1995년도

추계종합학술발표회 논문집

일 시 : 1995년 11월 11일(토) 10:00
• 장 소 : 고 려 대 학 교 과 학 도 서 관
• 주 최 : 사단법인 한 국 통 신 학 회
• 후 원 : 고 려 대 학 교
• 협 찬 : 현대전자, 대우통신, LG전자, 아남산업, 단암산업, LG전선, 콤택시스템, 삼성휴렛팩커드, KDC정보통신, 에이아이소프트, 모아통신

충격성 잡음환경에서 격자부호변조 직접수열 대역확산 다중접속 계통의 분석

김광순*, 김홍길*, 이성로*, 이용업*, 이민수*, 송익호*
*한국과학기술원 전기및전자공학과

An Analysis of DS/SSMA Systems Using Trellis Coded Modulation in Impulsive Noise Environment

Kwang Soon Kim*, Hong Gil Kim*, Seong Ro Lee*, Yong Up Lee*, Min Sou Lee*, Iickho Song*
*Dept. of EE, Korea Advanced Institute of Science & Technology

Abstract The performance of DS/SSMA systems using TCM under impulsive noise environment is analyzed. We obtain the bound of the probability of bit error. It is shown that the bit error probability is dominated by background noise variance when SNR is small and by tail noise variance when SNR is large.

1 Introduction

The developments in communications technology these days have changed our daily life profoundly and rapidly. With mobile communication, we can overcome the restrictions on time and space, which were inevitable in traditional communication. As is evident, the demand for mobile communication is increased rapidly. Recently, code division multiple access (CDMA) becomes an interesting research area for its potential applications in mobile communication.

A lot of modulation and coding techniques have been developed to get better communication systems. Ungerboeck [1] has shown that optimally designed rate $n/(n+1)$ trellis codes mapped into the conventional 2^{n+1} point signal sets can provide some coding gain without bandwidth expansion. After his work, many researchers have studied various aspects of the trellis coded modulation (TCM). For example, the performance of DS/SSMA system using TCM has been studied in [2][3]. Most of the work on TCM has been considered under the additive white Gaussian noise (AWGN) assumption [1][3][4][5].

It is well-known that sometimes Gaussian noise assumption cannot be entirely justified. The non-Gaussian nature of atmospheric noise, impulses caused by turning-on some electrical devices, etc, are among the typical examples. Therefore, it is worthwhile to study the effects of impulsive noise on communication systems. In this paper, we will investigate the effect of impulsive noise on the DS/SSMA system using TCM. We will derive the probability of bit error for the 2^{n+1}-PSK trellis coded DS/SSMA system under impulsive noise environment. We will then compare our result with that for the uncoded DS/SSMA system for various channel states.

2 System Model

The block diagram of the DS/SSMA system is illustrated in Fig. 1, which has been used frequently in previous studies of SSMA systems [3].

But the difference is that the noise $n(t)$ is impulsive noise. Each user transmits using different spreading code and the signal transmitted by the kth user $s^k(t)$ is assumed to be delayed randomly by τ^k. The received signal is composed of the desired signal, interference due to the noise, and signal transmitted by other users.

The transmitter system model for trellis-coded DS/SSMA is illustrated in Fig. 2.

The information bits select a coset of the signal constellation through the $n/(n+1)$ convolutional code. The selected signal multiplied by DS sequence is modulated by the carrier and transmitted. The signal constellation we are concerned in this paper is antipodal 2^n -PSK. The receiver model is the conventional I and Q type demodulators.

The notation in complex form in this paper are based on the work by Pursley [3], Kavenhrad [6], and Boudreau [2].

Let the kth user's complex baseband information signal be

$$x^k(t) = \sum_{p=-\infty}^{\infty} x_p^k P_T(t-pT), \quad (1)$$

where T is the symbol period, $P_T(\cdot)$ is a rectangular pulse with duration T, and x_p^k is the complex baseband symbol of the kth user during the pth symbol period which is determined by coset selection.

