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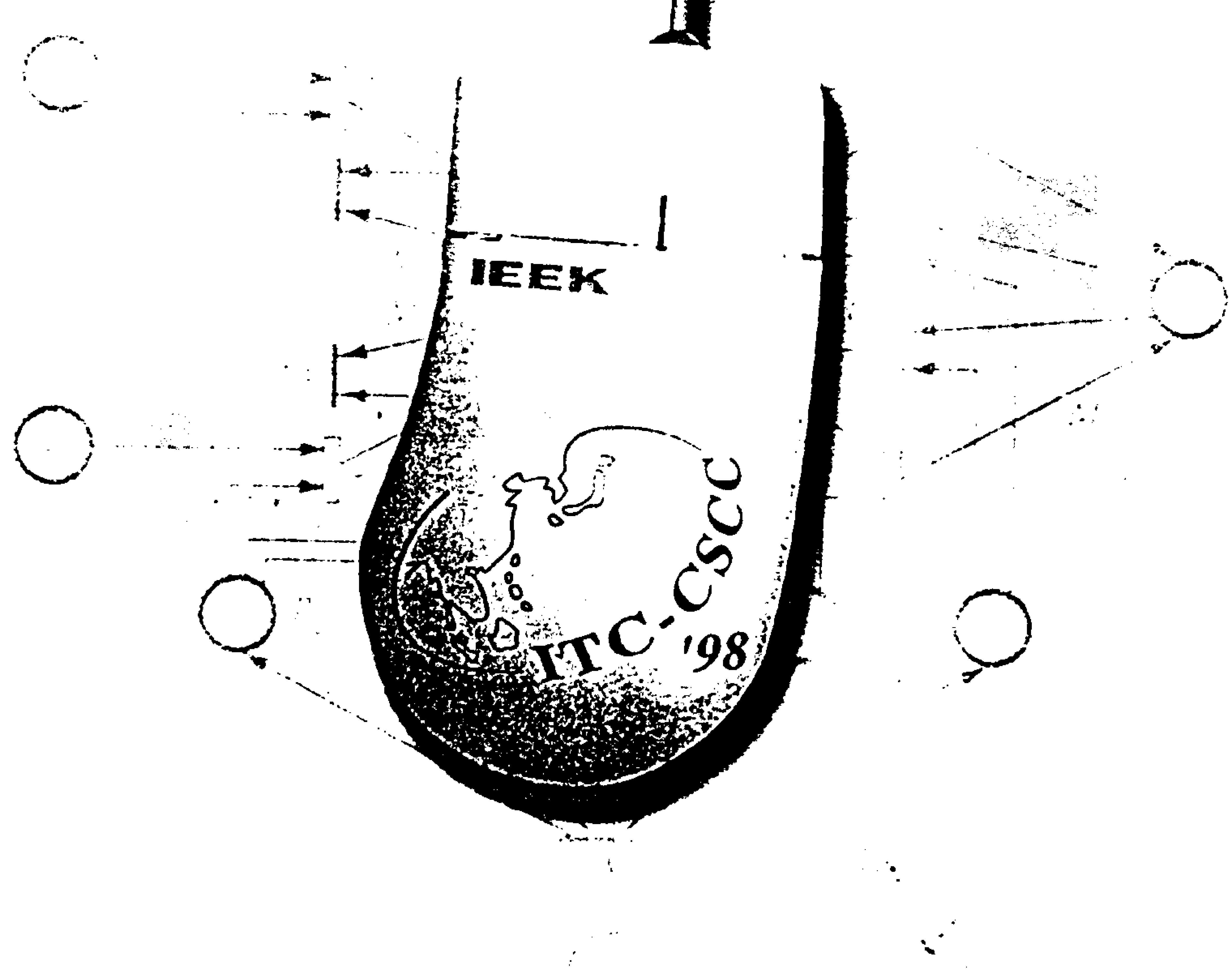

