PERFORMANCE ENHANCEMENT OF THE DECORRELATING DETECTOR USING ANTENNA ARRAYS

Kwang Soon Kim, Seong Ill Park, Hong Gil Kim, Yun Hee Kim, and Iickho Song

Department of Electrical Engineering
Korea Advanced Institute of Science and Technology (KAIST)
373-1 Guseong Dong, Yuseong Gu
Daejeon 305-701, Korea
Tel : +82-42-869-3445
Fax : +82-42-869-3410
e-mail : isong@Sejong.kaist.ac.kr

ABSTRACT

In this paper, a vector channel model is proposed and some statistical properties of the asymptotic efficiency of the decorrelating detector with base-station antenna arrays are investigated. It is shown that we can get gain over the conventional decorrelator by employing antenna arrays and the gain increases as the number of antennas increases and the angle dispersion decreases. It is also shown that we can increase the asymptotic efficiency of the decorrelating detector with base-station antenna arrays up to unity if we use infinite number of antennas when the channel is angle-nondispersive: we cannot, however, increase the asymptotic efficiency of the decorrelating detector by employing base-station antenna arrays if the dispersion is infinite.

1. INTRODUCTION

Recently, code division multiple access (CDMA) techniques and multiuser detection became highlighted research areas in mobile communication systems in order to share an additive white Gaussian multiple access channel among many users. It has been shown that CDMA techniques can offer higher spectral efficiency than others and we can reduce interuser interferences and the near-far problem which are the major drawbacks of CDMA techniques by employing multiuser detection [1]-[3].

The optimum multiuser detector and its asymptotic efficiency have been investigated in [1][2]. While the performance of an optimum multiuser detector gives the lower bound of the error probability, the complexity of the decision algorithm increases exponentially with the number of users. Because of this complexity, a decorrelating approach is considered as a suboptimum multiuser detector [3].

In [4]-[6], a decorrelating multiuser detector with base station antenna arrays has been investigated: we can get not only a combining gain from antenna array diversity but also a higher asymptotic efficiency by using space diversity.

In mobile communication systems, the signal received at base station antenna array includes not only a direct path signal, but also angularly spread signals that are coherent, phase-delayed, and amplitude-weighted replicas of the direct path signal. In other words, the signal observed from the array can be regarded as a superposition of a number of plane waves: some such vector channel models were investigated in [7]-[9].

In this paper, we will propose a vector channel model and investigate the statistical properties of the asymptotic efficiency of the decorrelating detector with base-station antenna arrays in the proposed model.

2. SYSTEM AND CHANNEL MODEL

We consider the reverse link from a mobile station to a base station with antenna arrays in a synchronous channel. We assume that we use a uniform linear array with M elements equally spaced at interval d. Let the impulse response of the mth antenna be

$$h_m(t) = \sum_{i=1}^{N} h_i \delta\left(t - \frac{l_{i,m}}{c}\right), \tag{1}$$

where N is the number of multipaths, h_i is the ith attenuation factor, $l_{i,m}$ is the path length of the ith path from a mobile to the mth antenna, and c is the speed of light. Then, the frequency response of the mth antenna is

$$H_m(f) = \sum_{i=1}^{N} h_i e^{-j2\pi f \frac{l_{i,m}}{c}}. \tag{2}$$

Let us assume that the array and each wave arriving at the array are at the same plane and that the wave is planar. We further assume that the antenna frequency response is flat over the bandwidth of the transmitted signal, i.e., we can use a narrowband signal model. Then, the frequency response of the base-station antenna array is

$$H(f) = e^{\frac{-j2\pi l_c}{\lambda}} \sum_{i=1}^{N} h_i e^{j\phi_i} v(\theta_i), \tag{3}$$

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where $\phi_i = e^{\frac{-j2\pi(l_{i,1}-l_c)}{\lambda}}$, l_c is the distance between the mobile and the first antenna element, $v(\theta_i) = [1\ e^{\frac{-2\pi d \sin\theta_i}{\lambda}} \cdots e^{\frac{-2\pi(M-1)d \sin\theta_i}{\lambda}}]^T$, and θ_i is the angle of arrival of the ith wave. Then, the vector channel can be characterized by

$$\tilde{a} = \sum_{i=1}^{N} h_i e^{j\phi_i} v(\theta_i), \quad (4)$$

and the received signal at the base-station antenna array is

$$r(t) = \tilde{a} u(t-\tau). \quad (5)$$

Now, let us consider a statistical model for $\tilde{a}$. First, we assume the followings.

