An Analysis of DOA Estimation Method under a Dispersive Signal Model

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Abstract - In direction of arrival estimation, the signal sources are usually assumed to be point sources. If the signal sources are dispersive, however, direction of arrival estimation methods based on the point source assumption may result in poor performance. In this paper, we consider direction of arrival estimation under a parametric dispersive signal modeling. An estimation method based on the well-known conditional maximum likelihood method in the parametric model are considered.

I. INTRODUCTION

Conventional DOA estimation methods are based on the assumption that the signal sources are point sources: i.e., if a DOA is θ, then there is no other signal at $\theta+\epsilon$, for a very small value of ϵ. It is a reasonable assumption if the signal sources are located far enough from the receivers and are not dispersive. Under this assumption the DOA's can be estimated using the steering vector which is a vector function of the DOA's. The array output vector is a weighted sum of these vectors with the weighting dependent on the signal sources, and corrupted by spatially white noise vector.

On the other hand, if the signal sources are dispersive, the array output is not a weighted sum of the *finite number of steering vectors*. In addition, although the DOA estimation methods for point signal sources can be directly applied to the DOA estimation for dispersive signal sources, we do not have confidence that the methods would provide us with good estimates of the DOA's.

In this paper we address the parametric dispersive signal source modeling problem and show that the conventional conditional maximum likelihood method with some modification can be applied to obtaining the DOA's of the dispersive signal sources.

II. PARAMETRIC DISPERSIVE SIGNAL MODEL

For M point sources, the output of an array with L elements can be written as

$$\underline{y}(t)=\sum_{i=1}^{M}\underline{a}(\omega_i)x_i(t)+\underline{n}(t), \qquad (1)$$

where $\underline{y}(t)\in C^{L\times 1}$, $x_i(t)$ represents the ith point signal source, $\underline{a}(\theta)\in C^{L\times 1}$ is the steering vector, ω_i is the DOA of the ith source, and $\underline{n}(t)\in C^{L\times 1}$ is the noise vector. Here $C^{L\times 1}$ denotes the space of $L\times 1$ complex-valued vectors. Denoting $A=[\underline{a}(\omega_1),\underline{a}(\omega_2),\ldots,\underline{a}(\omega_M)]$ and $\underline{x}(t)=[x_1(t),x_2(t),\ldots,x_M(t)]^T$,

$$\underline{y}(t)=A\underline{x}(t)+\underline{n}(t). \qquad (2)$$

Let the dispersive signal source density be $s(\theta,t)$ and assume that the complex normal noise vector $\underline{n}(t)$ is a spatially and temporally uncorrelated random vector with zero-mean and covariance matrix σI. Then the output of the array can be expressed as

$$\underline{y}(t)=\underline{z}(t)+\underline{n}(t), \qquad (3)$$

where $\underline{z}(t)\triangleq\frac{1}{2\pi}\int_0^{2\pi}\underline{a}(\theta)s(\theta,t)d\theta$. As a special case, if $s(\theta,t)=2\pi\sum_{i=1}^{M}x_i(t)\delta(\theta-\omega_i)$, then we obtain (1) from (3).

The signal source density $s(\theta,t)$ can be written as

$$s(\theta,t)=\sum_{m=0}^{\infty}c_m(t)e^{-jm\theta}. \qquad (4)$$

In (4) $c_m(t)$ is a complex normal random variable with $E[c_m(t)]=0$, $E[c_m(t)c_n^*(s)]=r_{mn}\delta_{ts}$, and $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}|r_{mn}|<\infty$.

Let us assume that $c_m(t)=\sum_{i=1}^{M}\alpha_i(t)p^m(\omega_i,\eta_i)$, where $\alpha_i(t)$ is a zero-mean complex random variable

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with covariance function $E[\alpha_m(t)\alpha_n^*(s)] = \sigma_{mn}\delta_{ts}$, and $p(\theta,\rho) = \rho e^{j\theta}$ is defined by the dispersion parameter ρ, $0 \le \rho < 1$, and DOA θ, $0 \le \theta < 2\pi$. Since

$$s(\theta,t) = \sum_{i=1}^{M} \alpha_i(t)/(1 - p(\omega_i,\eta_i)e^{-j\theta}), \quad (5)$$

the array output is

$$\begin{aligned} \underline{y}(t) &= \sum_{i=1}^{M} \frac{\alpha_i(t)}{2\pi} \int_0^{2\pi} \frac{\underline{a}(\theta)}{1 - p(\omega_i,\eta_i)e^{-j\theta}} d\theta + \underline{n}(t) \\ &= \sum_{i=1}^{M} \alpha_i(t)\underline{b}(\omega_i,\eta_i) + \underline{n}(t), \end{aligned} \quad (6)$$

where

$$\underline{b}(\omega_i,\eta_i) \triangleq \frac{1}{2\pi}\int_0^{2\pi} \frac{\underline{a}(\theta)}{1 - p(\omega_i,\eta_i)e^{-j\theta}} d\theta, \quad i = 1,\ldots,M. \quad (7)$$