Similarly, the DS chip signal is defined as

$$a^k(t) = \sum_{m=-\infty}^{\infty} a_m^k \Psi(t-mT_c), \quad (2)$$

where a_m^k is the mth chip of the kth user and Ψ is the chip waveform with duration T_c. Then, the transmitted signal modulated by carrier of frequency f_c is

$$s^k(t) = \sqrt{\frac{2E_s}{T}} Re\{\zeta^k(t) exp[j(w_c t + \psi^k)\}, \quad (3)$$

where $\zeta^k(t) = a^k(t)x^k(t)$ and ψ^k is the random phase of the kth carrier.

Then, the received signal can be written as

$$r(t) = \sqrt{\frac{E_s}{T}} Re\{\sum_{k=1}^{K} \zeta^k(t-\tau^k) expj(w_c t + \beta^k)\} + n(t), \quad (4)$$

where τ^k is the random delay of the kth user at the receiver, $\beta^k = \psi^k - w_c\tau^k$ is the random phase of kth user at the receiver, and $n(t)$ is the impulsive noise.

After demodulation through the I and Q type demodulaters, it can be shown that the sampled received sequence Y_p^i can be written as [2]

$$Y_p^i = X_p^i + Z_p^i + \eta_p^i, \tag{5}$$

where $X_p^i = \sqrt{E_s}x_p^i$, Z_p^i is the inter-user interference, and η_p^i is the interference by impulsive noise. The work by Boudreau was accomplished under AWGN environment assumption : in this paper we conside the impulsive noise environment.

3 Impulsive Noise Environment

In our communication channel, source noise has impulsiveness. Nowadays, we are surrounded by many electrical devices: thus the impulsive noise environment is a proper noise model for some cases. We can model impulsive noise with the ϵ-contaminated mixture noise model [7][8][9] , for which the pdf is

$$f(x) = (1-\epsilon)f_B(x) + \epsilon f_T(x). \tag{6}$$

In (6), f_B is a Gaussian pdf with mean zero and variance $\sigma_B{}^2$, and f_T is in general a zero-mean pdf which has a variance σ_T^2 much larger than $\sigma_B{}^2$. In this paper, we assume that f_T is a Gaussian pdf. The mean and variance of the impulsive noise are

$$\begin{aligned} E\{\eta\} &= 0 && (7)\\ E\{\eta^2\} &= (1-\epsilon)\sigma_B{}^2 + \epsilon\sigma_T{}^2 \\ &= (1+(M-1)\epsilon)\sigma_B{}^2 && (8)\end{aligned}$$

where $M = \frac{\sigma_T{}^2}{\sigma_B{}^2}$. Then we can define signal to noise ratio as

$$\begin{aligned} SNR &= \frac{E_s}{2E\{\eta^2\}} \\ &= \frac{nE_b}{2(1+(M-1)\epsilon)\sigma_B{}^2}. \end{aligned} \tag{9}$$

The characteristics of the impulsive noise are different for different ϵ and M which result the same SNR.

Now, we define the signal to background noise ratio (SBR) as

$$SBR = \frac{nE_b}{2\sigma_B{}^2} \tag{10}$$

As we can see, SBR is independent of M and ϵ. In practical situations, ϵ is very small, i.e. $(M-1)\epsilon \ll 1$. So, SBR can be a good approximation of SNR, and is useful when we want to know some result for various ϵ.

4 Performance Analysis

Let the complex coded symbol sequence X as

$$X = (X_1, X_2, \cdots, X_n), \tag{11}$$

the corresponding complex received sequence Y as

$$Y = (Y_1, Y_2, \cdots, Y_n), \tag{12}$$

and the complex inter-user interference sequence Z as

$$Z = (Z_1, Z_2, \cdots, Z_n). \tag{13}$$

Then the output signal at time p is

$$Y_p = X_p + Z_p + \eta_p. \tag{14}$$

where η_p is a sample of complex impulsive noise.

