A DECORRELATING MULTIUSER RECEIVER FOR DS/CDMA SYSTEMS USING TCM

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Abstract

In this paper, we propose and analyze a multiuser receiver using a decorrelating filter and Viterbi decoders for trellis coded DS/CDMA systems with biorthgonal signal constellation in asynchronous channels. The biorthogonality is implemented by user signature waveforms and the decorrelating filter. The performance of the proposed system is investigated: it is shown that the proposed system can provide us with some coding gain and near-far resistance.

1 INTRODUCTION

Multiuser receivers for the DS/CDMA systems becomes an interesting research area since they can reduce the interuser interference and resist the near-far effect which are among the major defects of DS/CDMA systems [1]-[3].

Most of the work on multiuser receiver has been investigated for uncoded systems: coding has been frequently used, however, for reliable communication systems. Recently, multiuser receivers for convolutionally coded DS/CDMA and those for trellis coded DS/CDMA using phase shift keying (PSK) signal constellation were investigated in [4][5] and [6], respectively: it was shown that we could achieve some coding gain and near-far resistance simultaneously.

In [7], TCM with biorthogonal signal constellation was proposed for DS/CDMA systems: it was shown that the system has better performance than the trellis coded DS/CDMA system using C-ary PSK signal constellation. In addition, the biorthogonality could be implemented by using binary PSK and $\frac{C}{2}$ signature waveforms for each user. Thus, it is more suitable to employ biorthogonal signal constellation (than to employ PSK) in trellis coded DS/CDMA systems, and employing multiuser receivers for the systems are more adequate because of the nature of the biorthogonal signal constellation.

In this paper, we propose a multiuser receiver for trellis coded DS/CDMA systems using biorthogonal signal constellation and analyze the performance of the proposed system.

2 SYSTEM MODEL

The transmitter scheme for the trellis coded DS/CDMA system considered in this paper is shown in Fig. 1. First, m bits of an information signal is divided into m_1 and m_2 bits. The m_1 bits enter the $\frac{m_1}{m_1+1}$ convolutional encoder. Then, m_1+1 output bits from the convolutional encoder and the m_2 bits decide the signal point in the signal mapper. A signature waveform is then selected according to the signal mapper output in the signature waveform selector. After the selected signature waveform is modulated by a carrier, it is transmitted. Let $\{c_{k,l}(t) | k = 1, 2, \cdots, K,\ l = 1, 2, \cdots, L\}$ be the set of signature waveforms with period T_s, where K is the number of users and $L = 2^m$.

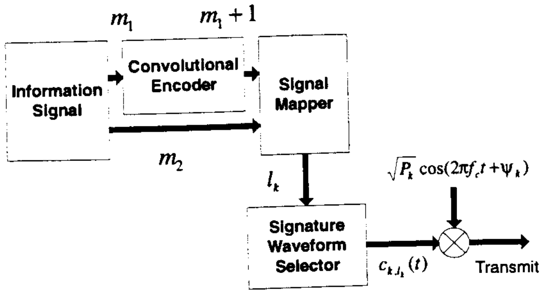

Figure 1: Transmitter structure of the trellis coded DS/CDMA systems.

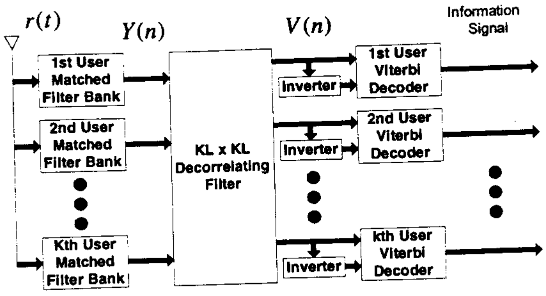

Figure 2: The receiver structure of the proposed system.

Then, we can construct the biorthogonal signal set of the kth user as $\{c_{k,l}(t)|l = 1,2,\cdots,2L\} = \{c_{k,1}(t),\cdots,c_{k,L}(t),-c_{k,1}(t),\cdots,-c_{k,L}(t)\}$. The transmitted signal from the kth mobile is then

$$u_k(t) = Re\{\sqrt{P_k}c_{k,l_k}(t)e^{j(2\pi f_c t+\psi_k)}\}, \quad (1)$$

where P_k is the transmitted power of the kth user signal, f_c is the carrier frequency, ψ_k is the random phase of the kth carrier, and l_k is the output of the signal mapper of the kth user.