Assumption 1. The quantities h_i, ϕ_i, and θ_i are independent random variables.

Assumption 2. The sequences $\{h_i\}_{i=1}^N$, $\{\phi_i\}_{i=1}^N$, and $\{\theta_i\}_{i=1}^N$ are i.i.d random variables with $E\{\theta_i\} = \theta_c$ and $E\{e^{j\phi_i}\} = 0$.

Assumption 3. $E\{Nh_i\} < \infty$ for arbitrary N.

Then, the following theorem holds.

Theorem 1. Let $a = E\{v(\theta)|\theta_c\}$ and $\alpha e^{j\phi} = \sum_{i=1}^N h_i e^{j\phi_i}$. Then, $\tilde{a}$ converges to $\alpha e^{j\phi} a$ in probability as $N \to \infty$ and α is a Rayleigh random variable and ϕ is uniformly distributed random variable over $[0, 2\pi]$.

Proof of the theorem is given in [10]. In practical situations in huge cities, there are a lot of reflected rays received at the base-station antenna array. Thus, we can get a statistical model for the vector channel from Theorem 1.

The distribution models of θ_i have been considered in [7]-[9]. In this paper, we will assume that $\frac{d \sin\theta_i}{\lambda}$ is a normally distributed random variable with mean $\frac{d\sin\theta_c}{\lambda}$ and variance η. Then, the channel vector a can be obtained as

$$\begin{aligned} a &= \frac{1}{\sqrt{2\pi\eta^2}} \int_{-\infty}^{\infty} v(\theta) e^{\frac{\left(\frac{d\sin\theta}{\lambda} - \frac{d\sin\theta_c}{\lambda}\right)^2}{2\eta^2}} d\theta \\ &= \left[1\ e^{-\eta^2 - \frac{jd\sin\theta_c}{\lambda}} \cdots e^{-((M-1)\eta)^2 - \frac{j(M-1)d\sin\theta_c}{\lambda}}\right]^T. \end{aligned} \quad (6)$$

After beamforming and decorrelating, the output of the detector is [4]-[6]

$$\begin{aligned} \rho(n) &= \Lambda^{-1} Y_a(n) \\ &= WX(n) + \Lambda^{-1} N(n) \\ &= WX(n) + N_d(n), \end{aligned} \quad (7)$$

where $X(n)$ is the user information vector, Λ is the decorrelating matrix whose ijth element is $\gamma_{i,j}(a_i^H a_j)/(\|a_i\|\|a_j\|)$, $\gamma_{i,j}$ is the cross-correlation between the ith and the jth user's signature waveforms, a_k is the channel vector of the kth user, $Y_a(n)$ is the beamformer output column vector whose ith element is $a_i^H y_i/\|a_i\|$, W is a diagonal matrix whose kth diagonal is $\sqrt{P_k}\alpha_k e^{j\phi_k}\ \|a_k\|$, T_s is the symbol period, and the covariance matrix of $N(n)$ is $\frac{\sigma_n^2}{T_s}\Lambda$. Thus, the covariance matrix of $N_d(n)$ is $\frac{\sigma_n^2}{T_s}\Lambda^{-1}$ and the asymptotic efficiency (AE) of the kth user is

$$\begin{aligned} AE_k &= [\Lambda^{-1}]_{k,k}^{-1} \\ &= \frac{Det(\Lambda)}{[Adj(\Lambda)]_{k,k}}, \end{aligned} \quad (8)$$

where $[R]_{i,j}$ is the ijth element of R. We can easily see that the AE is dependent on the channel vectors of the users. In the following sections, we will consider the statistical property of the asymptotic efficiency.