Now, we assume that the inter-user interference Z_p is the Gaussian random variable independent of η_p by the central limit theorem. Consider the interference term

$$\tilde{\eta_p} = Z_p + \eta_p. \tag{15}$$

In order to obtain the pairwise error probability, we use Chernoff bound. The pairwise error probability can be written as

$$P(x \to \tilde{x}) = Pr\{\sum_{p\in\nu}(\|Y_p - X_p\|^2 - \|Y_p - \tilde{X_p}\|^2) \geq 0\} \tag{16}$$

where ν is the set of p such that $X_p \neq \tilde{X_p}$. We can simplify (16) as

$$\begin{aligned} &P(x \to \tilde{x}) \\ &= Pr\{\sum_{p\in\nu}(-E_s\|x_p - \tilde{x_p}\|^2 - 2\sqrt{E_s}Re\{\eta_p(x_p - \tilde{x_p})^*\}) \geq 0\} \end{aligned} \tag{17}$$

where $x_p = \frac{X_p}{\sqrt{E_s}}$. Define the pth distance d_p as

$$\begin{aligned} d_p &= x_p - \tilde{x_p} \\ &= d_{pi} + id_{pq} \end{aligned} \tag{18}$$

and the impulsive interference term η_p as

$$\eta_p = \eta_{pi} + i\eta_{pq} \tag{19}$$

where $d_{pi}, d_{pq}, \eta_{pi}$, and η_{pq} are real. Since η_{pi} and η_{pq} are the in-phase and quadrature components of η_p, η_{pi} and η_{pq} have the ϵ-contaminated pdf with background variance $\sigma_B{}^2$ and tail variance $\sigma_T{}^2$ and independent of each other.

Now, let us define some new variable d and v as follows.

$$d^2 = \sum_{p\in\nu} d_p{}^2 \tag{20}$$

and

$$v = -\sum_{p\in\nu} d_{pi}\eta_{pi} + d_{pq}\eta_{pq} \tag{21}$$

Then we can rewrite (17) as follows.

$$P(x \to \tilde{x}) = Pr\{-E_s d^2 + 2\sqrt{E_s}v \geq 0\}. \tag{22}$$

After some calculation, the pairwise error probability can be bounded as

$$P(x \to \tilde{x}) \leq Pr\{-E_s d^2 + 2\sqrt{E_s}\bar{v} \geq 0\} \tag{23}$$

where $\bar{v}$ is a random variable with pdf $f_v(x) = (1-\epsilon)f_{B'}(x) + \epsilon f_{T'}(x)$, $f_B(\cdot)$ and $f_T(\cdot)$ are Gaussian pdf with mean zero and variance $d^2\sigma_B{}^2$ and $d^2\sigma_T{}^2$, respectively. Let $\hat{v} = -E_s d^2 + 2\sqrt{E_s}\bar{v}$, then the pdf of $\hat{v}$ can be obtained as

$$f_{\hat{v}}(x) = (1-\epsilon) f_{B'}(x) + \epsilon f_{T'}(x) \quad (24)$$

where

$$f_{B'}(x) = \frac{1}{\sqrt{8\pi E_s \sigma_B{}^2 d^2}} exp[-\frac{(x + E_s d^2)}{8 E_s \sigma_B{}^2 d^2}] \quad (25)$$

and

$$f_{T'}(x) = \frac{1}{\sqrt{8\pi E_s \sigma_T{}^2 d^2}} exp[-\frac{(x + E_s d^2)}{8 E_s \sigma_T{}^2 d^2}]. \quad (26)$$

Then (23) can be rewritten as follows.

$$\begin{aligned} P(x \to \hat{x}) &\leq Pr\{\hat{v} \geq 0\} \\ &= (1-\epsilon)\int_0^\infty f_{B'}(x)dx + \epsilon \int_0^\infty f_{T'}(x)dx \quad (27) \end{aligned}$$