Since the covariance function of $s(\theta,t)$ is

$$C_s(\theta,\phi) = \sum_{m=1}^{M}\sum_{n=1}^{M} \frac{\sigma_{mn}\delta_{ts}}{(1 - p(\omega_m,\eta_m)e^{-j\theta})(1 - p(\omega_n,\eta_n)e^{-j\phi})^*}, \quad (8)$$

the covariance matrix R_o of the output of array is

$$R_o = B(\underline{\theta},\underline{\rho})\Delta B^H(\underline{\theta},\underline{\rho}) + \sigma I, \quad (9)$$

where $B(\underline{\theta},\underline{\rho}) \triangleq [\underline{b}(\omega_1,\eta_1), \underline{b}(\omega_2,\eta_2), \cdots, \underline{b}(\omega_M,\eta_M)] \in C^{L\times M}$ is the steering matrix with $\underline{\theta} = [\theta_1,\theta_2,\ldots,\theta_M]^T$ and $\underline{\rho} = [\rho_1,\rho_2,\ldots,\rho_M]^T$, and $[\Delta]_{mn} = \sigma_{mn}$. For notational convenience, we will write B instead of $B(\underline{\theta},\underline{\rho})$. Equation (9) is quite similar to the equation of the covariance matrix of the output of array in the point model, $R_y = AR_xA^H + \sigma I$, where $R_x = E[\underline{x}(t)\underline{x}^H(t)]$.

III. DOA ESTIMATION UNDER PARAMETRIC DISPERSIVE SIGNAL MODEL

In the parametric model (5), we can use the eigenstructure-based or maximum likelihood methods for DOA estimation [1,3]. In this section we consider a DOA estimation method based on the conditional maximum likelihood (CML) method.

Let $P^s \triangleq B[B^HB]^{-1}B^H$ be the projection operator onto the space spanned by the columns of B, $P^n \triangleq I - P^s$ with I the identity matrix, and $\underline{\alpha}(t) \triangleq [\alpha_1(t),\alpha_2(t),\ldots,\alpha_M(t)]^T$. From (6), the conditional log-likelihood function of the observed data can be derived to be

$$\ln L(Y|\underline{\alpha}(t)) = -N\ln\sigma - \frac{1}{N\sigma}\sum_{t=1}^{N}[\underline{y}(t) - B\underline{\alpha}(t)]^H[\underline{y}(t) - B\underline{\alpha}(t)], \quad (10)$$

where $Y = [\underline{y}(1),\underline{y}(2),\ldots,\underline{y}(N)]$. In CML principle, the problem is to maximize (10) with respect to $\underline{\theta},\underline{\rho},\underline{\alpha}(t)$, and σ. Let us first maximize (10) with respect to $\underline{\alpha}(t)$ for given $\underline{\theta},\underline{\rho}$, and σ. Then we obtain

$$\underline{\alpha}(t) = (B^HB)^{-1}B^H\underline{y}(t). \quad (11)$$

Next, let us maximize (10) with respect to σ. Then we get,

$$\sigma = \frac{1}{L-M}tr[P^n\hat{R}_o], \quad (12)$$

where $\hat{R}_o = \frac{1}{N}\sum_{t=1}^{N}\underline{y}(t)\underline{y}^H(t)$ is the sample covariance matrix. Thus, the CML cost function is, from (10)-(12),

$$V_c(\underline{\theta},\underline{\rho}) = tr[P^n\hat{R}_o]. \quad (13)$$

Therefore, the estimate $(\hat{\omega}_i,\hat{\eta}_i)$ of (ω_i,η_i) can be obtained from

$$(\hat{\omega}_i,\hat{\rho}_i) = \arg\max_{\theta_i,\rho_i} tr[P^n\hat{R}_o], \quad i = 1,2,\ldots,M. \quad (14)$$

The optimization problem can be solved with the Newton [4], alternating projection [6], and expectation maximization [2] algorithms.