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Analysis of CDMA Using New Orthogonal Sequences

Seong Ill Park, Seok Ho Yoon, Kwang Soon Kim, Suk Chan Kim,
Yun Hee Kim, and Iickho Song *

Department of Electrical Engineering
Korea Advanced Institute of Science and Technology (KAIST)
373-1 Guseong Dong, Yuseong Gu
Daejeon 305-701, Korea
Tel : +82-42-869-3445
Fax : +82-42-869-3410
e-mail : isong@Sejong.kaist.ac.kr

Abstract— In this paper, new polyphase sequences and a sequence generation method are suggested. The correlation properties of the sequences are investigated. These sequences have good correlation properties. Since the suggested generation method consists only of integer sums and modular techniques, sequence generation is also easy. The performance of the sequences is investigated for QS-CDMA systems in frequency selective, time nonselective, slow Nakagami fading channel with additive white Gaussian noise.

I. Introduction

For direct sequence code division multiple access (DS/CDMA) systems, some sequences are suggested: among the examples are the $m-$sequences [1] or Gold's sequences [2]. These sequences, however, have some co-channel interference. For example, the variance of inter-user interference is $\frac{K-1}{3N}$, where K is the number of users and N is the spreading ratio [3]. The co-channel interference in a system apparently lowers to some degree the performance of the system. Some methods of interference cancellation for such sequences are investigated for several applications. However, since interferences in the systems using such sequences are produced from the correlation properties of the user code sequences, it is not possible to completely eliminate the inter-user interferences in multiple access systems.

In [4], an orthogonal sequence is proposed. The sequence, however, has some disadvantages. One is that since the sequence is generated by the DFT method, it is very complex to generate sequences. Among the other disadvantages are that there exist some non-zero cross-correlations among the sequences and that the sequence is useful only for synchronous channels.

In this paper, we suggest a semi-orthogonal sequence named *PS sequence*, which has several good correlation properties. The auto correlation function of the sequence is 0 except at some periodic intervals and that of the cross-correlation function between properly selected sequences is 0. Thus, we can completely reject the inter-user interference in a system with the PS sequence. Since a restriction for using the PS sequence in CDMA systems is that the first chips of the information symbols of the users are nearly synchronized, the PS sequence can be used in such practical situations as the down-link of DS/CDMA systems and both up-link and down-link of *quasi-synchronous* CDMA (QS-CDMA) systems.

*All correspondence should be addressed to this author.

II. Definition and generation of the sequence

A. The new sequence

Definition 1. Let us define the $N \times N$ DFT matrix as

$$F_{N,m} = \left[W_N^{-klm}\right], \tag{1}$$

where $k, l = 0, 1, ..., N-1$, $W_N = e^{2\pi j/N}$, $j = \sqrt{-1}$, and $m = 1, 2, ..., N-1$.

Definition 2. The diagonalized matrix $D(\{x_l\})$ of a sequence $\{x_l\ ,\ l = 0, 1, ..., H\}$ is defined as

$$D(\{x_l\}) = diag(\{x_l\}). \tag{2}$$

Definition 3. Let the quotient and residual functions Q and R be defined as

$$Q(\alpha, \beta) = q,\ R(\alpha, \beta) = r, \tag{3}$$

where $\alpha = q\beta + r$, $0 \leq r < \beta$, and $q \geq 0$.

Let us define the *basic* complex symbols as a set of N_b symbols β_i, $i = 0, 1, 2, ..., N_b - 1$, all with equal magnitude (not necessarily distinct): without loss of generality, we assume β_i's are located on the unit circle of the complex plane. For example, $\{W_3^0 = 1, W_3^1, W_3^2\}$ is a set of *basic* complex symbols.