Applying Chernoff bound to (27), we can obtain

$$\begin{aligned} P(x \to \hat{x}) &\leq (1-\epsilon)\cdot exp[-E_s d^2 \lambda_1 + 2E_s \sigma_B{}^2 d^2 \lambda_1{}^2] \\ &\quad + \epsilon \cdot exp[-E_s d^2 \lambda_2 + 2E_s \sigma_T{}^2 d^2 \lambda_2{}^2] \\ &= (1-\epsilon) D_B^{d^2}(\lambda_1) + \epsilon D_T^{d^2}(\lambda_2) \quad (28) \end{aligned}$$

where

$$D_B(\lambda_1) = exp[-E_s \lambda_1 + 2E_s \sigma_B{}^2 \lambda_1{}^2] \quad (29)$$

and

$$D_T(\lambda_2) = exp[-E_s \lambda_2 + 2E_s \sigma_T{}^2 \lambda_2{}^2]. \quad (30)$$

Then we can obtain the optimum λ_1, λ_2, and the minimum bound D_B and D_T as

$$\begin{aligned} \lambda_1 &= \frac{1}{4\sigma_B{}^2}, \quad (31) \\ \lambda_2 &= \frac{1}{4\sigma_T{}^2}, \quad (32) \\ D_B &= exp[-\frac{E_s}{8\sigma_B{}^2}], \quad (33) \\ D_T &= exp[-\frac{E_s}{8\sigma_T{}^2}]. \quad (34) \end{aligned}$$

Substituting (29)-(32) into (28) gives

$$P(x \to \hat{x}) \leq (1-\epsilon) D_B^{d^2} + \epsilon D_T^{d^2}. \quad (35)$$

From the transfer funtion [5] the bit error probability is

$$P_b \leq (1-\epsilon)\cdot \frac{1}{n}\frac{\partial T(D,I)}{\partial I}\Big|_{D=D_B, I=1} + \epsilon \cdot \frac{1}{n}\frac{\partial T(D,I)}{\partial I}\Big|_{D=D_T, I=1} \quad (36)$$

where $T(D,I)$ is the transfer function of the TCM scheme. To get a tighter bound, we can rewrite (24) as

$$P(x \to \hat{x}) \leq \frac{1}{2}(1-\epsilon)\cdot erfc[\sqrt{\frac{E_s d^2}{8\sigma_B{}^2}}] + \frac{1}{2}\epsilon \cdot erfc[\sqrt{\frac{E_s d^2}{8\sigma_T{}^2}}] \quad (37)$$

Using the inequality $erfc(\sqrt{\frac{x+y}{2}}) \leq erfc(\sqrt{\frac{x}{2}}) exp[-\frac{y}{2}]$, we can get another pairwise error probability bound as

$$\begin{aligned} P(x \to \hat{x}) &\leq \frac{1}{2}(1-\epsilon)\cdot erfc[\sqrt{\frac{E_s d_f{}^2}{8\sigma_B{}^2}}] exp[\frac{E_s d_f{}^2}{8\sigma_B{}^2}] exp[-\frac{E_s d^2}{8\sigma_B{}^2}] \\ &\quad + \frac{1}{2}\epsilon \cdot erfc[\sqrt{\frac{E_s d_f{}^2}{8\sigma_T{}^2}}] exp[\frac{E_s d_f{}^2}{8\sigma_T{}^2}] exp[-\frac{E_s d^2}{8\sigma_T{}^2}] \quad (38) \end{aligned}$$

where d_f is the free distance.

Then, the bit error probability can be bounded as

$$\begin{aligned} P_b &\leq \frac{1}{2n}(1-\epsilon)\cdot erfc[\sqrt{\frac{E_s d_f{}^2}{8\sigma_B{}^2}}] exp[\frac{E_s d_f{}^2}{8\sigma_B{}^2}] \cdot \frac{\partial T(D,I)}{\partial I}\Big|_{D=D_B, I=1} \\ &\quad + \frac{1}{2n}\epsilon \cdot erfc[\sqrt{\frac{E_s d_f{}^2}{8\sigma_T{}^2}}] exp[\frac{E_s d_f{}^2}{8\sigma_T{}^2}] \cdot \frac{\partial T(D,I)}{\partial I}\Big|_{D=D_T, I=1}. \quad (39) \end{aligned}$$

Now, let us analyze (34) asymptotically. For small SNR, the bit error probability becomes large. If the bit error probability becomes very larger than ϵ, the first term in (34) will dominate since the second term cannot be larger than ϵ. For large SNR, the bit error probability becomes small. If the bit error probability becomes very smaller than ϵ, the second term in (34) will dominate. It should be noted that for $\epsilon = 0$, the impulsive noise become Gaussian noise and (34) and (37) become the same result to that shown in [3][5].