We assume that the channel is slowly-varying nonselective Rayleigh and we use coherent reception. Then, the equivalent received baseband signal at the base-station is

$$r(t) = \sum_{k=1}^{K}\sum_{l=1}^{2L}\sqrt{P_k}\alpha_k e^{j\phi_k}\cdot I_{k,l}(t-\tau_k)c_{k,l}(t-\tau_k)+n(t), \quad (2)$$

where $\alpha_k e^{j\phi_k}$ is the complex fading process of the kth user, τ_k is the time delay of the kth user, $n(t)$ is additive white Gaussian noise with variance σ_n^2, and $I_{k,l}(t)$ is the indicator function of the kth user defined as

$$I_{k,l}(t) = \begin{cases} 1, & \text{if } l_k = l \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

The receiver structure considered in this paper is shown in Fig. 2. It is easily seen that the $(L+l)$th matched filter output can be obtained by multiplying -1 with the output of the lth matched filter output at each filter bank. Therefore, K filter banks each with L matched filters and a $KL \times KL$ decorrelator are necessary: the received signal passes through the bank of KL matched filters and the matched filter outputs are decorrelated by the decorrelating filter and then decoded by Viterbi decoders. The matched filter output vector can be written as

$$Y(n) = [y_{1,1}(n)\cdots y_{1,L}(n)\cdots y_{K,L}(n)]^T, \quad (4)$$

where

$$y_{p,q}(n) = \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=-1}^{1}\sqrt{P_k}\alpha_k e^{j\phi_k}\cdot x_{k,l}(n)\gamma_{p,q,k,l}^{(m)} + n_{p,q}(n), \quad (5)$$

$x_{k,l}(t) = I_{k,l}(t) - I_{k,L+l}(t)$, $l = 1,2,\cdots,L$, $\gamma_{p,q,k,l}^{(-1)}$, $\gamma_{p,q,k,l}^{(0)}$, and $\gamma_{p,q,k,l}^{(1)}$ are the cross correlations between the qth signature waveform of the pth user during the nth symbol and the lth signature waveform of the kth user during the $(n-1)$th, nth, and $(n+1)$th symbol, respectively, and $n_{p,q}(n)$ is a zero mean Gaussian random variable with

$$E\{n_{p,q}(n)n_{k,l}^*(n+m)\} = \begin{cases} \frac{\sigma_n^2\gamma_{p,q,k,l}^{(m)}}{T_s} & m=-1,0,1, \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

3 THE RECEIVER STRUCTURE

In this section, we will consider the decorrelating filter and Viterbi decoders for trellis coded DS/CDMA systems. We can rewrite the output of the bank of matched filters in vector notation as

$$Y(n) = \sum_{m=-1}^{1}\Gamma^{(m)}WX(n+m)+N(n), \quad (7)$$

where W is the $KL\times KL$ diagonal matrix whose $k\circ l$th diagonal is $w_{k,l} = \sqrt{P_k}\alpha_k e^{j\phi_k}$, $X(n)$ is the $KL\times 1$ column vector whose $k\circ l$th element is $x_{k,l}(n)$, $\Gamma^{(m)}$ is the $KL\times KL$ matrix whose $(p\circ q, k\circ l)$th element is $\gamma_{p,q,k,l}^{(m)}$, and $k \circ l = (k-1)L + l$. Then, the Z-transform $Y(Z)$ of $Y(n)$ is

$$\begin{aligned} Y(Z) &= \left(\Gamma^{(-1)}Z^{-1} + \Gamma^{(0)} + \Gamma^{(1)}Z\right) WX(Z) \\ &\quad + N(Z), \end{aligned} \tag{8}$$

where $X(Z)$ and $N(Z)$ are the Z-transforms of $X(n)$ and $N(n)$, respectively. Let $G(Z) = \sum_{k=-\infty}^{\infty} T(k)Z^{-k} = \left(\Gamma^{(-1)}Z^{-1} + \Gamma^{(0)} + \Gamma^{(1)}Z\right)^{-1}$, then the decorrelating filter output $V(Z)$ is

