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An Analysis of DS/SSMA Systems Using Trellis Coded Modulation in Impulsive Noise Environment

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Abstract The performance of DS/SSMA systems using TCM under impulsive noise environment is analyzed. We obtain the bound of the probability of bit error. It is shown that the bit error probability is dominated by background noise variance when SNR is small and by tail noise variance when SNR is large.

1 Introduction

The developments in communications technology these days have changed our daily life profoundly and rapidly. With mobile communication, we can overcome the restrictions on time and space, which were inevitable in traditional communication. As is evident, the demand for mobile communication is increased rapidly. Recently, code division multiple access (CDMA) becomes an interesting research area for its potential applications in mobile communication.

A lot of modulation and coding techniques have been developed to get better communication systems. Ungerboeck [1] has shown that optimally designed rate $n/(n+1)$ trellis codes mapped into the conventional $2^{n+1}$ point signal sets can provide some coding gain without bandwidth expansion. After his work, many researchers have studied various aspects of the trellis coded modulation (TCM). For example, the performance of DS/SSMA system using TCM has been studied in [2][3]. Most of the work on TCM has been considered under the additive white Gaussian noise (AWGN) assumption [1][3][4][5].

It is well-known that sometimes Gaussian noise assumption cannot be entirely justified. The non-Gaussian nature of atmospheric noise, impulses caused by turning-on some electrical devices, etc. are among the typical examples. Therefore, it is worthwhile to study the effects of impulsive noise on communication systems. In this paper, we will investigate the effect of impulsive noise on the DS/SSMA system using TCM. We will derive the probability of bit error for the $2^{n+1}$ PSK trellis coded DS/SSMA system under impulsive noise environment. We will then compare our result with that for the uncoded DS/SSMA system for various channel states.

2 System Model

The block diagram of the DS/SSMA system is illustrated in Fig. 1, which has been used frequently in previous studies of SSMA systems [3].

But the difference is that the noise $n(t)$ is impulsive noise. Each user transmits using different spreading code and the signal transmitted by the $k$th user $s^k(t)$ is assumed to be delayed randomly by $\tau^k$. The received signal is composed of the desired signal, interference due to the noise, and signal transmitted by other users.

The transmitter system model for trellis-coded DS/SSMA is illustrated in Fig. 2.

The information bits select a cost of the signal constellation through the $n/(n+1)$ convolutional code. The selected signal multiplied by DS sequence is modulated by the carrier and transmitted. The signal constellation we are concerned in this paper is antipodal $2^n$. PSK. The receiver model is the conventional I and Q type demodulators.

The notation in complex form in this paper is based on the work by Pursley [3], Ravenhard [6], and Boudreaux [2].

Let the $k$th user's complex baseband information signal be

$$x^k(t) = \sum_{p \in \mathbb{Z}} x^p_p(t - pT),$$

where $T$ is the symbol period, $P(t)$ is a rectangular pulse with duration $T$, and $x^p_p$ is the complex baseband symbol of the $k$th user during the $p$th symbol period which is determined by coset selection.

Similarly, the DS chip signal is defined as

$$a^k(t) = \sum_{m \in \mathbb{Z}} a_{m}^k \Psi(t - mT_c),$$

where $a_{m}^k$ is the $m$th chip of the $k$th user and $\Psi$ is the chip waveform with duration $T_c$. Then, the transmitted signal modulated by carrier of frequency $f_c$ is

$$s^k(t) = \sqrt{\frac{2E}{T}} \Re\{e^{j(\phi^k(t) + \phi^k)}\},$$

where $\phi^k(t) = a^k(t)\phi^k(t)$ and $\phi^k$ is the random phase of the $k$th carrier.

Then, the received signal can be written as

$$r(t) = \sqrt{\frac{2E}{T}} \Re\{\sum_{k=1}^{K} (s^k(t - r^k)exp(j\phi^k + \phi^k)) + n(t),$$

where $r^k$ is the random delay of the $k$th user at the receiver, $\phi^k = \phi^k - \omega_c r^k$ is the random phase of $k$th user at the receiver, and $n(t)$ is the impulsive noise.
After demodulation through the I and Q type demodulators, it can be shown that the sampled received sequence $Y_p^*$ can be written as [2]

$$Y_p^* = X_p^* + Z_p^* + \eta_p,$$  \hspace{1cm} (5)

where $X_p^* = \sqrt{E_x}^*$, $Z_p^*$ is the inter-user interference, and $\eta_p$ is the interference by impulsive noise. The work by Boudreau was accomplished under AWGN environment assumption: in this paper we consider the impulsive noise environment.

