Suboptimum Multiuser Detection of DS/CDMA Systems Using Antenna Arrays in Asynchronous Channels

Kwang Soon Kim*, Yun Hee Kim*, Jooshik Lee*, Iickho Song*†, and Suk Chan Kim*

* Department of Electrical Engineering
Korea Advanced Institute of Science and Technology (KAIST)
373-1 Guseong Dong, Yuseong Gu
Daejeon 305-701, Korea
Tel : +82-42-869-3445
Fax : +82-42-869-3410
e-mail : isong@sejong.kaist.ac.kr

† Communications Research Laboratory
Department of Electrical and Computer Engineering
McMaster University
Hamilton, ON, L8S 4K1, Canada

Abstract

In this paper, we consider quasi maximum likelihood (quasi-ML) detection in the reverse link system which uses antenna arrays in asynchronous channels. It is shown that the proposed quasi-ML detector can be regarded as a beamformer followed by a decorrelator, that the proposed system performs better than the conventional decorrelator, and that the performance gain over the conventional decorrelator system increases as the number of active users and number of antenna arrays increase.

1 Introduction

Although the CDMA technology offers higher spectral efficiency than others [1] and has many other desirable features, DS/CDMA system has major drawbacks of interuser interference and near-far problem. In addition, even its large capacity would not satisfy the explosive demands for mobile communication in the near future. Thus, some additional method to increase the spectral efficiency should be considered. Antenna array diversity techniques are investigated in [2][3] to increase the capacity of DS/CDMA systems. Diversity reception is an effective way to reduce the fading effects of wireless systems: however, it is insufficient to eliminate the interuser interference and restore fully the CDMA system capacity [4].

As a technique to eliminate interuser interference and to resist the near-far problem, multiuser detection has been considered in [5][6][7][8]. While the performance of an optimum multiuser detector gives the lower bound of the error probability, the complexity of the decision algorithm increases exponentially as the number of users. Because of the complexity of the optimum multiuser detector, the decorrelating approach is considered as a suboptimum multiuser detector in [5], and a system that uses a decorrelator and an antenna array is introduced in [9]. The performances of these systems are better than those of conventional systems because the multiuser interference can be eliminated at the expense of noise correlation and enhancement if the cross correlation matrix of signature waveforms is perfectly known. These systems, however, have a drawback that decorrelating enhances and correlates the noise, and the enhancement and correlation of noise gets larger as the cross-correlation gets larger or the number of users gets larger.

In this paper, we propose a quasi maximum likelihood (quasi-ML) detector in the reverse link from a mobile to a base station which uses antenna arrays in asynchronous channels. The performance of the proposed system will be investigated.

2 System model

We consider the reverse link from a mobile station to a base station with an antenna array in asynchronous channels. The channel is assumed to be frequency se-

0-7803-4198-8/97/$10.00

lective fading described by a wide sense stationary uncorrelated scattering (WSSUS) model and the characteristic of the channel is slowly varying: in other words, the coherence time of the channel is much larger than the data symbol duration. In mobile station, information bits are multiplied by a spreading sequence. After multiplied by a carrier, the signal is transmitted. We assume that we use binary phase shift keying (BPSK). Then, the transmitter signal modulated by a carrier frequency f_c is

$$u_k(t) = \sqrt{P_k} Re\{x_k(t)c_k(t)\exp[j(\omega_c t + \phi_k)]\}, \quad (1)$$

where $x_k(t)$ is the kth user's baseband information signal, $c_k(t)$ is the signature waveform of the kth user, ϕ_k is the random phase of the kth carrier, and P_k is the kth user's transmitted power.

Then, the equivalent complex baseband received signal vector at the receiver of the base station with antenna array is

$$r(t) = \sum_{k=1}^{K} s_k(t-\tau_k)c_k(t-\tau_k)e^{j\phi_k} + n(t), \quad (2)$$

where $s_k(t) = \sqrt{P_k}\alpha_k x_k(t)$, K is the number of users, α_k is the attenuation factor of the kth user, $\phi_k = -\omega_c\tau_k$, a_k is the $M\times 1$ channel vector of the kth user, and $n(t)$ is the $M \times 1$ additive temporally and spatially white complex Gaussian noise vector with covariance matrix ${\sigma_n}^2 I$. We assume that the channel is Rayleigh fading and the channel vectors remain constant during the symbol period. In other words, the channel vector of the kth user can vary only at the beginning of the symbol period and remains constant during the symbol period.