$$\begin{aligned} V(Z) &= [G(Z)Y(Z) \;\; -G(Z)Y(Z)]^T \\ &= \begin{bmatrix} WX(Z) + G(Z)N(Z) \\ -WX(Z) - G(Z)N(Z) \end{bmatrix} \\ &= W'X'(Z) + N'(Z), \end{aligned} \tag{9}$$

where $W' = diag\,[w_{1,1} \cdots w_{1,2L} \cdots w_{K,1} \cdots w_{K,2L}]$, $X'(Z) = \left[X^T(Z) \;\; -X^T(Z)\right]^T$, and $N'(Z) = \left[N^T(Z)G^T(Z) \;\; -N^T(Z)G^T(Z)\right]^T$. Then, the lth output $v_{k,l}(n)$ of the kth user decorrelating filter multiplied by $e^{-j\phi_k}$ is, since coherent reception is assumed,

$$v_{k,l}(n) = \sqrt{P_k}\alpha_k x'_{k,l}(n) + e^{-j\phi_k} n'_{k,l}(n), \tag{10}$$

where $x'_{k,l}(n)$ and $n'_{k,l}(n)$ are the $(k \circ l)$th elements of the inverse Z-transforms of $X'(Z)$ and $N'(Z)$, respectively.

Let the decoding depth of the trellis code be η, and Ξ be the set of all paths whose length is η. Then, the kth user's metric of $\xi_i \in \Xi$ is

$$\Delta_{k,\xi_i} = \sum_{j=1}^{\eta} v_{k,l_j^i}(j), \tag{11}$$

where $\xi_i = (l_1^i, \cdots, l_\eta^i)$ and l_j^i is the index of the signature waveform during the jth symbol of the ith path of the kth user. Then, we can choose the largest Δ_{k,ξ_i} among all $\xi_i \in \Xi$ by Viterbi soft decision algorithm.

4 PERFORMANCE ANALYSIS

Since we assume a slowly-varying channel, the fading process α_k is constant during an error event. Let ξ_d be the correct path and ξ_c be an error event with distance $D_{\xi_d,\xi_c} = \sum_{j=1}^{\eta} D_{\xi_d,\xi_c}(j) = \sum_{j=1}^{\eta}\left(x'_{k,l_j^d}(j) - x'_{k,l_j^c}(j)\right)$. Then, the pairwise error probability is

$$\begin{aligned} P_{\xi_d \to \xi_c} &= Pr\{\Delta_{k,\xi_d} < \Delta_{k,\xi_c}\} \\ &= Pr\{\sqrt{P_k}\alpha_k D_{\xi_d,\xi_c} + \bar{n}_{k,\xi_d,\xi_c} < 0\}, \end{aligned} \tag{12}$$

where $\bar{n}_{k,\xi_d,\xi_c} = e^{-j\phi_k}\sum_{j=1}^{\eta}\left(n'_{k,l_j^d}(j) - n'_{k,l_j^c}(j)\right)$. Let us define Q_a^b and R_a^b as the quotient and remainder of l_a^b divided by L, respectively. Then, after slight manipulations (see [8]), it is easily seen that $\bar{n}_{k,\xi_d,\xi_c}$ is a Gaussian random variable with mean zero and variance $\frac{\sigma_n^2 \beta_{\xi_d,\xi_c}}{T_s}$, where

$$\beta_{\xi_d,\xi_c} = \sum_{j_1=1}^{\eta}\sum_{j_2=1}^{\eta} b_{\xi_d,\xi_c}(j_1,j_2), \tag{13}$$

$$\begin{aligned} & b_{\xi_d,\xi_c}(j_1,j_2) \\ &= (-1)^{Q_{j_1}^d + Q_{j_2}^d}\left[T^k(j_2-j_1)\right]_{R_{j_1}^d, R_{j_2}^d} \\ &+ (-1)^{Q_{j_1}^c + Q_{j_2}^c}\left[T^k(j_2-j_1)\right]_{R_{j_1}^c, R_{j_2}^c} \\ &- (-1)^{Q_{j_1}^d + Q_{j_2}^c}\left[T^k(j_2-j_1)\right]_{R_{j_1}^d, R_{j_2}^c} \\ &- (-1)^{Q_{j_1}^c + Q_{j_2}^d}\left[T^k(j_2-j_1)\right]_{R_{j_1}^c, R_{j_2}^d}, \end{aligned} \tag{14}$$

$[\cdot]_{i,j}$ denotes the ijth element of a matrix, and $T^k(\cdot)$ is the kth $L \times L$ submatrix of the $KL \times KL$ matrix $T(\cdot)$ such that $[T^k(\cdot)]_{i,j} = [T(\cdot)]_{k\circ i,k\circ j}$.