3 Impulsive Noise Environment

In our communication channel, source noise has impulsiveness. Nowadays, we are surrounded by many electrical devices: thus the impulsive noise environment is a proper noise model for some cases. We can model impulsive noise with the $c$-contaminated mixture noise model [7][8][9], for which the pdf is

$$f(x) = (1 - \epsilon) f_B(x) + \epsilon f_r(x).$$ \hspace{1cm} (6)

In (6), $f_B$ is a Gaussian pdf with mean zero and variance $\sigma_B^2$, and $f_r$ is in general a zero-mean pdf which has a variance $\sigma_r^2$ much larger than $\sigma_B^2$. In this paper, we assume that $f_r$ is a Gaussian pdf. The mean and variance of the impulsive noise are

$$E(\eta) = 0$$ \hspace{1cm} (7)

$$E(\eta^2) = (1 - \epsilon)\sigma_B^2 + \epsilon \sigma_r^2$$ \hspace{1cm} (8)

where $M = \frac{\epsilon}{1-\epsilon} \sigma_r^2$. Then we can define signal to noise ratio as

$$SNR = \frac{E_x}{2E(\eta^2)}$$ \hspace{1cm} (9)

The characteristics of the impulsive noise are different for different $\epsilon$ and $M$ which result the same $SNR$.

Now, we define the signal to background noise ratio ($SBR$) as

$$SBR = \frac{E_x}{2\sigma_B^2}$$ \hspace{1cm} (10)

As we can see, $SBR$ is independent of $M$ and $\epsilon$. In practical situations, $\epsilon$ is very small, i.e. $(M - 1)\epsilon \ll 1$. So, $SBR$ can be a good approximation of $SNR$, and is useful when we want to know some result for various $\epsilon$.

4 Performance Analysis

Let the complex coded symbol sequence $X$ as

$$X = (X_1, X_2, \cdots, X_n),$$ \hspace{1cm} (11)

the corresponding complex received sequence $Y$ as

$$Y = (Y_1, Y_2, \cdots, Y_n),$$ \hspace{1cm} (12)

and the complex inter-user interference sequence $Z$ as

$$Z = (Z_1, Z_2, \cdots, Z_n).$$ \hspace{1cm} (13)

Then the output signal at time $p$ is

$$Y_p = X_p + Z_p + \eta_p,$$ \hspace{1cm} (14)

where $\eta_p$ is a sample of complex impulsive noise.

Now, we assume that the inter-user interference $Z_p$ is the Gaussian random variable independent of $\eta_p$ by the central limit theorem. Consider the interference term

$$\eta_p = Z_p + \eta_p.$$ \hspace{1cm} (15)

In order to obtain the pairwise error probability, we use Chernoff bound. The pairwise error probability can be written as

$$P(z \rightarrow \delta) = \Pr\{\sum_{p \in p} (|Y_p - X_p|^2 - |Y_p - X_p|^2) \geq 0\}$$ \hspace{1cm} (16)

where $\nu$ is the set of $p$ such that $X_p \neq \lambda_p$. We can simplify (16) as

$$P(z \rightarrow \delta) = \Pr\{\sum_{p \in p} (-E_p||x_p - \bar{x}_p||^2 \geq 2\sqrt{E_r} R_e(\eta_p - \bar{x}_p)^2) \geq 0\}$$ \hspace{1cm} (17)

where $x_p = \frac{x_p}{\sqrt{E_r}}$. Define the $p$th distance $d_p$ as

$$d_p = d_p = \frac{x_p - \bar{x}_p}{\delta_p}$$ \hspace{1cm} (18)

and the impulsive interference term $\eta_p$ as

$$\eta_p = \eta_p + \eta_p$$ \hspace{1cm} (19)

where $d_p, \eta_p, \eta_p, \eta_p$ are real. Since $\eta_p$ and $\eta_p$ are the in-phase and quadrature components of $\eta_p$, $\eta_p$ and $\eta_p$ have the $c$-contaminated pdf with background variance $\sigma_B^2$ and tail variance $\sigma_r^2$ and independent of each other.

