2], except that a turbo code is used instead of a convolutional code, each of the turbo code symbols is replicated by a repetition code of different rate, and we employ only an I modu-

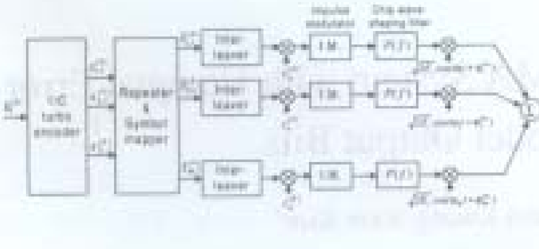

Fig. 1: The transmitter.

Fig. 2: An example of symbol mapping when $M = 12$ and a rate 1/4 turbo code is used with a repetition vector $(M_1, M_2, M_3, M_4) = (2, 5, 3, 2)$.

lator rather than I and Q modulators in order to simplify the problem.

The data bit $b_l^{(k)}$ at time l for the kth user with bit duration T_b is encoded by a rate $1/C$ turbo encoder whose output symbols are denoted by $\{x_{j,l}^{(k)}, j = 1, 2, \ldots, C\}$. Generally, the encoder is constructed from recursive systematic convolutional (RSC) encoders connected in parallel and separated by interleavers. Even though more than two RSC encoders can be concatenated to form a turbo code, we consider only the case of two RSC codes whose rates are, respectively, $1/C_1$ and $1/C_2$. For the encoder outputs, $x_{1,l}^{(k)}$ denotes the systematically transmitted binary data symbol, $\{x_{j,l}^{(k)}\}_{P_1} = \{x_{2,l}^{(k)}, \ldots, x_{C_1,l}^{(k)}\}$ denote the parity check symbols from the first RSC encoder, and $\{x_{j,l}^{(k)}\}_{P_2} = \{x_{C_1+1,l}^{(k)}, \ldots, x_{C,l}^{(k)}\}$ denote the parity check symbols from the second RSC encoder.

The symbol $x_{j,l}^{(k)}$ is repetition coded at a rate $1/M_j$ and mapped into the subchannels by the symbol mapper to properly separate, in frequency, the repetition code symbols of each turbo code symbol. For example, Fig. 2 shows a possible mapping of 4 turbo code symbols to 12 subchannels when the repetition vector (M_1, M_2, M_3, M_4) is $(2, 5, 3, 2)$. Let $\{\tilde{x}_{m,l}^{(k)}, m = 1, 2, \ldots, M\}$ be the symbol mapper outputs, where $\tilde{x}_{m,l}^{(k)}$ is the symbol transmitted through the mth subchannel at time l. More specifically, $\tilde{x}_{m,l}^{(k)} = x_{j,l}^{(k)}$ if m belongs to the index set A_j whose elements are the indices of the subchannels where $x_{j,l}^{(k)}$ is transmitted. A channel interleaver is also utilized to distribute the turbo code symbols in time such that the effects of deep fades are distributed among codewords separated in time.

The transmitted signal of the kth user is

$$s^{(k)}(t) = \sqrt{2E_c} \sum_{m=1}^{M} \sum_{n=-\infty}^{\infty} \tilde{x}_{m,\lfloor n/N \rfloor}^{(k)} c_n^{(k)} \cdot p(t - nT_c) \cos(\omega_m t + \varphi_m^{(k)}), \quad (1)$$

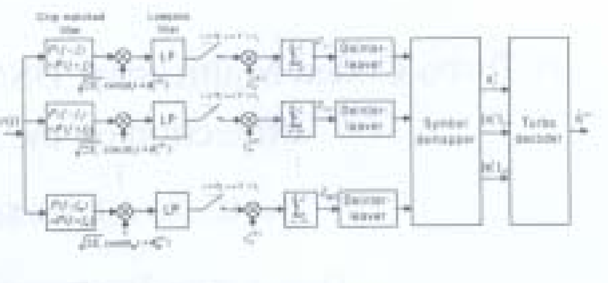

Fig. 3: The receiver.

where E_c is the energy-per-chip, $\{c_n^{(k)}\}$ are random binary signature sequences with period N, $p(t)$ is the impulse response of the chip wave-shaping filter, $1/T_c$ is the chip rate, ω_m is the mth subcarrier angular frequency, and $\{\varphi_m^{(k)}\}$ are independent identically distributed (iid) random variables uniformly distributed over $[0, 2\pi)$.