We first generate an orthogonal sequence from β_i's. For a set $\{\beta_i\}$ of basic complex symbols and $1 \leq m \leq N_b - 1$, the *basic orthogonal sequence matrix* G of size $N_b \times N_b$ is defined as

$$G = F_{N_b,m}^{-1} D(\{\beta_i\}). \quad (4)$$

Next, the *basic orthogonal sequence* $\{g_p\}$ of length N_b^2 is defined by, for $p = 0, ..., N_b^2 - 1$,

$$\begin{aligned} g_p &= G_{Q(p,N_b),R(p,N_b)} \\ &= \beta_{R(p,N_b)} W_{N_b}^{Q(p,N_b)R(p,N_b)m}, \end{aligned} \quad (5)$$

where G_{ab} means the ath row, bth column element of G. Using the basic orthogonal sequence $\{g_p\}$, we make an $N_s \times K$ matrix H as

$$H = [h_{ik}], \quad (6)$$

where

$$h_{ik} = \sum_{p=0}^{N_b^2-1} g_p \delta(i - k - pK), \quad (7)$$

$N_s = KN_b^2$, and K is a natural number. The first column of H starts with g_0 followed by $K-1$ 0's, then g_1 followed by $K-1$ 0's, ..., and $g_{N_b^2-1}$ followed by $K-1$ 0's. Other columns of H are shifted vectors of the first column.

Then the *PS sequence matrix* C of size $N_s \times K$ is defined as

$$C = \frac{1}{N_b} F_{N_s,1}^{-1} H = [c_{l,k}], \quad (8)$$

where, as shown in [5],

$$c_{l,k} = W_{N_s}^{lk} \sum_{p=0}^{N_b-1} \beta_p W_{N_b^2}^{lp} \delta(R(l + mp, N_b)). \quad (9)$$

The sequence $\{c_{l,k}, l = 0, 1, ..., N_s - 1\}$ (a column of C) will be called a *PS sequence.*

B. Simpler generation of the PS sequence

In the above subsection, we have introduced the PS sequence. The generation, however, involves a number of matrix multiplications: in this subsection, a method for simpler generation is introduced.

Without loss of generality, we can assume $\beta_p \in \{W_V^i, i = 0, 1, ..., V-1\}$, where V is a positive integer. Consider a function P defined by $P(\beta_p) = v_p$ when $\beta_p = W_V^{v_p}$. Then (9) becomes

$$\begin{aligned} c_{l,k} &= \sum_{p=0}^{N_b-1} W_{N_s}^{lk} W_V^{v_p} W_{N_b^2}^{lp} \delta(R(l + mp, N_b)) \\ &= W_{V_s}^{i_s}, \end{aligned} \quad (10)$$

where $i_s = Vl(k + Kp_s) + P(\beta_{p_s})N_s$, $V_s = VN_s$, and p_s satisfies $R(l + mp_s, N_b) = 0$.

Since the simpler method does not require DFT calculations and matrix multiplications, the sequences can be generated quite easily.

