A Modulated Orthogonal Sequence and Noise Reduction

Seong Ill Park, Jeongsoon Park, Hong Gil Kim, Iickho Song, and Kwang Soon Kim

Department of Electrical Engineering
Korea Advanced Institute of Science and Technology (KAIST)
373-1 Guseong Dong, Yuseong Gu
Daejeon 305-701, Korea
Tel : +82-42-869-3445
Fax : +82-42-869-3410
e-mail : isong@sejong.kaist.ac.kr

Abstract

In this paper, we investigate a noise reduction scheme based on the inherent noise reduction characteristic of a modulated orthogonal sequence. The modulated orthogonal sequence generates length N^2 sequences from N information symbols. Using the amplitudes of the received symbols, we first estimate the location of the smallest amplitude noise. Then the noise is reduced by the suggested system.

1 Introduction

For code division multiple access (CDMA) systems, some sequences are suggested, such as the $m-$sequences [1] and Gold's sequences [2]. These sequences, however, have co-channel interference whose value exceeds $1/\sqrt{L}$, where L is the spreading ratio of the CDMA systems. The co-channel interference in these systems lowers to some degree the performance of the systems. In [3], an orthogonal sequence is proposed: when the period is L, the autocorrelation function of the code sequence is 0 except for every Lth term, and the absolute value of the cross-correlation is $1/L$. This absolute value $1/L$ of the cross-correlation function is the mathematical lower bound for orthogonal sequences. In this paper, we suggest a noise reduction method for the orthogonal code sequence proposed in [3], when information symbols have constant absolute values.

In Section 2, we describe the suggested system model which consists of a code sequence generator, a matched filter, and an FFT processor. A code sequence of length N^2 is generated from N information symbols by the code sequence generator. After being passed through the channel, this sequence is processed by the matched filter and FFT processor. In Section 3, using the inherent characteristic of the code generator system, we describe a noise reduction method: an example is also given. Some simulation results are discussed in Section 4.

2 System model

A block diagram of the suggested noise reduction scheme is shown in Figure 1. The code sequence generator makes length N^2 codes from length N information symbols. This code sequence has orthogonality and a good property of cross-correlation [3]. In the channel, noise is added to the code sequence. The received signal passes through the matched filter and FFT processor. In the amplitude estimator, we choose the indices of the received symbols having the smallest amplitudes. Finally, we reduce the amplitude of the noise in the noise reducer using the results of the amplitude estimator and the inherent structure of the code generator.

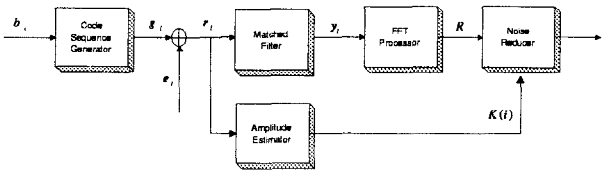

Figure 1: A block diagram of the suggested system

Definition 1. Let the quotient and residue functions Q and R be defined as

$$Q(\alpha, \beta) = q, \;\; R(\alpha, \beta) = r, \qquad (1)$$

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where $\alpha = q\beta + r$, $0 \le r < \beta$, and $q \ge 0$. For notational convenience, let

$$\begin{array}{ll} l_q = Q(l,N), & l_r = R(l,N), \\ n_q = Q(n,N), & n_r = R(n,N), \\ i_q = Q(k,N), & i_r = R(k,N). \end{array} \tag{2}$$

Definition 2. Let us define the $N \times N$ DFT matrix with index m as

$$F_{N,m} = \left[W_N^{-klm}\right], \tag{3}$$

where $W_N = \exp\left(\frac{2\pi j}{N}\right)$ with $j = \sqrt{-1}$.