IV. STATISTICAL PROPERTIES

Under the assumption that the estimate $(\hat{\underline{\omega}},\hat{\underline{\eta}})$ is sufficiently close to $(\underline{\omega},\underline{\eta})$, the estimation error vector is

$$\begin{bmatrix} \hat{\underline{\omega}} - \underline{\omega} \\ \hat{\underline{\eta}} - \underline{\eta} \end{bmatrix} \simeq -H_c^{-1}(\underline{\omega},\underline{\eta})V_c'(\underline{\omega},\underline{\eta}), \quad (15)$$

where

$$V_c'(\underline{\omega},\underline{\eta}) = \begin{bmatrix} \frac{\partial}{\partial\underline{\theta}}V_c(\underline{\omega},\underline{\eta}) \\ \frac{\partial}{\partial\underline{\rho}}V_c(\underline{\omega},\underline{\eta}) \end{bmatrix}$$

and

$$H_c(\underline{\omega},\underline{\eta}) = \begin{bmatrix} \frac{\partial}{\partial\underline{\theta}}(\frac{\partial}{\partial\underline{\theta}}V_c(\underline{\omega},\underline{\eta}))^T & \frac{\partial}{\partial\underline{\theta}}(\frac{\partial}{\partial\underline{\rho}}V_c(\underline{\omega},\underline{\eta}))^T \\ \frac{\partial}{\partial\underline{\rho}}(\frac{\partial}{\partial\underline{\theta}}V_c(\underline{\omega},\underline{\eta}))^T & \frac{\partial}{\partial\underline{\rho}}(\frac{\partial}{\partial\underline{\rho}}V_c(\underline{\omega},\underline{\eta}))^T \end{bmatrix}.$$

From (15) and by use of the statistical results of [5], we can show that the asymptotic distribution of the estimation error vector $[(\hat{\underline{\omega}} - \underline{\omega})^T, (\hat{\underline{\eta}} - \underline{\eta})^T]^T$ is zero-mean normal with covariance matrix

$$\begin{aligned} C_c &= E\left[\begin{bmatrix} \hat{\underline{\omega}} - \underline{\omega} \\ \hat{\underline{\eta}} - \underline{\eta} \end{bmatrix}[\hat{\underline{\omega}} - \underline{\omega}, \hat{\underline{\eta}} - \underline{\eta}]\right] \\ &= \bar{H}_c^{-1}\bar{C}_c\bar{H}_c^{-1}, \end{aligned} \quad (16)$$

where

$$\bar{H}_c = 2\begin{bmatrix} Re(h_{\underline{\omega}\underline{\omega}} \odot \Delta^T) & Re(h_{\underline{\omega}\underline{\eta}} \odot \Delta^T) \\ Re(h_{\underline{\eta}\underline{\omega}} \odot \Delta^T) & Re(h_{\underline{\eta}\underline{\eta}} \odot \Delta^T) \end{bmatrix}, \quad (17)$$

and

$$\bar{C}_c = \frac{\sigma}{2N} \cdot \begin{bmatrix} Re(h_{\underline{\omega}\underline{\omega}} \odot (\Delta W_c \Delta)^T) & Re(h_{\underline{\omega}\underline{\eta}} \odot (\Delta W_c \Delta)^T) \\ Re(h_{\underline{\eta}\underline{\omega}} \odot (\Delta W_c \Delta)^T) & Re(h_{\underline{\eta}\underline{\eta}} \odot (\Delta W_c \Delta)^T) \end{bmatrix} \quad (18)$$

with $W_c \triangleq \Delta^{-1} + \sigma\Delta^{-1}(B^H B)^{-1}\Delta^{-1}$, $[A \odot B]_{ij} \triangleq [A]_{ij}[B]_{ij}$, $h_{\underline{\omega}\underline{\omega}} \triangleq [\frac{\partial}{\partial \underline{\theta}} B^H(\underline{\omega}, \underline{\eta})]P^n[\frac{\partial}{\partial \underline{\theta}^T} B(\underline{\omega}, \underline{\eta})]$,
$h_{\underline{\omega}\underline{\eta}} \triangleq [\frac{\partial}{\partial \underline{\theta}} B^H(\underline{\omega}, \underline{\eta})]P^n[\frac{\partial}{\partial \underline{\rho}^T} B(\underline{\omega}, \underline{\eta})]$,
$h_{\underline{\eta}\underline{\omega}} \triangleq [\frac{\partial}{\partial \underline{\rho}} B^H(\underline{\omega}, \underline{\eta})]P^n[\frac{\partial}{\partial \underline{\theta}^T} B(\underline{\omega}, \underline{\eta})]$, and
$h_{\underline{\eta}\underline{\eta}} \triangleq [\frac{\partial}{\partial \underline{\rho}} B^H(\underline{\omega}, \underline{\eta})]P^n[\frac{\partial}{\partial \underline{\rho}^T} B(\underline{\omega}, \underline{\eta})]$.