III. Characteristics of the PS sequence

A. Autocorrelation

As shown in [5], the autocorrelation function of the PS sequence is

$$A(\tau) = N_s W_{N_b^2}^{\tau k} \delta(R(\tau, N_b^2)). \quad (11)$$

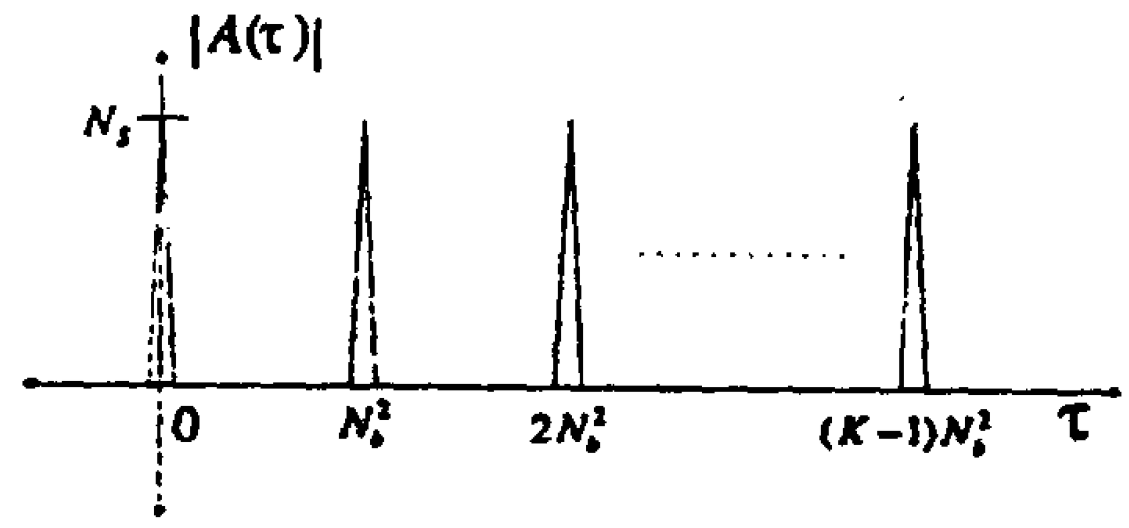

Fig. 1. The autoautocorrcorrelation function of the PS sequence.

As we can see in (11) and Figure 1, the autocorrelation function has a nonzero value only when $\tau = iN_b^2$, $i = 0, 1, ..., K-1$. That is, delayed PS sequences will have no autocorrelation at some intervals: we can control the interval by choosing the value of N_b. On the other hand, the PN sequence has nonzero values of the autocorrelation function at all intervals.

B. Cross-correlation

Let us denote two PS sequences as $\left\{c_{l,k}^{I}\right\}$ and $\left\{c_{l,k}^{II}\right\}$ Then, as shown in [5], the cross-correlation function o the two sequences is

$$\begin{aligned} C(\tau) = KN_b W_{N_s}^{\tau k^I} \sum_{p=0}^{N_b-1} \sum_{q=0}^{N_b-1} \beta_p^I \beta_q^{II*} W_{N_b^2}^{\tau p} \\ \delta(R(\tau + m^I p - m^{II} q, N_b)) \\ \delta(\{k^I - k^{II} + K(p-q)\}), \end{aligned} \quad (1$$

where $*$ represent the complex conjugate and the superscripts I and II of k, m, and β are used to distingui the two sequences.

In Table 1, the normalized absolute values of the cross correlation function are shown for 3 distinct cases.

Case	$\|C(\tau)\|/N_s, \tau = 0, 1, ..., N_s - 1$
$k^I \neq k^{II}$	0
$k^I = k^{II} = k, m^I \neq m^{II}$	$\frac{1}{N_b}$
$k^I = k^{II} = k, m^I = m^{II}, \{\beta^I\} \neq \{\beta^{II}\}$	$\frac{1}{N_b}\left\|\sum_{p=0}^{N_b-1} \beta_p^I \beta_p^{II*} W_{N_s}^{\tau p}\right\| \delta(R(\tau, N_b))$

TABLE I

A SUMMARY OF THE CROSS-CORRELATION FUNCTION NORMALIZED ABSOLUTE VALUES.

the first case ($k^I \neq k^{II}$), we see the normalized absolute cross-correlation function is 0. In this case, the cross-correlation is not dependent on the values of the other parameters, m and β. When $k^I = k^{II} = k$ and $m^I \neq m^{II}$, the absolute value of the normalized cross-correlation function is $1/N_b$. Finally, when $k^I = k^{II} = k$, $m^I = m^{II}$, and $\{\beta^I\} \neq \{\beta^{II}\}$ (in other words, when only the β's for the two sequences are different), the normalized absolute value of the cross-correlation function is 0 except for $\tau = nN_b, n = 0, \pm1, \pm2, ..., \pm(KN_b - 1)$. In this case, if we focus on the zero-shifted interval ($\tau = 0$) of the cross-correlation, the normalized absolute value can be made to be 0 at $\tau = 0$ by choosing β_i's to satisfy $\sum_{p=0}^{N_b-1} \beta_p^I \beta_p^{II*} = 0$.