The code considered for multiple access and noise reduction in this paper is as follows [3]. Let b_i, $i = 0, 1, 2, ..., N-1$, be the information symbols. To maintain the property of orthogonality and to have no cross-correlation, b_i's must have the same absolute value [4]. The output symbol g_l of the code sequence generator can be expressed as

$$g_l = b_{l_r} W_N^{l_q l_r m},\ 0 \le l \le N^2 - 1, \tag{4}$$

where $m \ge 1$ is the user index: a different number is given to each user. Then, the received symbol can be expressed as

$$r_l = g_l + e_l,\ 0 \le l \le N^2 - 1, \tag{5}$$

where e_l is the noise symbol. The received symbol r_l will pass through the matched filter with coefficients

$$p_l = W_N^{-l_q l_r m},\ 0 \le l \le N^2 - 1. \tag{6}$$

The matched filter output is therefore

$$\begin{aligned} y_l &= \sum_{i=0}^{N^2-1} g_{R(i-l,N^2)} p_i + \sum_{i=0}^{N^2-1} e_{R(i-l,N^2)} p_i \\ &= y_l^s + y_l^e,\ 0 \le l \le N^2 - 1, \end{aligned} \tag{7}$$

where y_l^s and y_l^e are the matched filter outputs of the signal and noise components, respectively. As shown in [5], the signal and noise components of the matched filter output can be obtained as

$$y_l^s = \begin{cases} N \sum_{i=0}^{N-1} b_i W_N^{-l_q i m} & \text{if } l_r = 0, \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

and

$$y_l^e = \sum_{i=0}^{N^2-1} e_i W_N^{-f_1(i,l)m}, \tag{9}$$

where

$$f_1(i,l) = \begin{cases} (i_q + l_q)(i_r + l_r), \\ \quad \text{if } 0 \le i_r + l_r \le N-1, \\ (i_q + l_q + 1)(i_r + l_r), \\ \quad \text{if } N \le i_r + l_r \le 2(N-1). \end{cases} \tag{10}$$

Let us now define the matched filter output matrix as

$$Y = \begin{bmatrix} y_0 & y_1 & \cdots & y_{N-1} \\ y_N & y_{N+1} & \cdots & y_{2N-1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{(N-1)N} & y_{(N-1)N+1} & \cdots & y_{N^2-1} \end{bmatrix}. \tag{11}$$

Then the FFT processor outputs are given by

$$R = F_{N,m}^{-1} Y = F_{N,m}^{-1}(Y^s + Y^e) = S + E, \tag{12}$$

where Y^s is the signal component of Y, Y^e is the noise component of Y, $S = F_{N,m}^{-1} Y^s$, and $E = F_{N,m}^{-1} Y^e$. Then, as shown in [5], the FFT processor output matrix S of the signal component is

$$S = N^2 \begin{bmatrix} b_0 & 0 & \cdots & 0 \\ b_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_{N-1} & 0 & \cdots & 0 \end{bmatrix}, \tag{13}$$

and the noise component E of the FFT processor output is

$$E = [E_{kl}], \tag{14}$$

where

$$E_{kl} = N \sum_{n=0}^{N^2-1} e_n W_N^{-f_2(n,k)m} \delta(k - l - n_r), \tag{15}$$

with

$$f_2(n,l) = \begin{cases} n_q(n_r + l), \\ \quad \text{if } 0 \le l \le N-1-n_r, \\ (n_q + 1)(n_r + l), \\ \quad \text{if } N - n_r \le l \le N-1. \end{cases} \tag{16}$$

From (12), (13), and (14), the FFT processor output can be expressed as

$$R = N \begin{bmatrix} Nb_0 + E_{00} & E_{01} & \cdots & E_{0,N-1} \\ Nb_1 + E_{10} & E_{11} & \cdots & E_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ Nb_{N-1} + E_{N-1,0} & E_{N-1,1} & \cdots & E_{N-1,N-1} \end{bmatrix}. \tag{17}$$

3 Noise reduction method

From (13) and (17), it is easy to see that the signal components can be represented by only the first column of R. On the other hand, if there is some noise in the received sequences, then the noise component will appear in all the columns.

The noise reduction method suggested is as follows. As we can see in (4), (5), and (17), each received symbol r_l consists of an information symbol and a noise component. Therefore, each element R_{i0} of the first column of the FFT processor output contains one information symbol b_i and N noise elements, e_{nN+i}, $n = 0, 1, ..., N-1$. The main idea of the proposed noise reduction method is to remove all the noise elements except for the smallest amplitude noise element in each row of R. We thus first have to estimate the smallest noise element for each row of R. One simple way to do that is to estimate the smallest amplitude noise element by the received symbol with the smallest amplitude. Apparently, the received symbol so chosen does not always contain the smallest amplitude noise. As shown in [5], however, the probability that the noise element so chosen has the smallest amplitude in fact is equal to or greater than 1/2. In addition, we can show that, as the SNR decreases, the probability bound becomes higher. Let $K(i)$ be the amplitude estimator output for the ith information symbol, where $K(i) = k$, $i = 1, 2, ...N-1$, if the smallest amplitude element of the set $\{r_i, r_{N+i}, r_{2N+i}, ..., r_{(N-1)N+i}\}$ is the $(k+1)$th element r_{kN+i}.