Next, let us obtain the Cramer Rao bound (CRB) of the variance of the DOA estimation error vector. From (10) and by extending the statistical results of [5], the asymptotic CRB is obtained as,

$$CRB = \frac{\sigma}{2N} \cdot \begin{bmatrix} Re(h_{\underline{\omega}\underline{\omega}} \odot \Delta^T) & Re(h_{\underline{\omega}\underline{\eta}} \odot \Delta^T) \\ Re(h_{\underline{\eta}\underline{\omega}} \odot \Delta^T) & Re(h_{\underline{\eta}\underline{\eta}} \odot \Delta^T) \end{bmatrix}^{-1}. \quad (19)$$

V. NUMERICAL EXAMPLES

Let us assume that $L = 5$, $M = 2$, and the number of snapshots $N = 100$.

Example 1: In this example, we compare the variances of the estimation errors of DOA's and dispersion parameters with CRB at SNR = 10, 20 dB. The comparison between the variance (16) and the CRB (19) of the DOA estimation errors is shown in Figure 1 when one signal is located at 30° with $\eta_1 = 0.99$ and the DOA of the other source is changed with fixed $\eta_2 = 0.95$. From Figure 1, we observe that the difference between the variance and the CRB of the DOA estimation errors approaches zero as the difference between the two DOA's become larger.

Example 2: In this example, let us evaluate the relative efficiency defined as the ratio of the CRB to the variance of estimation error. The relative efficiencies of the DOA estimation errors are shown in Figure 2(a) under the same environment as in Figure 1. Since the slopes in Figure 2(a) are larger values than those in Figure 2(b), we may conclude that the DOA estimation error is more sensitive to the difference of DOA than that of dispersion parameter. Also, we observe that when SNR is larger than 20 dB, the variance of the DOA estimation error is essentially equal to the CRB.

When one signal is located at 30° and the DOA of the other signal is changed, and the two dispersion parameters have the same values 0.5, 0.7, 0.9, 1, the relative efficiency is shown in Figure 2(c). From Figure 2(c) we observe that the relative efficiency increases as the dispersion parameter increases.

VI. CONCLUDING REMARKS

When the signal sources are not point sources, but dispersed over an area, we cannot use the well-known direction of arrival estimation methods which are based on the point source assumption. We consider a direction of arrival estimation method for dispersive signal sources. The dispersive signal source is modeled by the parametric method, and under the model we consider the CML-based direction of arrival estimation method.

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REFERENCES

[1] J. F. Böhme, "Estimation of spectral parameters of correlated signals in wavefields," *Signal Proc.*, vol. 11, pp. 329-337, December 1986.

[2] M. I. Miller and D. R. Fuhrmann, "Maximum-likelihood narrow-band direction finding and the EM algorithm", *IEEE Trans. Acous., Speech, Signal Proc.*, vol. ASSP-38, pp. 1560-1577, September 1990.

[3] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Ant., Prop.*, vol. AP-34, pp. 276-280, March 1986.

[4] D. Starer and A. Nehorai, "Newton algorithms for conditional and unconditional maximum likelihood estimation of the parameters of exponential signals in noise," *IEEE Trans. Signal Proc.*, vol. SP-40, pp. 1528-1534, June 1992.

[5] P. Stoica and A. Nehorai, "MUSIC, maximum likelihood, and Cramer-Rao bound," *IEEE Trans. Acous., Speech, Signal Proc.*, vol. ASSP-37, pp. 720-740, May 1989.

[6] J. Ziskind and M. Wax, "Maximum likelihood localization of multiple sources by alternative projection", *IEEE Trans. Acous., Speech, Signal Proc.*, vol. ASSP-36, pp. 1553-1560, October 1988.