Now, we need some definitions and theorems for clearer explanations.

Definition 4. An m_k*-subset* of the *PS sequence* is the set of $\{c_{lk}\}$'s generated by the same values of k and m.

Definition 5. We define a k*-class* of the PS sequence as a collection of m_k*-subsets* which have the same value of k.

Note that the family of k*-classes* constitutes a partition of the PS sequence, and the family of m_k*-subsets* constitutes a partition of a k*-class*.

Theorem 1. The autocorrelation function of the *PS sequence* is zero except when the time difference is a multiple of the square of the number of the *basic symbols*.

Theorem 2. Two PS sequences chosen from different k*-classes* have no cross-correlation.

Theorem 3. The absolute value of the cross-correlation between two PS sequences chosen from different m_k*-subsets* in the same k*-class* is $1/N_b$.

Theorem 4. If $\sum_{p=0}^{N_b-1} \beta_p^I \beta_p^{II*} = 0$, the absolute value of the cross-correlation $C(\tau)$ between two PS sequences chosen from the same m_k*-subset* is zero for $|\tau| < N_b$ and $iN_b < |\tau| < (i+1)N_b$, $i = 1, 2, ..., KN_b - 2$.

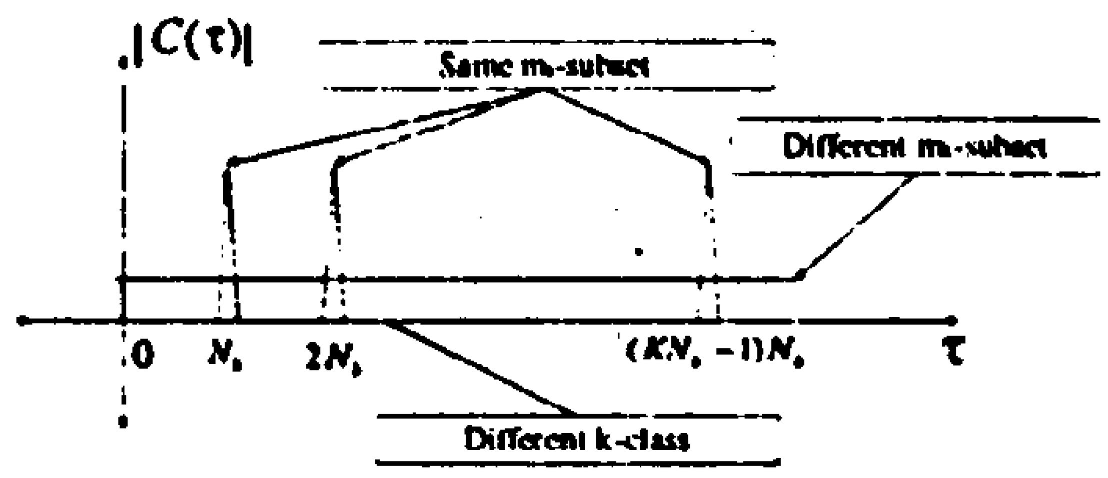

Fig. 2. The cross-correlation function of the PS sequence.

Proofs of Theorems 1-4 can be found in [5]. In Figure 2, we show the normalized cross-correlation function for various cases. Theorem 2 implies that we can choose K sequences which do not have any cross-correlation. Viewing the N_b basic symbols $\beta_i, i = 0, 1, .., N_b - 1$ as a complex vector $\vec{\beta}$, there exist (at most) N_b orthogonal vectors in the complex vector space C^{N_b}: this means that there are N_b sequences with $\sum_{p=0}^{N_b-1} \beta_p^I \beta_p^{II*} = 0$ in an m_k*-subset*. Thus, from Theorems 2 and 4, we can choose KN_b sequences which have no cross-correlation for $|\tau| < N_b$.