Now, let us define some new variable $d$ and $v$ as follows.

$$d^2 = \sum_{p \in p} d_p^2$$ \hspace{1cm} (20)

and

$$v = -\sum_{p \in p} d_p \eta_p + d_p \eta_p$$ \hspace{1cm} (21)

Then we can rewrite (17) as follows.

$$P(z \rightarrow \delta) \leq \Pr\{(-E_v d^2 + 2\sqrt{E_r} v \geq 0)\}.$$ \hspace{1cm} (22)

After some calculation, the pairwise error probability can be bounded as

$$P(z \rightarrow \delta) \leq \Pr\{(-E_v d^2 + 2\sqrt{E_r} v \geq 0)\}$$ \hspace{1cm} (23)

where $\theta$ is a random variable with pdf $f_{\theta}(\theta) = (1 - \epsilon)f_r(\theta) + \epsilon f_r(\theta)$, $f_B(\cdot)$ and $f_r(\cdot)$ are Gaussian pdf with mean zero and variance $\sigma_B^2$ and $\sigma_r^2$, respectively. Let $\bar{v} = -E_v d^2 + 2\sqrt{E_r} v$, then the pdf of $\bar{v}$ can be obtained as
where $d_f$ is the free distance.

Then, the bit error probability can be bounded as

$$ P_b \leq \frac{1}{2} \left( 1 - \epsilon \cdot \text{erf} \left( \frac{E_b}{2 \sigma_b^2} \right) \right) \cdot \frac{L(T,D,I)}{D_{D_b,D_f,l=1}} + \frac{1}{2} \epsilon \cdot \text{erf} \left( \frac{E_b}{2 \sigma_b^2} \right) \cdot \frac{L(T,D,I)}{D_{D_f,D_f,l=1}} \cdot \frac{1}{\sqrt{2}}. \tag{39} $$

Now, let us analyze (31) asymptotically. For small $SNR$, the bit error probability becomes large. If the bit error probability becomes very larger than $\epsilon$, the first term in (34) will dominate since the second term cannot be larger than $\epsilon$. For large $SNR$, the bit error probability becomes small. If the bit error probability becomes very smaller than $\epsilon$, the second term in (34) will dominate. It should be noted that for $\epsilon = 0$, the impulsive noise becomes Gaussian noise and (34) and (37) become the same result to that shown in [3],[5].

5 Numerical Results

Now we will consider the performance of the 2-state 1/2 rate 4-PSK trellis coded DS/SSMA system under the impulsive noise environment. The bit error probabilities of the 2-state 1/2 rate 4-PSK trellis coded DS/SSMA and uncoded BPSK DS/SSMA are shown in Fig. 3 and Fig. 4, respectively. In these figures, TPBI means the bound (30) and TPBS means the bound (39). We can see that the 2-state 1/2 rate 4-PSK trellis coded DS/SSMA has roughly 2dB gain over the uncoded BPSK DS/SSMA. We can also see that the bit error probability increases as $M$ increases for fixed $\epsilon$. The bit error probability of the system for various value of $\epsilon$ is shown in Fig. 5. For $\epsilon = 0$, the impulsive noise becomes Gaussian noise. Since SNR is a function of $\epsilon$, we plot the bit error probability versus SNR when $M = 10$. We can also see that the bit error probability increases as $\epsilon$ increases for fixed $M$. We can see that the bit error probability is dominated by the first term of (36) for small $SNR$ and by the second term for large $SNR$. The transition point is around the $SNR$ which makes the bit error probability $\epsilon$.

6 Conclusion

We have studied the performance of $2^{n+1}$-PSK trellis coded DS/SSMA system under impulsive noise environment. We analyze the system performance and obtain the bit error probability bound. From the result, we can see that SNR gain can be obtained by using TCM scheme in DS/SSMA system. For the 2 state 1/2 rate 4-PSK TCM scheme, roughly 2dB gain can be obtained. We also see that the system performance is degraded as $\epsilon$ increases or $M$ increases. In this case, we need more SNR to achieve the same error probability in AWGN case.

References


Figure 1: A block diagram of the general system architecture.

Figure 2: The DS/SSMA baseband system using TCM.

Figure 3: The bit error probability when $M = 10$ and $\epsilon = 0.01$.

Figure 4: The bit error probability when $M = 20$ and $\epsilon = 0.01$.

Figure 5: The bit error probability for various $\epsilon$ when $M = 10$. 

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