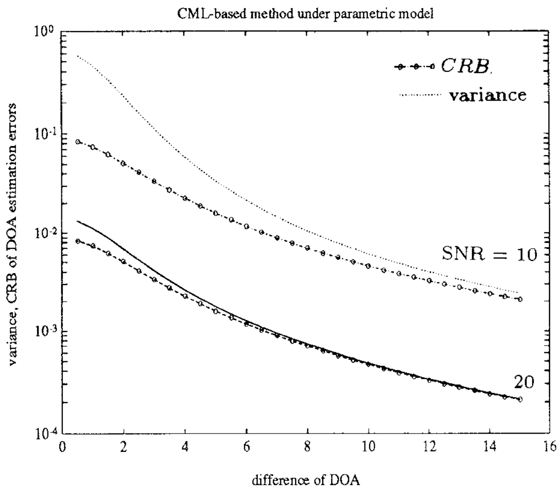

Figure 1: The variance and CRB of DOA estimation errors when $\omega_1 = 30°$, $\eta_1 = 0.99$, $\eta_2 = 0.95$, $L = 5$, $N = 100$, and SNR = 10, 20 dB.

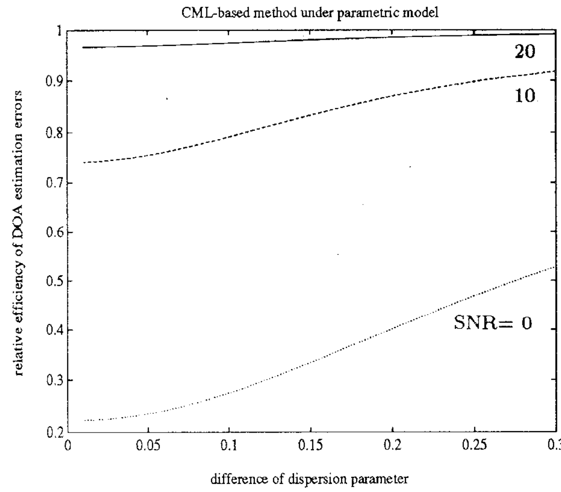

Figure 2(b): The relative efficiency of DOA estimation errors with changing the two dispersion parameter difference under $\omega_1 = 30°$, $\omega_2 = 40°$, $\eta_1 = 0.99$, $L = 5$, $N = 100$, and SNR= 0, 10, 20 dB.

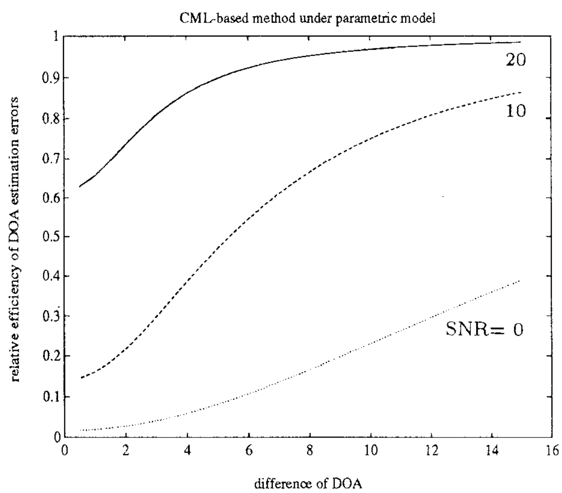

Figure 2(a): The relative efficiency of DOA estimation errors with changing the two DOA difference under $\omega_1 = 30°$, $\eta_1 = 0.99$, $\eta_2 = 0.95$, $L = 5$, $N = 100$, and SNR= 0, 10, 20 dB.

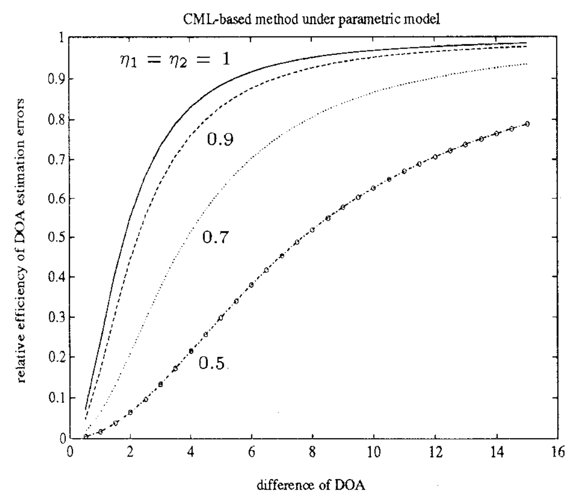

Figure 2(c): The relative efficiency of DOA estimation errors with changing the two DOA difference under $\omega_1 = 30°$, $\eta_1 = \eta_2 = 1, 0.9, 0.7, 0.5$, $L = 5$, and SNR=20 dB.