Now, we consider the linear combination $c_i^{K(i)}$ of E_{kl} defined by

$$c_i^{K(i)} = \sum_{q=1}^{N-1-i} E_{R(i+q,N)q} W_N^{K(i)qm} + \sum_{q=N-i}^{N-1} E_{R(i+q,N)q} W_N^{\{(K(i)+1)q+i\}m}. \quad (18)$$

We can show that

$$c_i^{K(i)} = N\left\{(N-1)\, e_{K(i)N+i} W_N^{-K(i)im} - \sum_{n=0, n\neq K(i)}^{N-1} e_{nN+i} W_N^{-K(i)im}\right\}, \quad (19)$$

and, if we add $c_i^{K(i)}$ to R_{i0}, we get from (15) and (19)

$$R_{i0} + c_i^{K(i)} = N^2 \left(b_i + e_{K(i)N+i} W_N^{-K(i)im}\right), \quad (20)$$

since R_{i0} can be expressed as $R_{i0} = N\{Nb_i + \sum_{n=0}^{N^2-1} e_n W_N^{-n_q n_r m}\, \delta(i-n_r)\} = N\{Nb_i + \sum_{n=0}^{N-1} e_{nN+i}\, W_N^{-nim}\}$.

As we can see from (20), we now have one information symbol added to only the smallest amplitude noise for each row of R: this implies the SNR is increased by the reducer. Intuitively, it is easily conceivable that this system will perform better under lower SNR environments.

As an example, assume that $N = 3$ and $m = 1$. Then, from (17), the FFT processor output is

$$R = 3\begin{bmatrix} 3b_0 + e_0 + e_3 + e_6 & e_2 + e_5 + e_8 & e_1 + e_4 + e_7 \\ 3b_1 + e_1 + e_4 W_3^2 + e_7 W_3 & e_0 + e_3 W_3^2 + e_6 W_3 & e_2 W_3^2 + e_5 W_3 + e_8 \\ 3b_2 + e_2 + e_5 W_3 + e_8 W_3^2 & e_1 + e_4 W_3 + e_7 W_3^2 & e_0 + e_3 W_3 + e_6 W_3^2 \end{bmatrix}. \quad (21)$$

Assume that $K(1) = 0$ for the second row or information symbol b_1: that is, among the received symbols r_1, r_4, and r_7 containing b_1, the absolute value of r_1 is assumed to be the smallest. Now

$$\begin{aligned} c_1^0 &= R_{2,1} W_3^0 + R_{0,2} W_3^3 \\ &= 2e_1 - \left(e_4 W_3^2 + e_7 W_3\right), \end{aligned} \quad (22)$$

and using (20), the output of the noise reducer is

$$\left(R_{1,0} + c_1^0\right) = 9\,(b_1 + e_1). \quad (23)$$

Similarly, we can obtain the output of the noise reducer for the first and last information symbols: if we assume $K(0) = 1$ and $K(2) = 2$, then the noise reducer outputs are $(R_{0,0} + c_0^0) = 9\,(b_0 + e_3)$ and $(R_{2,0} + c_2^2) = 9\,(b_2 + e_8 W_3^2)$ for the first and last information symbols, respectively.

4 Simulation results

In this section, we show some simulation results for the suggested noise reduction method. Assume that the pdf of the noise is the ϵ-contaminated pdf

$$f(x) = (1-\epsilon) f_b(x) + \epsilon f_i(x), \quad (24)$$

where ϵ is called the impulsiveness parameter or contamination parameter [6], and $f_b(x)$ and $f_i(x)$ are zero-mean Gaussian pdfs with variances σ_b^2 and σ_i^2, respectively.

To quantify the simulation results, let us define the noise reduction gain G in dB as

$$G = 10 \log_{10} \frac{P_{not\ reduced}}{P_{reduced}} (dB),$$

where $P_{not\ reduced}$ and $P_{reduced}$ are the noise powers before and after the reducer, respectively.