The receiver scheme considered in this paper is shown in Fig. 1. We use a uniform linear array which has M elements. For simplicity, we assume the symbol synchronous channel, which means the time delays are smaller than the symbol duration: the result, however, can be extended easily to the general case. Let $Y(n)$ be the $M \times K$ output matrix from the code correlator, then we can write $Y(n)$ as

$$Y(n) = [y_1(n)\ y_2(n)\ \cdots\ y_K(n)], \quad (3)$$

$$y_p(n) = \sum_{k=1}^{K}\sum_{m=-1}^{1} s_k(n+m)\gamma_{p,k}^{(m)} a_k(n+m) + n_p(n), \quad (4)$$

where

$$\gamma_{p,k}^{(-1)} = \begin{cases} \frac{1}{T_s}\int_{\tau_p}^{\tau_k} c_k(t-\tau_k)c_p(t-\tau_p)dt, & \tau_k > \tau_p \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$\gamma_{p,k}^{(0)} = \frac{1}{T_s}\int_{\max(\tau_p,\tau_k)}^{\min(\tau_p,\tau_k)+T_s} c_k(t-\tau_k)c_p(t-\tau_p)dt, \quad (6)$$

$$\gamma_{p,k}^{(1)} = \begin{cases} \frac{1}{T_s}\int_{\tau_k}^{\tau_p} c_k(t-\tau_k)c_p(t-\tau_p)dt, & \tau_k < \tau_p \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

and

$$n_p = \frac{1}{T_s}\int_{\tau_p+(n-1)T_s}^{\tau_p+nT_s} n(t)c_p(t-\tau_p)dt. \quad (8)$$

If we know the signature waveform and time delay of each user, we can compute $\gamma_{p,k}^{(-1)}$, $\gamma_{p,k}^{(0)}$, and $\gamma_{p,k}^{(1)}$. The signature waveforms are known a priori and the time delay of each user can be estimated by the method described in [3].

The covariance matrix of $n_p(n)$ can be obtained as

$$E\{n_p(n)n_p^H(n)\} = {\sigma'_n}^2 I, \quad (9)$$

where ${\sigma'_n}^2 = \frac{{\sigma_n}^2}{T_s}$.

3 Quasi-ML detection

In this section, we will investigate the quasi-ML detection of user signals. From the code correlated output, we can set the likelihood function $L_p(n)$ as

$$L_p(n) = C\exp\Bigg[-\Bigg(y_p(n) - \sum_{k=1}^{K}\sum_{m=-1}^{1} s_k(n+m)\cdot \gamma_{p,k}^{(m)} a_k(n+m)\Bigg)^H R_{nn,p}^{-1}\Bigg(y_p(n) - \sum_{k=1}^{K} \sum_{m=-1}^{1} s_k(n+m)\gamma_{p,k}^{(m)} a_k(n+m)\Bigg)\Bigg], \quad (10)$$

where $C = \|\pi R_{nn,p}\|^{-1}$, $R_{nn,p} = E\{n_p(n)n_p^H(n)\}$, $p = 1,\cdots,K$. Since the noise is spatially and temporally white, i.e. $R_{nn,p} = \sigma_n' I$, the ML estimates of $s_p(n)$ can be obtained by taking the partial derivative of $\log L_p(n)$ and setting the result to zero.

$$\hat{s}_p(n) = \left(\gamma_{p,p}^{(0)}\|a_p(n)\|\right)^{-1}\Bigg[\frac{a_p^H(n)y_p(n)}{\|a_p(n)\|} - \sum_{\substack{k=1\\k\neq p}}^{K}\sum_{m=-1}^{m} s_k(n+m)\gamma_{p,k}^{(m)}\|a_k(n+m)\|d_{p,k}^{(m)}(n)\Bigg], \quad (11)$$

where $d_{p,k}^{(m)}(n) = a_p^H(n)a_k(n+m)/\|a_p(n)\|\|a_k(n+m)\|$. In (11), we need to know $s_k(n+m)$, $k = 1,\cdots,p-1,p+1,\cdots,K$ and $m = -1,0,1$, in order to estimate $s_p(n)$.