3. ASYMPTOTIC EFFICIENCY FOR TWO USERS

In this section, we will consider the asymptotic efficiency when the number of active users is two. Asymptotic efficiency in this case is

$$\begin{aligned} AE &= 1 - \gamma_{1,2}\gamma_{2,1} d_{1,2} d_{2,1} \\ &= 1 - \gamma_{1,2}^2 \|d_{1,2}\|^2, \end{aligned} \quad (9)$$

where $d_{p,k} = (a_p^H a_k)/\|a_p\|\|a_k\|$. Since $\|d_{1,2}\|$ is a random variable, we will define the expected asymptotic efficiency (EAE) as

$$\begin{aligned} EAE &= E\{AE\} \\ &= 1 - \gamma_{1,2}^2 E\{\|d_{1,2}\|^2\}. \end{aligned} \quad (10)$$

Let us define $T_k = \|a_k\|^2 = \sum_{p=0}^{M-1} e^{-2(p\eta_k)^2}$. Then, $\|d_{1,2}\|^2$ can be obtained as

$$\begin{aligned} \|d_{1,2}\|^2 &= \frac{\|a_1^H a_2\|^2}{\|a_1\|^2\|a_2\|^2} \\ &= \frac{1}{T_1 T_2} \sum_{p=0}^{M-1} \beta_{1,2}(p) \cos p\theta, \end{aligned} \quad (11)$$

where

$$\beta_{i,j}(p) = \begin{cases} \sum_{q=0}^{M-1} e^{-2q^2(\eta_i^2+\eta_j^2)} & p = 0, \\ 2\sum_{q=0}^{M-p-1} e^{-((p+q)^2+q^2)(\eta_i^2+\eta_j^2)} & p > 0. \end{cases} \quad (12)$$

Thus, we have

$$\begin{aligned} E\{\|d_{1,2}\|^2\} &= \frac{1}{T_1 T_2} \sum_{p=0}^{M-1} \beta_{1,2}(p) \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos p\theta d\theta \\ &= \frac{\beta_{1,2}(0)}{T_1 T_2}. \end{aligned} \quad (13)$$

The EAE can then be obtained as

$$\begin{aligned} EAE &= 1 - \gamma_{1,2}^2 E\{\|d_{1,2}\|^2\} \\ &= 1 - \frac{\gamma_{1,2}^2 \beta_{1,2}(0)}{T_1 T_2}. \end{aligned} \quad (14)$$

In Fig. 1, the EAE is plotted, when the number of active users is two, and the number of antennas is $1, 2, \cdots, 5$. We can see that the EAE increases as the number of antennas increases or the dispersion decreases.

Let us consider the case where we use infinite number of antennas. To simplify the problem, we consider the case in which $\eta_1 = \eta_2 = \eta$. If we assume that η is small, then

$$\begin{aligned} \lim_{M\to\infty} T_1 = \lim_{M\to\infty} T_2 &= \sum_{p=0}^{\infty} e^{-2(p\eta)^2} \\ &\approx \frac{1}{\eta}\int_0^{\infty} e^{-2x^2}dx \\ &= \sqrt{\frac{\pi}{8\eta^2}}, \end{aligned} \tag{15}$$

$$\begin{aligned} \lim_{M\to\infty} \beta_{1,2}(0) &= \sum_{p=0}^{\infty} e^{-4(p\eta)^2} \\ &\approx \frac{1}{\eta}\int_0^{\infty} e^{-4x^2}dx \\ &= \sqrt{\frac{\pi}{16\eta^2}}. \end{aligned} \tag{16}$$

Thus, we obtain

$$\lim_{M\to\infty} EAE \approx 1 - \gamma_{1,2}^2 \frac{2\eta}{\sqrt{\pi}}. \tag{17}$$

We can see again that the EAE approaches unity as $\eta \to 0$ (no dispersion case) and the EAE cannot be unity if $\eta > 0$. Consider the case $\eta \to \infty$ (infinite dispersion case). In such a case, it is easy to see that $T_1 = T_2 = 1$ and $\beta_{1,2}(0) = 1$. Thus, we cannot get any gain from space diversity.