5 Numerical Results

Now we will consider the performance of the 2-state 1/2 rate 4-PSK trellis coded DS/SSMA system under the impulsive noise environment. The bit error probabilities of the 2-state 1/2 rate 4-PSK trellis coded DS/SSMA and uncoded BPSK DS/SSMA are shown in Fig. 3 and Fig. 4, respectively. In these figures, TFB1 means the bound (36) and TFB2 means the bound (39). We can see that the 2-state 1/2 rate 4-PSK trellis coded DS/SSMA has roughly 2dB gain over the uncoded BPSK DS/SSMA. We can also see that the bit error probability increases as M increases for fixed ϵ. The bit error probability of the system for various value of ϵ is shown in Fig. 5. For $\epsilon = 0$, the impulsive noise becomes Gaussian noise. Since SNR is a function of ϵ, we plot the bit error probability versus SBR when $M = 10$. We can also see that the bit error probability increases as ϵ increases for fixed M. We can see that the bit error probability is dominated by the first term of (36) for small SNR and by the second term for large SNR : the transition point is arround the SNR which makes the bit error probability ϵ.

6 Conclusion

We have studied the performance of 2^{n+1}-PSK trellis coded DS/SSMA system under impulsive noise environment. We analyze the system performance and obtain the bit error probability bound. From the result, we can see that SNR gain can be obtained by using TCM scheme in DS/SSMA system. For the 2 state 1/2 rate 4-PSK TCM scheme, roughly 2dB gain can be obtained. We also see that the system performance is degraded as ϵ increases or M increases. In this case, we need more SNR to achieve the same error probability in AWGN case.

References

[1] G. Ungerboeck, "Channel Coding with Multilevel Phase Signals," *IEEE Trans. Inform. Theory*, Vol. IT-28, pp. 55-67, Jan. 1982.

[2] G. D. Boudreau, D. D. Falconer, S. A. Mahmoud, "A Comparison of Trellis Coded Versus Convolutionally Coded Spread-Spectrum Multiple-Access Systems," *IEEE J. Select. Areas Comm.*, Vol. 8, pp. 628-640, May 1990.

[3] M. B. Pursley, "Performance Evaluation for Phase-Coded Spread Spectrum Multiple Access Communications - Part I: System analysis," *IEEE Trans. Comm.*, Vol. COM-25, pp. 795-799, Aug. 1977.

[4] E. Zehavi and J. K. Wolf, "On the Performance Evaluation of Trellis Codes," *IEEE Trans. Inform. Theory*, Vol. IT-33, No. 2, pp 196-202, Mar. 1987.

[5] E. Biglieri, D. Divsalar, P. J. McLane, and M. K. Simon, *Introduction to Trellis-Coded Modulation with Applications*. New York: Macmillan Publishing Company, 1991.

[6] M. Kavehrad and P. J. McLane, "Spread Spectrum for indoor digital radio," *IEEE Comm. Mag.*, Vol. 25, pp. 32-40, June 1987.

[7] J. H. Miller and J. B. Thomas, "The Detection of Signals in Impulsive Noise Modeled as a Mixture Process," *IEEE Trans. Comm.*, Vol. COM-24, pp. 559-563, May 1976.

[8] S. A. Kassam and H. V. Poor, "Robust Techniques for Signal Processing : A Survey," *IEEE Proc.*, Vol. 73, No. 3, pp. 433-481, Mar. 1985.

[9] S. A. Kassam and J. B. Thomas, "Asymptotically Robust Detection of a Signal in Contaminated Non-Gaussian noise," *IEEE Trans. Inform. Theory*, Vol. IT-22, pp. 22-26, Jan. 1976.