IV. Applications in QS-CDMA systems

Here, the channel is modeled as frequency selective, time nonselective, slow Nakagami fading channel with additive white Gaussian noise. To modulate information symbols, M-ary phase shift keying (MPSK) is used. Coherent reception is considered to analyze the performance of QS-CDMA systems. For the receiver, RAKE receiver model is assumed.

Here, Nakagami probability density function is $f_{l,k}(x) = M(x, p, \Omega_l^{(k)})$ with $M(r, p, \Omega) = \frac{2p^p r^{2p-1}}{\Gamma(p)\Omega^p} \exp\left(-\frac{p}{\Omega} r^2\right)$, where p is a positive real number, $\Omega_l^{(k)} = \Omega_0^{(k)} e^{-\delta l}, \delta \geq 0$, and δ is a positive real number. Gaussian noise has a variance of $\eta_0/2$.

Under these assumptions, the average symbol error probability is [5]

$$P_M = \sqrt{\frac{\gamma_s}{1+\gamma_s}} \frac{(1+\gamma_s)^{-p_s} \Gamma\left(p_s + \frac{1}{2}\right)}{\sqrt{\pi}\Gamma(p_s+1)} {}_2F_1\left(1, p_s + \frac{1}{2}; p_s + 1; \frac{1}{1+\gamma_s}\right), \quad (13)$$

where $\gamma_s = \frac{\gamma\, q(L-r,2\delta)}{2p\, q(L-r,\delta)} \sin^2 \frac{\pi}{M}$, $p_s = p\frac{q^2(L_r,\delta)}{q(L_r,2\delta)}$, $\Gamma(z)$ is the gamma function, ${}_2F_1(a, b; c; z)$ is a hypergeometric function defined as ${}_2F_1(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{(c)_k k!}$ with $(a)_k = a(a+1)\cdots(a+k-1), (a)_0 = 1, q(a, b) =$

$\frac{1-e^{-ab}}{1-e^{-b}}$, and $\gamma = \left(\frac{\eta_0}{2E\frac{N}{N}\Omega_0}\right)^{-1}$. From (13), we can see that there are some energy losses in the transmitted symbol by a factor of $2L/N$. However, it is clear that the inter-user interferences are fully rejected. Furthermore, since the factor $2L/N$ is relatively quite small in practical systems, we lose only a small fraction of energy, and could get high performance gain. Thus we can expect that the system performance will be highly improved when compared with the performance of other systems without PS sequence.