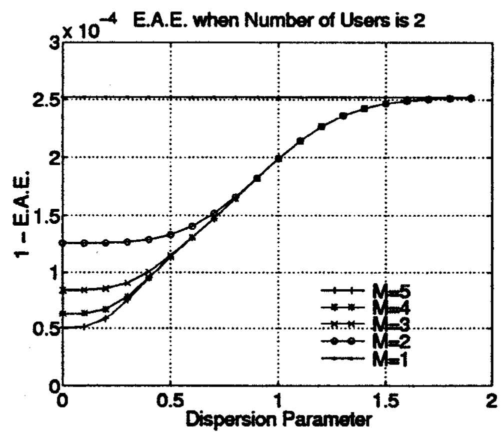

Figure 1: EAE when the number of active users is two, the period of Gold sequence is 63, and $\eta_1 = \eta_2 = \eta$.

4. ASYMPTOTIC EFFICIENCY FOR K USERS

In this section, we will consider the asymptotic efficiencies when the number of active users is K. In such a case, the kth user's EAE is

$$EAE_k = E\{[\Lambda^{-1}]_{k,k}^{-1}\}. \tag{18}$$

Let $\Lambda^{(l)}$ be an $l \times l$ submatrix of Λ whose ijth element is the ijth element of Λ for $l = 1, 2, \cdots, K$. For notational simplicity, let $Q_l = \left(\Lambda^{(l)}\right)^{-1}$. Since Λ is a Hermitian matrix, we can obtain Q_l as [11]

$$\begin{aligned} Q_l &= \begin{bmatrix} Q_{l-1} & 0 \\ 0^T & 0 \end{bmatrix} + \kappa_l^{-1} \cdot \\ & \begin{bmatrix} -Q_{l-1}\lambda_l \\ 1 \end{bmatrix} \begin{bmatrix} -\lambda_l^H Q_{l-1} \\ 1 \end{bmatrix}^T, \end{aligned} \tag{19}$$

where $\lambda_l = [\gamma_{1,l}d_{1,l}\ \gamma_{2,l}d_{2,l}\ \cdots\ \gamma_{l-1,l}d_{l-1,l}]^T$ and $\kappa_l = 1 - \lambda_l^H Q_{l-1}\lambda_l$. Then, the AE of the Kth user can be written as

$$AE_K = \kappa_K^{-1} \tag{20}$$

Now, we can assume that $\frac{d}{\lambda}\sin\theta_k \ mod\ 2\pi$, $k = 1, 2, \cdots, K$ are independent uniform random variables over $[-\pi, \pi]$. Then $\psi_{p,q} = \frac{d}{\lambda}(\sin\theta_p - \sin\theta_q)\ mod\ 2\pi$, $p = 1, 2, \cdots, q-1, q+1, \cdots, K$ are also independent uniform random variables over $[-\pi, \pi]$ and $d_{p,q}$, $p = 1, 2, \cdots, q-1, q+1, \cdots, K$ are independent random variables. Then, it can be easily seen that $d_{i,j}$, $i = 1, 2, \cdots, K-1$, $j = 1, 2, \cdots, K-1$ and $d_{l,K}$, $l = 1, 2, \cdots, K$ are independent, and Q_{l-1} and λ_K are independent. Thus, the EAE of the Kth user can be obtained as

$$\begin{aligned} EAE_K &= E\{\kappa_K^{-1}\} \\ &= 1 - E\left\{\lambda_K^H Q_{l-1}\lambda_K\right\} \\ &= 1 - \sum_{p=1}^{K-1}\sum_{q=1}^{K-1} \gamma_{p,K}\gamma_{q,K} \cdot \\ & \quad E\left\{[Q_{l-1}]_{p,q}\right\}\zeta_{p,q}, \end{aligned} \tag{21}$$

where

$$\begin{aligned} \zeta_{p,q} &= E\{d_{p,K}d_{q,K}^*\} \\ &= \begin{cases} \frac{1}{T_pT_qT_K^2} & p \neq q, \\ \frac{\beta_{p,K}(0)}{T_p^2T_K^2} & p = q. \end{cases} \end{aligned} \tag{22}$$