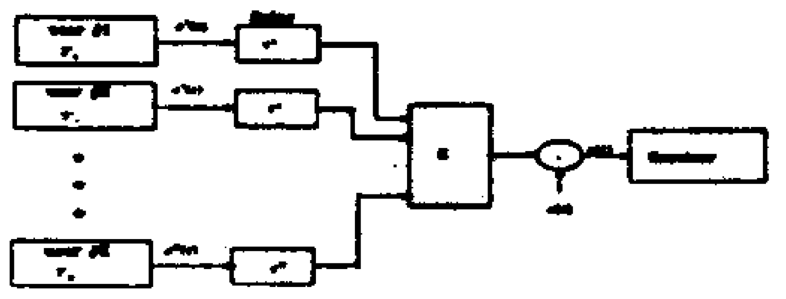

Figure 1: A block diagram of the general system architecture.

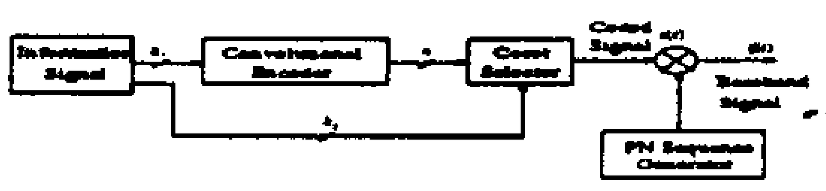

Figure 2: The DS/SSMA baseband system using TCM.

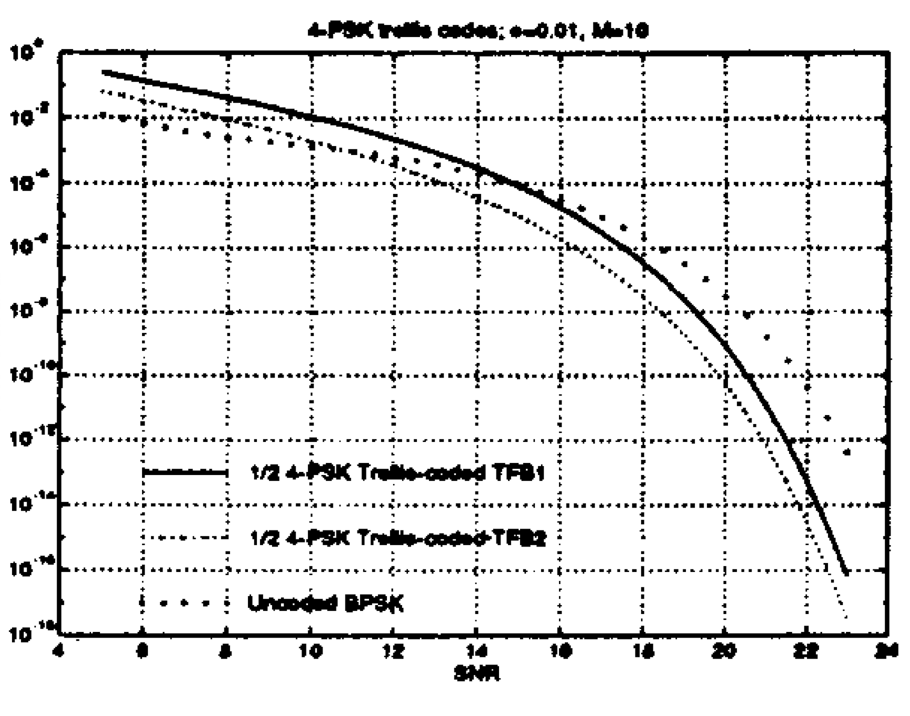

Figure 3: The bit error probability when $M = 10$ and $\epsilon = 0.01$.