If we replace $\hat{s}_k(n+m)$, $k = 1, \cdots, p-1, p+1, \cdots, K$ and $m = -1, 0, 1$, for $s_k(n+m)$, $k = 1, \cdots, p-1, p+1, \cdots, K$ and $m = -1, 0, 1$ and combine the likelihood function of code correlated output of each user, we can obtain the following equation:

$$Y_a(n) = Q(n)W\hat{X}_a(n), \tag{12}$$

where

$$Q(n) = \Gamma_n^{(-1)} \cdot D + \Gamma_n^{(0)} + \Gamma_n^{(1)} \cdot D^{-1}, \tag{13}$$

$$\Gamma_n^{(m)} = \begin{bmatrix} \epsilon_{1,1}^{(m)}(n) & \epsilon_{1,2}^{(m)}(n) & \cdots & \epsilon_{1,K}^{(m)}(n) \\ \epsilon_{2,1}^{(m)}(n) & \epsilon_{2,2}^{(m)}(n) & \cdots & \epsilon_{2,K}^{(m)}(n) \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{K,1}^{(m)}(n) & \epsilon_{K,2}^{(m)}(n) & \cdots & \epsilon_{K,K}^{(m)}(n) \end{bmatrix} \tag{14}$$

for $m = -1, 0, 1$, $\epsilon_{p,k}^{(m)}(n) = \gamma_{p,k}^{(m)} d_{p,k}^{(m)}(n)$, D is the delay operator (i.e., $Da(n) = a(n-1)$), $W = diag([w_1 \; w_2 \; \cdots \; w_K])$, $w_k = \sqrt{P_k}\alpha_k e^{j\phi_k}$, $X_a(n) = [x_{a,1}(n) \; x_{a,2}(n) \; \cdots \; x_{a,K}(n)]^T$, $x_{a,k}(n) = \|a_k(n)\|x_k$, $\hat{X}_a(n)$ is the estimate of $X_a(n)$, and $Y_a(n) = [a_1^H(n)y_1(n)/\|a_1(n)\| \; a_2^H(n)y_2(n)/\|a_2(n)\| \; \cdots \; a_K^H(n)y_K(n)/\|a_K(n)\|]^T$.

Since each element of $Q(n)$ is a polynomial of D, we can see that each element of $Q^{-1}(n)$ is a rational function of D. Thus, $W\hat{X}_a(n)$ is a linear combination of $Y_a(n)$, delayed version of $Y_a(n)$, and delayed version of $W\hat{X}_a(n)$. We can design a time varying inverse filter whose input is $Y_a(n)$ and output is $W\hat{X}_a(n)$: the coefficients of the filter is time-varying and can be computed if we know the signature waveforms and the sequence $d_{p,k}^{(m)}(n)$, $m = -1, 0, 1$. Since we can estimate the channel vector by the method in [3], we can compute $d_{p,k}^{(m)}(n)$, $m = -1, 0, 1$ and the filter coefficients. The inverse filter derived is a time-varying filter and we can treat it as a filter whose coefficient is varying with time epoch n. Let the impulse response of the filter at time epoch n be $H_n(k)$, and define the transfer function of the filter at time epoch n as $G_n(Z) = \sum_{k=-\infty}^{\infty} H_n(k)$. Then, it is easily seen that the inverse filter has the following form:

$$G_n(Z) = \left[\Lambda_n^{(-1)} Z^{-1} + \Lambda_n^{(0)} + \Lambda_n^{(1)} Z\right]^{-1}, \tag{15}$$