Each subchannel is assumed to be a slowly-varying frequency non-selective Rayleigh fading with impulse response function

$$h_m^{(k)}(t) = \alpha_m^{(k)} e^{j\theta_m^{(k)}} \delta(t), \text{ for } m = 1, 2, \ldots, M, \quad (2)$$

where $\{\alpha_m^{(k)}\}$ are the fading amplitudes and $\{\theta_m^{(k)}\}$ are the random phases. The fading amplitudes are, actually, correlated in time and frequency. However, the correlation can be reduced by using a proper channel interleaver and symbol mapper. Thus, we assume that the fading amplitudes are iid Rayleigh random variables with a unit second moment and the phases are iid random variables uniformly distributed over $[0, 2\pi)$.

Then the received signal is given by

$$r(t) = \sqrt{2E_c} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{n=-\infty}^{\infty} \alpha_m^{(k)} \tilde{x}_{m,\lfloor n/N \rfloor}^{(k)} c_n^{(k)} \cdot p(t - nT_c - \tau_k) \cos(\omega_m t + \phi_m^{(k)}) + n_W(t), \quad (3)$$

where the propagation delays $\{\tau_k\}$ are iid random variables uniformly distributed over $[0, T_b)$, $\phi_m^{(k)} = \varphi_m^{(k)} + \theta_m^{(k)} - \omega_m \tau_k$, and $n_W(t)$ is the AWGN with two-sided power spectral density (psd) $\eta_0/2$.

The receiver of the kth user is shown in Fig. 3. The received signal is chip-matched filtered, coherently demodulated, and lowpass filtered in each subchannel. The lowpass filter outputs are sampled every T_c, and then correlated by the desired user signature sequence. The correlator outputs are channel deinterleaved, demapped by the symbol demapper, and decoded by the turbo decoding method which will be described in Section 3. We assume that the chip wave-shaping filter $P(f)$ satisfies the Nyquist criterion and is of unit energy. We further define $G(f) = |P(f)|^2$ for the later use.

3 Turbo Decoding

3.1 Modeling of the Decoder Inputs

Here, we model the inputs to the iterative turbo decoder assuming that the first user ($k = 1$) is the desired user and that perfect carrier, code, and bit synchronization is obtained. The correlator output of the qth subchannel at time l can be written as

$$\begin{aligned} Z_{q,l}^{(1)} &= \sum_{n'=0}^{N-1} c_{n'}^{(1)} y_q(lT_b + n'T_c + \tau_1) \\ &= N\sqrt{E_c}\alpha_q^{(1)}\bar{x}_{q,l}^{(1)} + U_{q,l} + W_{q,l}, \end{aligned} \tag{4}$$

where $y_q(t)$ is the lowpass filter output on the qth subchannel after down-converting, and $U_{q,l}$ and $W_{q,l}$ are, respectively, the multiple access interference (MAI) and AWGN components of the qth correlator output at time l. It is shown in [1] that $Z_{q,l}^{(1)}$, conditioned on $\alpha_q^{(1)}$ and $\bar{x}_{q,l}^{(1)}$, is approximately Gaussian, where the approximation is valid for a large number of users, and is perfect for $K = 1$. The means of $U_{q,l}$ and $W_{q,l}$ are zero, and their variances are, respectively, given by [1]