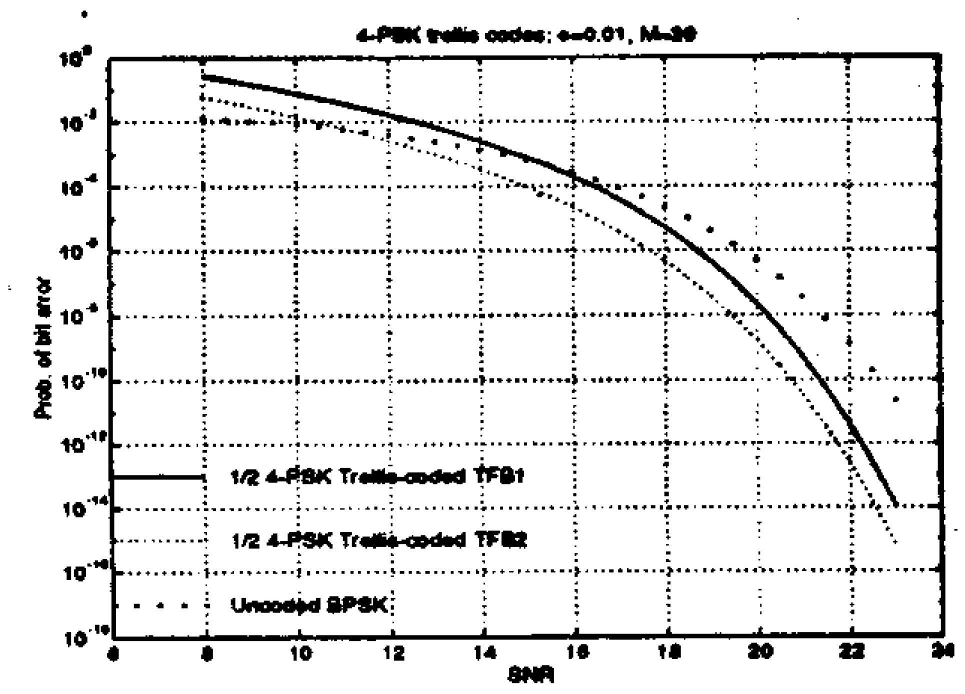

Figure 4: The bit error probability when $M = 20$ and $\epsilon = 0.01$.

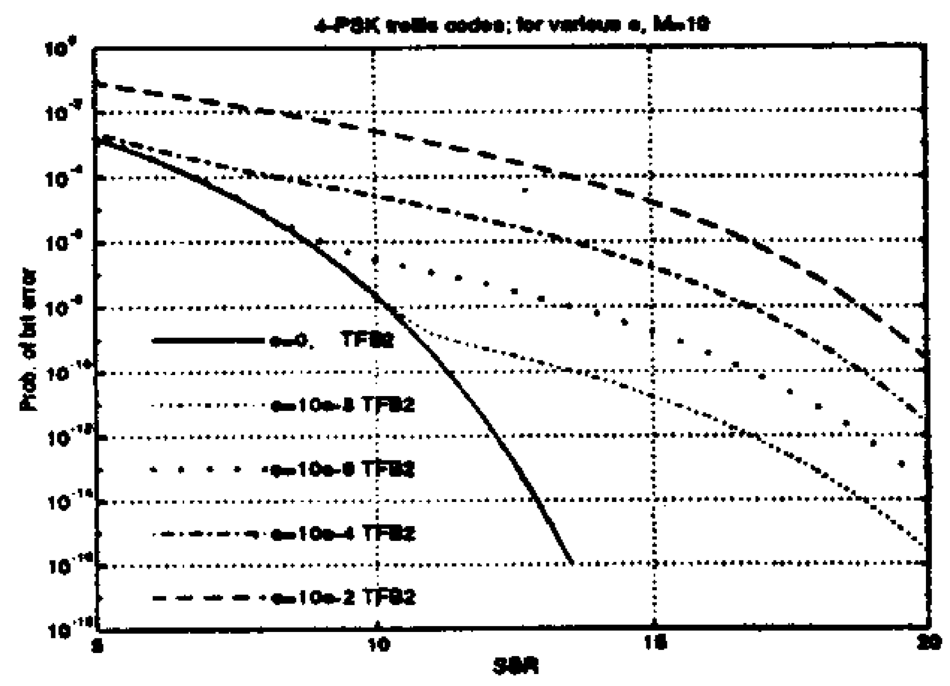

Figure 5: The bit error probability for various ϵ when $M = 10$.