where $\Lambda_n^{(m)}$ is a $K \times K$ matrix whose elements consist of $\gamma_{p,k}^{(m)}$ and $d_{p,k}^{(m)}(n)$, $m = -1, 0, 1$, $k = 1, 2, \cdots K$, and $p = 1, 2, \cdots, K$. Note that in the systems proposed in [5][9], the filter $G'(Z) = \left(R^{(-1)}Z^{-1} + R^{(0)} + R^{(1)}Z\right)^{-1}$ is used for the asynchronous multiuser detection, where $R^{(m)}$, $m = -1, 0, 1$, are the correlation matrices of user signature waveforms. In the proposed system, on the other hand, we use a filter with the same structure, but the coefficients of the filter depend on the channel vectors of each user. We can implement the proposed filter by adding some more memories and multipliers to the filter used in [5][9].

4 Comparison with the conventional decorrelating approach

In the conventional decorrelating approach with a base station antenna array, decorrelation precedes beamforming. In the proposed system, on the other hand, the received signal is beamformed first and decorrelated later. With the proposed approach, we can get not only diversity combining but also the advantage of reducing the inevitable noise correlation and enhancement due to decorrelation. The decorrelating filter proposed in this paper is similar to the conventional one: we can see that the former can be regarded as the later when the cross-correlation of signature waveforms are reduced. The reducing effect is due to the beamforming, and it can be seen that the space diversity is used not only to get diversity combining gain but also to reduce the effective cross-correlation of user signature waveforms (i.e., to reduce the noise enhancement and correlation).

The near-far resistance and asymptotic efficiency of the kth user of the conventional decorrelating detector is

$$\bar{\eta}_k^d = \left[\frac{1}{2\pi}\int_0^{2\pi} \left[G'(e^{j\omega})\right]_{k,k}^{-1} d\omega\right]^{-1}. \tag{16}$$

Similarly, the near-far resistance and asymptotic efficiency of the kth user of the proposed system can be written as

$$\bar{\eta}_k^p = \left[\frac{1}{2\pi}\int_0^{2\pi} \left[G_n(e^{j\omega})\right]_{k,k}^{-1} d\omega\right]^{-1}. \tag{17}$$

We have already seen that $\Lambda_n^{(1)}$, $\Lambda_n^{(0)}$, and $\Lambda_n^{(-1)}$ have smaller off-diagonal terms than $R^{(-1)}$, $R^{(0)}$, and $R^{(1)}$, respectively. Thus, it is easily seen that the near-far resistance and asymptotic efficiency of the proposed system is higher than those of the conventional system.

5 Performance analysis

In this section, we analyze the performance of the proposed system in the sense of bit error probability. Without loss of generality, we assume that the first user signal is the desired signal.

The filter output can be written as

$$\begin{aligned} V_n(Z) &= WX_a(Z) + N_g(Z) \\ &= WX_a(Z) + G_n(Z)N(Z), \end{aligned} \quad (18)$$

where $N(Z)$ is the Z-transform of a Gaussian noise vector sequence. The Z-transform of the noise covariance matrix is $R_n(Z) = \Lambda_n^{(-1)}Z^{-1} + \Lambda_n^{(0)} + \Lambda_n^{(1)}Z$. Then, the output sequence of the first user is

$$v_1(n) = \sqrt{P_1 a_1^H(n)a_1(n)}\alpha_1 e^{j\phi_1}x_1(n) + n_{g,1}(n), \quad (19)$$

where $n_g(n) = Z^{-1}\{N_g(Z)\}$ and $n_g(n) = [n_{g,1}(n)\ n_{g,2}(n)\ \cdots\ n_{g,K}(n)]^T$. Let $S_n(Z) = G_n(Z) \cdot R_n(z)G_n^*(1/Z^*) = \sum_{k=-\infty}^{\infty} T_n(k)Z^{-k}$, then the noise covariance matrix can be written as

$$E\{n_g(n)n_g^H(n)\} = \sigma'^2_n T_n(0). \quad (20)$$

If we assume coherent reception, we can get the decision variable $\rho_1(n)$ of the first user as

$$\begin{aligned} \rho_1(n) &= \sqrt{P_1 a_1^H(n)a_1(n)}\alpha_1 e^{-j\phi_1}v_1(n) \\ &= P_1 a_1^H(n)a_1(n)\alpha_1^2 x_1(n) \\ &\quad + \sqrt{P_1 a_1^H(n)a_1(n)}\alpha_1 e^{-j\phi_1}n_{g,1}(n). \end{aligned} \quad (21)$$