In Figure 2, we show the reduction gain versus the impulsiveness parameter, where it is assumed that $N = 3$, $\sigma_i^2/\sigma_b^2 = 100$, SNR=0, 2.5, 4.5, 7dB, and the number of Monte-Carlo runs is 10000 for each point. Apparently, as is anticipated from the probabilistic analysis [5], we get more gain as the SNR decreases. It is interesting to see that, when ϵ is approximately $0.3 - 0.4$, we get the maximum gain.

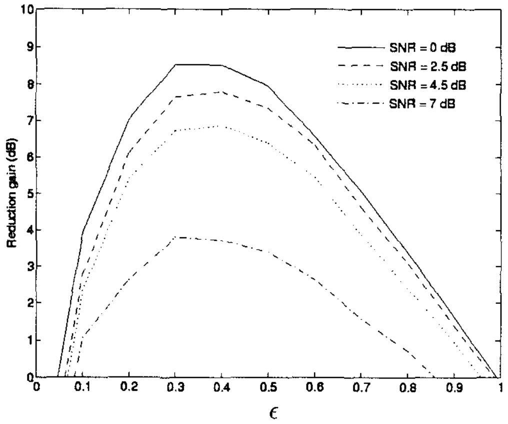

Figure 2: Noise reduction gain versus ϵ of the suggested method when $N = 3$ and $\sigma_i^2/\sigma_b^2 = 100$.

To more clearly show the noise reduction gain versus the SNR, we obtain another set of curves as shown in Figure 3, where $N = 3$ and $\sigma_i^2/\sigma_b^2 = 100$.

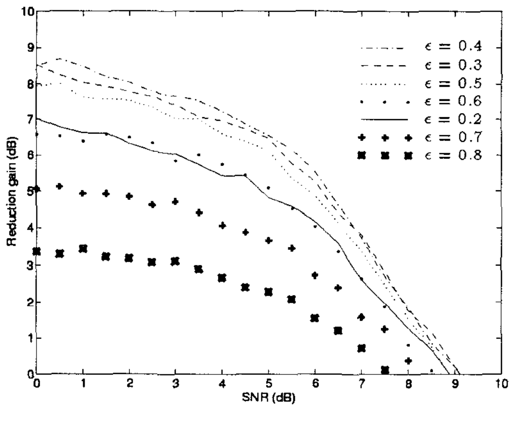

Figure 3: Noise reduction gain versus SNR of the suggested method when $N = 3$ and $\sigma_i^2/\sigma_b^2 = 100$.

Clearly, we see that the noise reduction gain is high when the SNR is low and when the impulsiveness of the noise is intermediate for the ϵ−contaminated pdf.

5 Concluding remarks

We have suggested a noise reduction scheme which can reduce noise power for an orthogonal sequence. The code sequence considered in this paper has not only the orthogonality after modulated by the information symbols but also good cross-correlation property. This code also has an inherent noise reduction characteristic, which is exploited in the reduction scheme. The amplitude of noise can be reduced to the level of the smallest one: the reduction is expected and found to be better for lower SNR cases.

Acknowledgements

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References

[1] S. W. Golomb, *Shift-Register Sequences,* San Francisco, CA: Holden-Day, 1967.

[2] D. V. Sarwate and W. B. Pursley, "Cross-correlation properties of pseudo-random and related sequences," *Proc. IEEE,* vol. 68, pp. 593-619, May 1980.

[3] N. Suehiro and M. Hatori, "Modulatable orthogonal sequences and their application to SSMA systems," *IEEE Trans. Inform. Theory,* vol. IT-34, pp. 93-100, January 1988.

[4] S. I. Park, J. Bae, N. Suehiro, and I. Song, "A new method for the generation of an orthogonal sequence," *IEEE Trans. Inform. Theory* (submitted).

[5] S. I. Park, *Optimum modulation and demodulation techniques and channel coding schemes for spread spectrum multiple access systems*, Ph. D. Dissertation, Daejeon, Korea Advanced Institute of Science and Technology (in preparation).

[6] S. A. Kassam, *Signal Detection in Non-Gaussian Noise*, Springer-Verlag, New York, NY, 1988.