In order to get the EAE_K from (21), we should know $E\{Q_{l-1}\}$: it is, however, almost impossible to get an exact value, and we need some approximation. The ijth element of $\Lambda^{(K-1)}$ is $\gamma_{i,j}d_{i,j}$, in which $d_{i,j}$ can be regarded as a reducing factor of the cross-correlation of user signature waveforms due to the space diversity. Then, we can consider the effective cross-correlation $\gamma_{p,q}^e$ of the user signature waveforms as

$$\begin{aligned} \gamma_{p,q}^e &= \gamma_{p,q}\sqrt{E\{\|d_{p,q}\|^2\}} \\ &= \gamma_{p,q} \cdot \frac{\sqrt{\beta_{p,q}(0)}}{T_pT_q}. \end{aligned} \tag{23}$$

Then, we can approximate $E\{Q_{l-1}\}$ to

$$E\{Q_{l-1}\} \approx \left(\Lambda_e^{(K-1)}\right)^{-1}, \qquad (24)$$

where $\Lambda_e^{(K-1)}$ is a $(K-1)\times(K-1)$ matrix whose ijth element is $\gamma_{i,j}^e$. Hence, we can obtain the approximate of the EAE of the Kth user is

$$EAE_K \approx 1-\sum_{p=1}^{K-1}\sum_{q=1}^{K-1}\gamma_{p,K}\gamma_{q,K}\left[\left(\Lambda_e^{(K-1)}\right)^{-1}\right]_{p,q}\zeta_{p,q}. \qquad (25)$$

Note that we can obtain the EAE of the kth user when the number of active users is K by exchanging the kth low with the Kth low and the kth column with the Kth column in Λ.

In Fig. 2, the EAE's from (25) and from simulation results are plotted when the number (M) of antennas is 1, 2, 3 and the number (K) of active users is 50. In simulation, $\frac{d}{\lambda}\sin\theta_k$, $k=1,2,\cdots,K$ are randomly generated with the uniform pdf over $[0,2\pi]$, and $\eta_k=\eta$ for $k=1,2,\cdots,K$. We use Gold sequence with period 63 and get each result from 100 trials. We can see that the EAE increases as the number of antennas increases or the dispersion decreases, and that the approximation in (25) is quite close to the exact value.

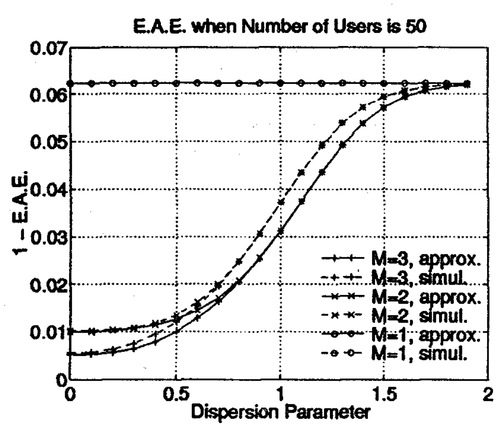

Figure 2: EAE when the number of active users is 50, the period of Gold sequence is 63, and $\eta_k=\eta$ for $k=1,2,\cdots,K$.

5. CONCLUDING REMARKS

In this paper, we proposed a statistical model for vector channels and investigated the asymptotic efficiencies of the decorrelating detector with base-station antenna arrays. It was shown that the asymptotic efficiencies can be increased by employing antenna arrays and the gain increases as the number of antennas increases and the angle dispersion decreases. It was also shown that we can increase the asymptotic efficiency of the decorrelating detector with base-station antenna array up to unity with infinite number of antennas if the channel is angle-nondispersive and that we cannot increase the asymptotic efficiency of the decorrelating detector by employing base-station antenna array even if we use infinite number of antennas when the angle dispersion is infinite.

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