Then the instantaneous SNR ν_1 of the first user is

$$\nu_1 = \frac{E_1 a_1^H(n)a_1(n)\alpha_1^2}{\sigma_n{}^2[T_n(0)]_{1,1}}, \quad (22)$$

where E_1 is the transmitted symbol energy of the first user and $[T_n(0)]_{i,j}$ is the ijth element of $T_n(0)$. Since α_1 is a Rayleigh random variable, ν_1 is a chi-square random variable with 2 degrees of freedom. Hence, we can get the bit error probability as

$$\begin{aligned} P_{b,1} &= \int_0^\infty \frac{1}{2} erfc(x) f_{\nu_1}(x)dx \\ &= \frac{1}{2}\left[1 - \sqrt{\frac{\kappa_1}{1+\kappa_1}}\right], \end{aligned} \quad (23)$$

where $\kappa_1 = \frac{E_1 a_1^H(n)a_1(n)E\{\alpha_1^2\}}{\sigma_n{}^2[T_n(0)]_{1,1}}$.

6 Simulation results

In this section, we use randomly generated channel vectors. For each resolvable path, we set the channel response vector as the linear combination of 20 array response vectors whose angles are uniformly distributed in $[0, 2\pi]$. The coefficients are randomly generated with a complex Gaussian pdf and normalized, i.e. the sum of the squares of the absolute values of the coefficients is equal to 1. The time delay of each resolvable path is also uniformly distributed. We use a Gold code of length 63 and set $E\{\alpha^2\} = 0.2$.

In Figs. 2-4, the bit error probabilities of the two systems are plotted when the Rake diversity is not combined and the number of active users is 10, 30, and 50. In each figure, we have three solid lines and three dotted lines. The solid lines are the bit error probability curves when the number M of antennas is 2, 3, and 4 for the proposed system, and the dotted lines are the bit error probability curves for the conventional system. We can see that we can get more gain over the conventional system as the number of antennas gets larger: the gain also gets larger as the number of active users increases.

It is clearly seen that higher spectral efficiency can be achieved by using the proposed system and the performance gain of the proposed system over the conventional decorrelator system increases as the number of active users and number of antennas used increase.

7 Concluding remarks

In this paper, we proposed a quasi-ML detector employing antenna arrays in asynchronous channels. We analyzed the performance of the proposed system and saw that the enhancement and correlation of noise which was the major drawback of the conventional decorrelator system could be reduced. The proposed quasi-ML detector can be considered as a system which performs beamforming first and decorrelating later. It was shown that we can get some gain over the conventional decorrelator system and the gain got larger as the number of active users and the number of antennas increased. Thus, we can get considerably higher capacity with the proposed system than that of the conventional decorrelator system.

Acknowledgements

This research was supported by the Ministry of Information and Communications under a Grant from University Basic Research Fund, Korea, and the Natural Sciences and Engineering Research Council (NSERC), Canada, for which the authors would like to express their thanks.

References

[1] K.S. Gilhousen, I.M. Jacobs, R. Padovani, A.J. Viterbi, L.A. Weaver, and C. Wheatly, "On the capacity of a cellular CDMA system," *IEEE Tr. Veh. Tech.*, Vol. VT-40, pp. 303-312, May 1991.

[2] A.F. Naguib, A. Paulaj, and T. Kailath, "Capacity improvement with base station antenna array in cellular CDMA," *IEEE Tr. Veh. Tech.*, Vol. VT-43, pp. 691-689, August 1994.

[3] A.F. Naguib, *Adaptive Antennas for CDMA Wireless Network,* Ph. D. Dissertation, Stanford University, 1996.

[4] C. Kchao and G. Stuber, "Performance analysis of a single cell direct sequence mobile radio system," *IEEE Trans. Commun.*, Vol. COM-41, pp. 1507-1516, October 1993.