$$Var(U_{q,l}) = \frac{N(K-1)E_c}{2T_c}\int_{-\infty}^{\infty} |G(f)|^2 df \tag{5}$$

and $Var(W_{q,l}) = \frac{N\eta_0}{2}$. Thus we can model the decoder inputs as

$$z_{q,l} = \alpha_{q,l}\bar{x}_{q,l} + I_{q,l}, \tag{6}$$

where $z_{q,l}$ and $I_{q,l}$ are, respectively, the correlator output and its total noise component, normalized by $N\sqrt{E_c}$, on the qth subchannel at time l after channel deinterleaving. Here, we have dropped the user index (1) in the superscript of symbols for notational convenience. The MAI plus AWGN term, $I_{q,l}$, is approximately a zero-mean Gaussian random variable, and its variance is given by

$$\sigma_q^2 = \frac{K-1}{2NT_c}\int_{-\infty}^{\infty} |G(f)|^2 df + \frac{\eta_0 M}{2E_b}, \tag{7}$$

where $E_b = NME_c$ is the energy-per-bit.

Before turbo decoding is performed, the normalized correlator outputs are partitioned into the C subsets $\{z_l^j, j = 1, 2, \ldots, C\}$ by the symbol demapper, where $z_l^j = \{z_{q,l}, q \in A_j\}$ is the subset composed of the correlator outputs for the subchannels where the turbo code symbol $x_{j,l}$ is transmitted. The C subsets are again divided into three groups, z_l^1 corresponding to the systematic data symbol, $\{z_l^j\}_{p_1} = \{z_l^2, \ldots, z_l^{C_1}\}$ corresponding to the parity check symbols for the first RSC decoder, and $\{z_l^j\}_{p_2} = \{z_l^{C_1+1}, \ldots, z_l^C\}$ corresponding to the parity check symbols for the second RSC decoder. These sets are the inputs to the turbo decoder, along with side information on the fading amplitudes and noise variances.

3.2 Decoding Metric

Now we describe the turbo decoding metric for our system model. The turbo decoder uses an iterative, suboptimal, soft-decoding rule where each constituent RSC code is separately decoded by its decoder. However, the constituent decoders share bit-likelihood information through an iterative process. We utilize the maximum a posteriori (MAP) algorithm for both the first and second RSC decoders as in [6] with some modification on the branch transition metric to incorporate the appropriate channel statistics for our system model.

For a fully interleaved channel with known fading amplitudes $\boldsymbol{\alpha} = \{\alpha_{1,1}, \ldots, \alpha_{M,1}, \ldots, \alpha_{1,W}, \ldots, \alpha_{M,W}\}$, the branch transition metric, defined in [4] and [6], can be written as

$$\begin{aligned} &\gamma_i((z_l^1, \{z_l^j\}_{p_1}), S_{l-1}, S_l|\boldsymbol{\alpha}) \\ &= q(b_l = i|S_{l-1}, S_l) \\ &\quad \cdot p(z_l^1, \{z_l^j\}_{p_1}|b_l = i, S_{l-1}, S_l, \boldsymbol{\alpha})\Pr\{S_l|S_{l-1}\}, \end{aligned} \tag{8}$$

where S_l denotes the encoder state at time l, which can take values between 0 and $2^\nu - 1$ for an RSC encoder with ν memory elements. The value of $q(b_l = i|S_{l-1}, S_l)$ is 1 if a transition from states S_{l-1} to S_l occurs, and is 0 otherwise. The probability $\Pr\{S_l|S_{l-1}\}$ is $\Pr\{b_l = 1\}$ if $q(b_l = 1|S_{l-1}, S_l) = 1$, is $\Pr\{b_l = 0\}$ if $q(b_l = 0|S_{l-1}, S_l) = 1$, and is 0 otherwise. Here, *a-priori* probabilities $\Pr\{b_l = i\}$ are estimated by the other decoder through the iterative decoding process. The conditional probability $p(\cdot|\cdot)$ in (8) is given by