Then, the instantaneous signal to noise ratio (SNR) ν_k of the kth user is

$$\nu_k = \frac{E_k \alpha_k^2 D_{\xi_d,\xi_c}^2}{2\sigma_n^2 \beta_{\xi_d,\xi_c}}, \tag{15}$$

where $E_k = P_k T_s$ is the transmitted symbol energy, and the conditional pairwise error probability is

$$P_{\xi_d \to \xi_c}(\nu_k) = \frac{1}{2} erfc(\nu_k). \tag{16}$$

Therefore, the pairwise error probability can be obtained as

$$P_{\xi_d \to \xi_c} = \frac{1}{2}\left(1 - \sqrt{\frac{\kappa}{\kappa+1}}\right), \tag{17}$$

where $\kappa = \frac{E_k E\{\alpha_k^2\} D_{\xi_d,\xi_c}^2}{2\sigma_n^2 \beta_{\xi_d,\xi_c}}$. We can approximate (17) as

$$P_{\xi_d \to \xi_c} \approx \frac{1}{4\kappa}, \tag{18}$$

when the SNR is high. We can see from (17) or (18) that the performance of the kth user is independent of the energy level of the other users. In other words, the proposed system is near-far resistant.

Next, let $g(\xi_i, \xi_j)$ be the number of different information bits between ξ_i and ξ_j. Then, the bit error probability P_b of the proposed system satisfies

$$P_b \le \sum_{\xi_i \in \Xi} g(\xi_i, \xi_d) P_{\xi_d \to \xi_i}. \tag{19}$$

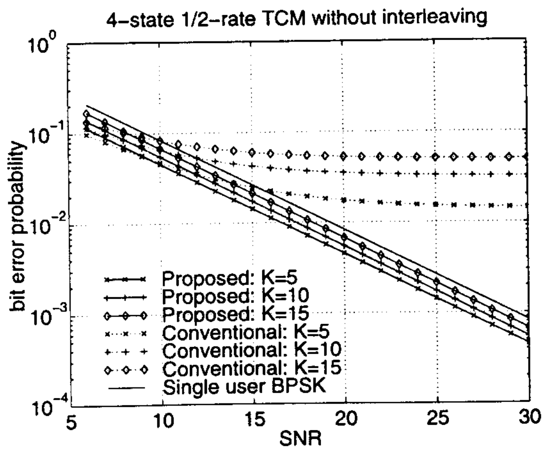

Figure 3: Bit error probabilities of the proposed and conventional systems

At moderate to high values of SNR, the bit error probability is dominated by the free-distance error events [9]. Thus, the bit error probability can be approximated as

$$P_b \approx g_f P_{\xi_d \to \xi_f}, \qquad (20)$$

where $g_f = \sum_{\xi_i \in \Xi_f} g(\xi_i, \xi_d)$ and Ξ_f is the set of the free-distance error events.

In Fig. 3, bit error probabilities of the proposed and conventional systems are plotted when we use 4-state $\frac{1}{2}$-rate TCM. We use Gold sequences of period 63 as the user signature sequences and the time delays (τ_k) of users are uniform over $[0, T_s]$. It is clearly seen that the performance of the conventional system suffers from the error floor and gets severely worse as the number of users increases, while the performance of the proposed system is affected only a little by the increase of the number of interferers.

5 CONCLUSION

In this paper, we proposed a multiuser receiver using a decorrelating filter and Viterbi decoders for trellis coded DS/CDMA systems with biorthgonal signal constellation in asynchronous channels, and investigated the performance of the proposed system. It is shown that the proposed system is near-far resistant and has some coding gain over uncoded systems. The performance of the proposed system naturally degrades as the number of active users increases: the performance is, however, still much higher than that of conventional trellis coded DS/CDMA systems.

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