[5] S. Verdu, "Multiuser Detection," in *Advances in Signal Processing* (H.V. Poor and J.B. Thomas, eds.), vol. 2: Signal Detection, pp. 369-409, CT: JAI Press, 1992

[6] S. Verdu, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Trans. Inform. Theory*, Vol. IT-32, pp. 85-96, January 1986.

[7] R. Lupas and S. Verdu, "Near-far resistance of multiuser detector in asynchronous channels," *IEEE Trans. Commun.*, Vol. COM-34, pp. 496-508, April 1986.

[8] M. Varanasi, "Noncoherent detection in asynchronous multiuser channels," *IEEE Trans. Inform. Theory*, Vol. IT-39, pp. 157-176, January 1993.

[9] Z. Zvonar, "Combined multiuser detection and diversity reception for wireless CDMA systems," *IEEE Tr. Veh. Tech.*, Vol. VT-45, pp. 205-211, February 1996.

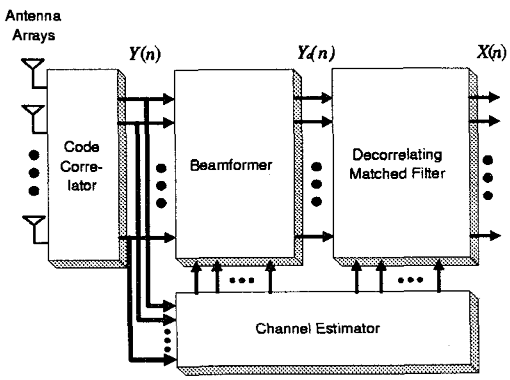

Figure 1: The receiver system architecture.

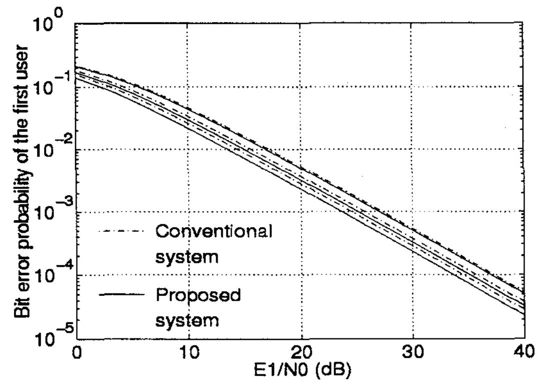

Figure 2: The bit error probabilities of the two systems when the number (K) of active users is 10.

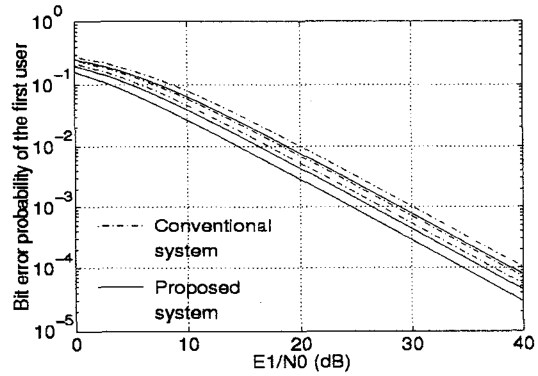

Figure 3: The bit error probabilities of the two systems when the number (K) of active users is 30.

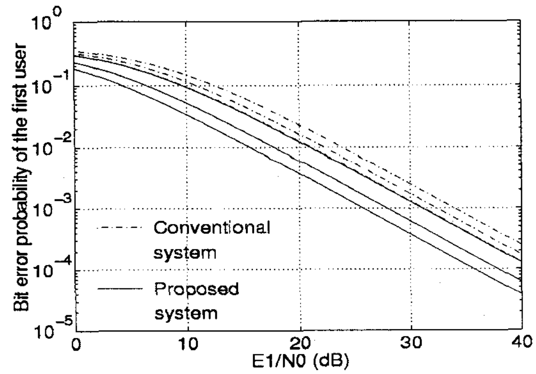

Figure 4: The bit error probabilities of the two systems when the number (K) of active users is 50.