$$\begin{aligned} &p(z_l^1, \{z_l^j\}_{p_1}|b_l = i, S_{l-1}, S_l, \boldsymbol{\alpha}) \\ &= B_l \exp\left\{\left(\sum_{q\in A_1} \frac{\alpha_{q,l} z_{q,l}}{\sigma_q^2}\right) x_{1,l}(i) \right. \\ &\quad \left. + \sum_{j=2}^{C_1}\left(\sum_{q\in A_j} \frac{\alpha_{q,l} z_{q,l}}{\sigma_q^2}\right) x_{j,l}(i, S_{l-1}, S_l)\right\}, \end{aligned} \tag{9}$$

where $x_{1,l}(i) = \pm 1$ according to $b_l = i$, $x_{j,l}(i, S_{l-1}, S_l) = \pm 1$ according to the value of the jth code symbol generated when the state transits from S_{l-1} to S_l with input $b_l = i$, and B_l is a constant which has no influence on the decoding process. In a similar way, the transition metric for the second RSC decoder can be obtained with the interleaved version of z_l^1 and the other sets of parity check outputs $\{z_l^j\}_{p_2}$.

With the transition metrics, the log-likelihood ratio (LLR) of the lth data bit b_l is computed at each decoder and code symbols are then decoded through the iterative process as described in [6].

4 Performance Evaluation

The performance of the proposed system is investigated for various repetition vectors $(M_1, M_2, \ldots, M_C)$ under the following conditions.

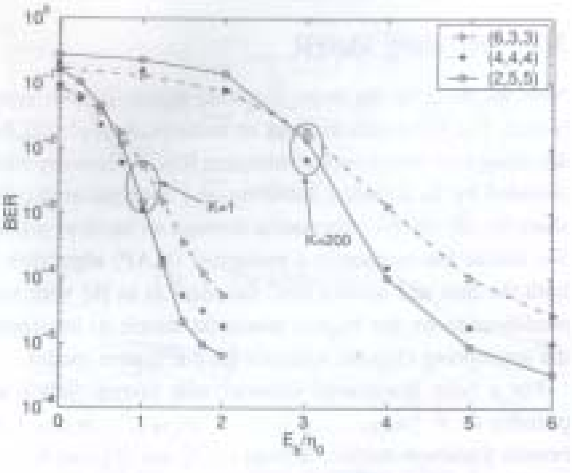

Fig. 4: The BER versus E_b/η_0 when $M = 12$, $K = 1$ and 200, $W = 1000$, and a rate 1/3 turbo code is used.

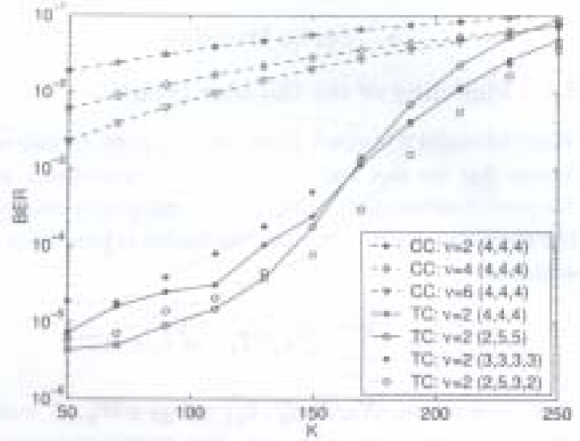

Fig. 5: The BER versus K when $M = 12$, $W = 1000$, and $E_b/\eta_0 = 3$dB.

- A random interleaver of size $W = 1000$ is used.
- The number of decoding iterations is 15.
- The total processing gain is $NM = 512$.
- The number of subcarriers is $N = 12$.
- A raised-cosine filter with rolloff factor 0.5 is used for $G(f)$.