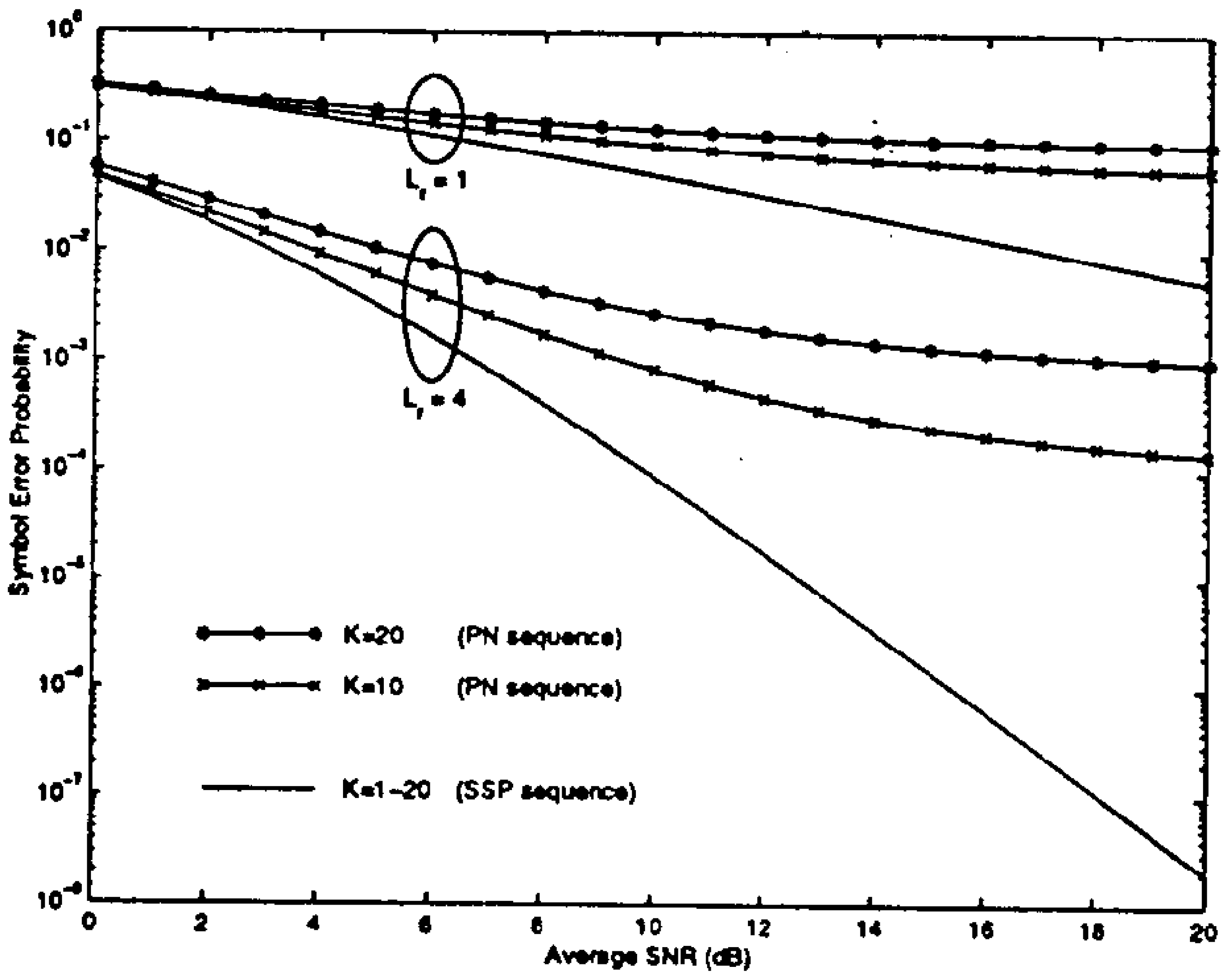

Fig. 3. The symbol error probabilities of the system by using PS sequences and PN sequences, when $p = 1$, $L_p = 4$, $\delta = 0.2$, $M = 2$, $N = 194$, $L = 7$, $N_s = 180$ and $L_r = 1, 4$.

Let us define the average signal to noise ratio (SNR) as $\gamma_0 \equiv \frac{E\Omega_0}{\eta_0}$. In Figure 3, the symbol error probabilities of the systems with PN sequences and those with PS sequence are shown: parameters are $p = 1$, $L_p = 4$, $\delta = 0.2$, $M = 2$, $N = 194$, $L = 7$, and $N_s = 180$. It is clear to see that the symbol error probabilities of system with PN sequence deteriorate when the number of users increases. When $L_r = 4$, we need SNR $\simeq$ 10 dB to get $P_b = 10^{-3}$ with the PN sequence for 10 users, while, with the PS sequence, SNR $\simeq$ 6 dB is needed even for 20 users.

V. Concluding remark

We have suggested a new class of sequences and a simple method of generating the sequence. Systems with the suggested sequences will have no inter-user interference because of the correlation properties of the sequence. In addition, the suggested sequence can be generated by a method consisting only of integer sums and modular techniques, which makes the system easy to implement. The performance of QS-CDMA systems with RAKE receiver in selective slow Nakagami fading channels using the suggested sequences was investigated. Since there is no inter-user interference, the system using the PS sequence far outperforms systems using other sequences.

Acknowledgments

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