Fig. 4 shows the simulation results versus E_b/η_0 for a rate 1/3 turbo code having generator $(1, 5/7, 5/7)$ with $\nu = 2$ memory elements for each constituent RSC encoder when $K = 1$ and 200. It is observed the system with a $(2, 5, 5)$ repetition vector performs better in the error floor region (at high SNRs). As noted in [7], the performance of a turbo code in the error floor region is dominated by the *effective free distance*, the minimum weight of codewords due to input data sequences of weight 2. The *total effective free distance*, defined in this paper by the resultant effective free distance after repetition coding is applied to the turbo code symbols, becomes $d_{t,ef} = 36$, $d_{t,ef} = 40$, and $d_{t,ef} = 44$ for the $(6, 3, 3)$, $(4, 4, 4)$, and $(2, 5, 5)$ repetition vectors, respectively. Thus, the BER at which the error floor begins can be lowered by assigning more diversity to the parity check symbols, thereby increasing the total effective free distance. Another observation is that the repetition method possessing better performance in the error floor region shows somewhat worse performance at very low SNRs. While we cannot prove it, we have a conjecture on why this happens. At low SNR values, one decoder outputs unreliable erroneous extrinsic information which adds erroneous information to the other decoder and thus it is hard for the decoders to converge on a decision of the transmitted data. Thus, if a higher diversity order is given to the systematic data, a lower diversity order is given to the parity check symbols, and the effect of erroneous information exchange will be reduced in the low SNR region. However, as the decoder begins to perform better as the SNR increases, the extrinsic information from one decoder enhances the ability of the other decoder to output more reliable LLR, and vice-versa. Thus, there is a trade-off between the performance in the low SNR region and that in the high SNR region according to how much diversity is given to the data symbols and how much is given to parity check symbols.

Fig. 5 compares the simulation results of the turbo coded system (denoted by *TC*) with different turbo code rates and repetition vectors as K varies, when $E_b/\eta_0 = 3$dB. We use the generator in octal $(1, 5/7, 5/7)$ for a rate 1/3 turbo code and $(1, 5/7, 5/7, 3/7)$ for a rate 1/4 turbo code, of which the constituent RSC codes have $\nu = 2$ memory elements. The total effective free distance for a rate 1/4 turbo code is given by $d_{t,ef} = 36$ and 40 for the $(3, 3, 3, 3)$ and $(2, 5, 3, 2)$ repetition methods, respectively. Note that the code fragment generated by the generator 3/7 has a smaller weight contribution to the effective free distance than that of the generator 5/7. Thus, as new code fragments generated by the generator 3/7 are added, the performance in the error floor region gets worse, while that in the low SNR region gets better. We also show the simulation results of convolutionally coded multicarrier DS/CDMA system (denoted by *CC*) when we use the rate 1/3 maximum free distance convolutional code having memory elements $\nu = 2$, 4, and 6, combined with a $(4, 4, 4)$ repetition vector. It is known that the complexity of decoding a turbo code is at least two times that of decoding a constituent convolutional code using the Viterbi algorithm. Thus the complexity of decoding a convolutional code with the number of memory elements more than 4 is comparable to that of decoding a turbo code constructed by RSC codes with 2 memory elements. It is observed that the performance is significantly improved by applying turbo codes instead of convolutional codes. However, the latency of the turbo coded system is larger than that of the convolutionally coded system. It is also observed that there exists a preferred combination of the repetition vector and code rate at each desirable BER for a turbo coded system.

5 Conclusions

In this paper, we investigated the performance of a turbo coded multicarrier DS/CDMA system, where the outputs of a turbo encoder were repetition coded at multiple rates and transmitted in parallel over a number of subchannels. It was observed that we could lower the error floor by assigning more diversity branches to the parity check symbols, so that the total effective free distance increased after repetition coding. However, this led to somewhat worse performance at low SNR values. Thus, we could increase the system capacity by assigning more diversity to the parity check symbols when the desirable BER was near the error floor region.

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